Fluid Mechanics
and Machinery

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Fundamentals of Fluid Mechanics

In this chapter, we discuss the properties that are encountered in the analysis of fluid flow. First we discuss the concept of a fluid and then classify the fluid through a rheological diagram. This is followed by a description of the properties such as density, specific volume, specific gravity, relative density, and thermodynamic properties. Then we treat the fluid as continuum and describe its viscosity property, which plays a dominant role in most aspects of fluid flow. Finally, other properties, such as vapor pressure, compressibility, capillarity, and surface tension are also considered.

1.1 INTRODUCTION

The science of mechanics of fluids based on the fundamental laws of motion (similar to those applied to mechanics of solids) is known as *fluid mechanics*. Thus, fluid mechanics is the study of fluids in motion or at rest and the subsequent effects of the fluids on the boundaries, which may be either solid surfaces or other fluids. In essence, fluid mechanics combines the rational equations of ideal fluid flow with empirical equations of real fluid flow and correlates the physical analysis with results from experiments. A great deal of theoretical treatment is available only in case of certain idealized situations, which may not be valid in real-life problems. Thus, recourse to experiments and numerical approaches is often found useful to deal with complex fluid flows. Traditionally, the engineering science of fluid mechanics has been developed through an understanding of fluid properties, the application of basic laws of mechanics and thermodynamics, and an orderly experimentation.

In this chapter, several fluid properties, such as density, viscosity, surface tension, and vapor pressure are described. Density and viscosity play major roles in open and closed channel flows and in the flow around immersed objects. The consideration of surface tension is important in the formation of droplets, in the flow of small jets, and in the formation of capillary waves. Vapor pressure accounts for changes from liquid to gas and is particularly important when reduced pressures are encountered.

1.2 CONCEPT OF A FLUID

A fluid is a substance that deforms continuously when subjected to a shear stress, no matter how small that shear stress may be. This property is important since it distinguishes a liquid from a solid, no matter how viscous the liquid may be. A shear force is the force component tangent to a surface, and this force divided by the area of the surface is the average shear stress over the area. Shear stress at a point is the limiting value of the shear force to area, as the area is reduced to the point or tends to zero.

A solid can resist a shear stress by a static deformation, whereas a fluid cannot. Under the action of shear stress on a solid, the amount of unit deformation will be proportional to the unit stress, and if the elastic limit is not exceeded, the solid returns to its original shape on removal of the stress. The molecules of a solid are more closely packed as compared to that of a fluid. Attractive forces between the molecules of a solid are much larger than those of a fluid. A solid body undergoes either a definite deformation or breaks completely when the shear stress is applied on it. The amount of deformation is proportional to the magnitude of the applied stress up to some limiting condition.

Any shear stress applied to a fluid, no matter how small, will result in motion of the fluid. The fluid moves and deforms continuously as long as the shear stress is applied. As a corollary, we can say that a fluid at rest must be in a state of zero shear stress, a state often called the hydrostatic stress condition in structural analysis. In this condition, Mohr's circle for stress reduces to a point and there is no shear stress on any plane cut through the element under the stress.

If a shear stress τ is applied at any location in a fluid, the element 011^1 (Fig. 1.1) that is initially at rest will move to 022^1 and to 033^1, etc. In other words, the tangential stress in a fluid body depends on the velocity of deformation and vanishes as this velocity approaches zero.

All liquids and gases are fluids, as they undergo deformation continuously when subjected to even the slightest shear force. We shall hereafter refer to liquids and gases only as fluids. A liquid has a definite volume and it takes the shape of

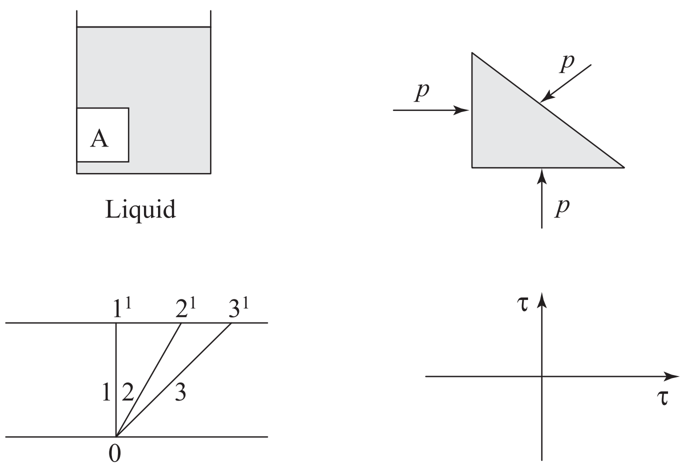

Fig. 1.1 Shear stress on a fluid body

the vessel containing it. It would occupy the vessel fully or partially depending on its content and it will have free surface. However, a gas has no definite shape, and it would expand and occupy the vessel fully and it cannot have a free surface. The volume of a liquid varies very slightly due to the change in temperature and pressure. This variation is so small that for all practical purposes it is often negligible, and hence, a liquid can be considered as incompressible. But a gas undergoes considerable change in volume due to changes in temperature and pressure, and hence, gas is a compressible fluid.

Rheology is a science of deformation and flow. Fluids may be classified as Newtonian and non-Newtonian. Figure 1.2 shows such a classification of fluids. In the case of solid, shear stress τ is proportional to the magnitude of the deformation, but in many fluids the shear stress is proportional to the time rate of angular deformation. For Newtonian fluids, the slope of the line is equal to the viscosity. Glycerin, air, water, kerosene, thin lubricating oil (under normal working conditions), etc., are some of the examples of Newtonian fluids. The ideal fluid, with no viscosity, is represented by the horizontal axis, whereas the true elastic solid is represented by the vertical axis. A plastic that sustains a certain amount of stress before suffering a plastic flow can be shown by a straight line intersecting the vertical axis at the yield stress. There are certain non-Newtonian fluids in which μ varies with the rate of deformation. Some examples of non-Newtonian fluids are human blood and thick lubricating oil. The viscous behavior of non-Newtonian fluid may be prescribed by the power law equation

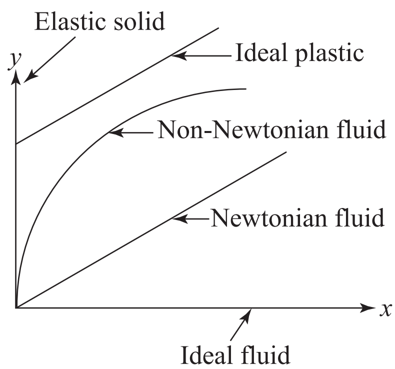

Fig. 1.2 Rheological diagram

$$\tau = k\left(\frac{du}{dy}\right)^n \qquad (1.1)$$

Here, n = flow behavior index and k = consistency index.

For Newtonian fluids, the consistency index k becomes dynamic viscosity μ and the flow behavior index n assumes a unity value.

Fluids such as milk, blood, clay, and liquid cement for which the flow behavior index $n < 1$, are called pseudoplastic. Fluids for which $n > 1$ are called dilatants. Concentrated solution of sugar and aqueous suspension of rice starch are examples of dilatants.

Example 1.1 Classify the substances that have the rates of deformation corresponding to shear stresses shown in Table 1.1.

Table 1.1

***du/dy* (rad/s)**	0	1	3	5
τ (kPa or kN/m^2)	15	20	30	40

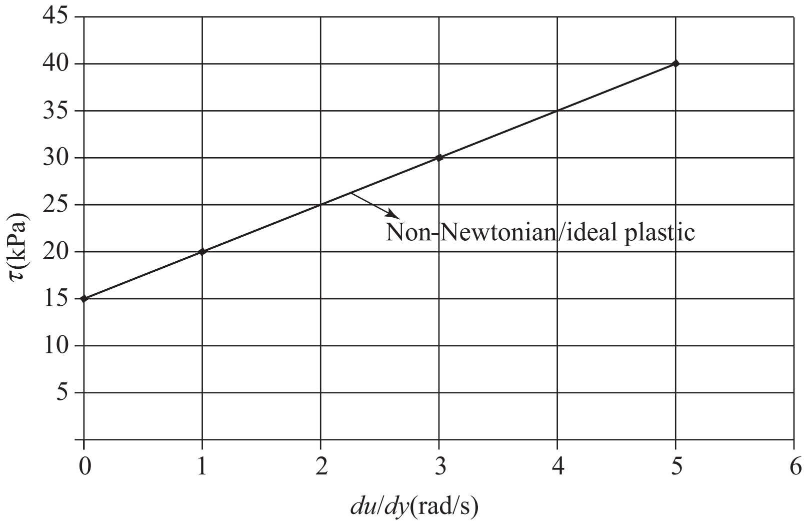

Fig. 1.3 Shear stress versus rate of deformation graph

Solution Figure 1.3 indicates the classification as non-Newtonian. Please note that at zero deformation, shear stress is not zero.

1.3 ENGINEERING SYSTEM OF UNITS

In this book, the International System of Units is utilized throughout. Four fundamental quantities of measurement (from which others can be derived) are length, mass, force, and time. In the International System, they are meter (m), kilogram (kg), newton (N), and second (s) (Table 1.2).

The two unit prefixes in the International System that are commonly encountered in fluid mechanics problems are kilo (k) and milli (m), which indicate factors of 1000 and 0.001, respectively. The prefixes used for SI units are detailed in Table 1.3.

Table 1.2 SI units of measurements

Quantities	International system
Length	Meter (m)
Mass	Kilogram (kg)
Force	Newton (N) (=kg-m/s^2)
Time	Second (s)
Weight	Newton (N)
Area	m^2
Volume	m^3
Velocity	m/s
Acceleration	m/s^2

1.4 PROPERTIES OF FLUID

Any characteristic of a system is called its property. Some familiar properties are pressure p, temperature T, volume V, and mass m. The list can be extended to include less familiar ones such as viscosity, thermal conductivity, modulus of elasticity, thermal expansion of coefficient, electric resistivity, and even velocity and elevation.

Two important parameters that tend to indicate the heaviness of substances are mass density and specific weight (unit weight). Mass density is typically used

Table 1.3 Prefixes for SI units

Factors by which unit is multiplied	Prefix	Symbol
10^{12}	Tera	T
10^{9}	Giga	G
10^{6}	Mega	M
10^{3}	Kilo	k
10^{2}	Hecto	h
10	Deka	da
10^{-1}	Deci	d
10^{-2}	Centi	c
10^{-3}	Milli	m
10^{-6}	Micro	μ
10^{-9}	Nano	n
10^{-12}	Pico	p
10^{-15}	Femto	f
10^{-18}	Atto	a

Table 1.4 Variation of mass density with respect to temperature

Temperature (°C)	Mass density (kg/m^3)
0	1000
10	1000
20	998
30	996
40	992
50	988
60	984
70	978
80	971
90	965
100	958

to characterize the mass of the fluid system, and specific weight is typically used to characterize the weight of the system.

The density of a substance, in general, depends on temperature and pressure. The density of most gases is proportional to pressure and inversely proportional to temperature.

Liquids and solids, on the other hand, are essentially incompressible substances and the variation of their density with pressure is usually negligible. For example, at 20°C the density of water changes from 998 kg/m^3 at 1 atm to 1003 kg/m^3 at 100 atm, a change of just 0.5%.

The density of liquids and solids depends more strongly on temperature than it does on pressure. For example, the density of water changes from 998 kg/m^3 at 20°C to 975 kg/m^3 at 75°C, a change of 2.3%. In view of this, the values of mass density reported in Table 1.4 are expected to be invariant with the alterations in pressure.

Mass density It is also known as specific mass of a liquid and may be defined as the mass per unit volume. It is usually denoted by ρ (rho).

$$\rho = \frac{m}{V} \text{ kg/m}^3 \tag{1.2}$$

Figure 1.4 shows a graphical representation of mass density variation with temperature. It can be seen that with an increase in temperature, mass density decreases.

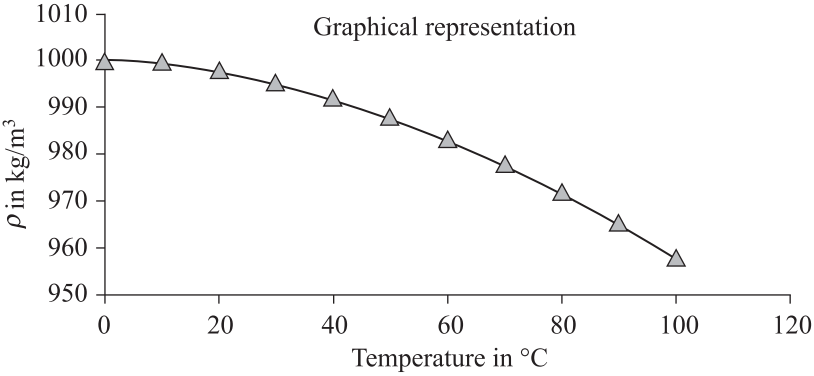

Fig. 1.4 Variation of mass density with respect to temperature

To compare the different fluids, Table 1.5 lists mass density at 20°C. It can be seen that mercury has a mass density that is 13.6 times that of water.

As the water temperature range of the data is considerable, one requires relationships for ρ as functions of temperature T. Streeter and Wylie (1979) have given the variation of ρ for water with T ranging from 0°C to 100°C in a tabular form. Using these data, the following best-fit equation is obtained in SI units.

$$\rho = 958.4 + 41.5\left\{\left(\frac{71}{100-T}\right)^5 + \left(\frac{415}{415+T}\right)^6\right\}^{-0.18} \qquad (1.2a)$$

The maximum percentage error in the use of Eqn (1.2a) is 1.0, which occurs in a very narrow band of temperature.

Weight density It (also known as specific weight or unit weight) is defined as the weight per unit volume. It is usually denoted by γ (gamma).

$$\gamma = \frac{W}{V}\ \text{N/m}^3 \qquad (1.3)$$

Figure 1.5 shows a graphical representation of specific weight with respect to temperature.

Note In engineering, we find use of specific weight as well as unit weight in lieu of each other.

Table 1.5 Approximate physical property (mass density) of some common liquids at 1 atmospheric pressure and at 20°C

Fluids	**ρ (kg/m³)**
Water	998
Sea water	1028
Mercury	13,570
Kerosene	819
Carbon tetrachloride	1588
Glycerin	1258
Gasoline	719
Benzene	879
Ammonia	829
Air	1.205

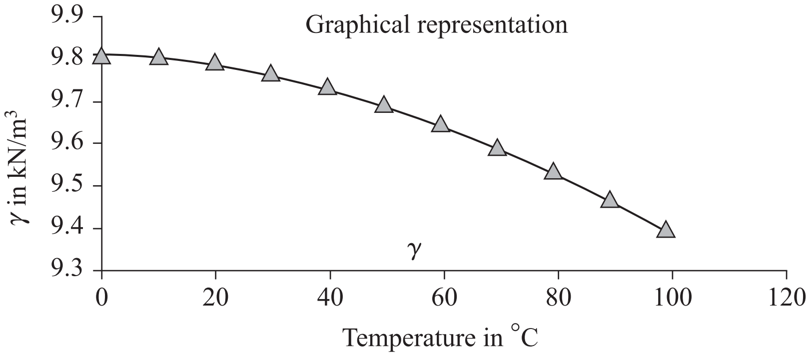

Fig. 1.5 Variation of specific weight with respect to temperature

The relationship between mass density, and specific weight is

$$\rho = \frac{\gamma}{g} \text{ or } \gamma = \rho g \tag{1.4}$$

Variations of specific weight are analogous to those of mass density, as described earlier. Table 1.6 shows variation of specific weight with respect to temperature and Table 1.7 shows specific weight at 20°C.

Table 1.6 Variation of specific weight with respect to temperature

Temperature (°C)	Specific weight (kN/m³)
0	9.81
10	9.81
20	9.79
30	9.77
40	9.73
50	9.69
60	9.65
70	9.59
80	9.53
90	9.47
100	9.40

Table 1.7 Approximate physical-property (specific weight) of some common liquids at 1 atmospheric pressure and at 20°C

Fluid	γ (kN/m³)
Water	9.81
Sea water	10.08
Mercury	133.1
Kerosene	8.03
Carbon tetrachloride	15.57
Glycerin	12.34
Gasoline	7.05
Benzene	8.62
Ammonia	8.13
Air	0.01182

Specific volume It is defined as the volume per unit mass of a fluid. It is usually denoted by V_s.

$$V_s = \frac{V}{m} = \frac{1}{\rho} \text{ m}^3/\text{kg} \tag{1.5}$$

Specific gravity It is a parameter that indicates how heavier is the given substance than water. It is defined as the ratio of the specific weight of the liquid to the specific weight of a standard fluid. It is denoted by *S*. For liquids, the standard fluid is pure water at 4°C. So,

$$S = \frac{\gamma_{\text{liquid}}}{\gamma_{\text{water}}} \text{ or } \frac{\rho_{\text{liquid}}}{\rho_{\text{water}}} \tag{1.6}$$

Specific gravity has no unit, i.e. it is a dimensionless quantity.

Though specific gravity is a dimensionless quantity, in SI units, the numerical value of the specific gravity of a substance is exactly equal to its density in g/cm^3 or kg/L (or 0.001 times density in kg/m^3). For example, density of water at 4°C is 1 g/cm^3 = 1 kg/L = 1000 kg/m^3. The specific gravity of mercury at 0°C is 13.6. Therefore, its density at 0°C is 13.6 g/cm^3 = 13.6 kg/L = 13,600 kg/m^3. Table 1.8 lists specific gravity at 20°C.

Relative density It is a dimensionless ratio of the densities of two materials. This term is similar to specific gravity except that the reference material is water. Mathematically, relative density is expressed as

$$G = \frac{\rho_{\text{object}}}{\rho_{\text{reference}}} \tag{1.7}$$

In Eqn (1.7), ρ is the density of the two materials in the same unit (e.g., kg/m^3, g/cm^3).

Relative density is a dimensionless term, since it is a ratio between two quantities of the same unit. When the reference material is not specified, it is usually understood to be water at 4°C.

It is to be noted that the relative density of an object relative to mercury is different from that with respect to water (specific gravity). The term specific gravity used in CGS and FPS units is the same as relative density. Relative densities for water and air are 1.00 and 1.204×10^{-3}, respectively.

Table 1.8 Approximate physical property (specific gravity) of some common liquids at 1 atmospheric pressure and at 20°C

Fluids	*S*
Water	1.0
Sea water	1.03
Mercury	13.6
Kerosene	0.82
Carbon tetrachloride	1.59
Glycerin	1.26
Gasoline	0.72
Benzene	0.88
Ammonia	0.83
Air	0.0013
Gold	19.2

Density of ideal gases (thermodynamic properties) Gases are highly compressible, and hence thermodynamic properties play an important role. With the change of pressure and temperature, the gases undergo large variation in their density. It is convenient to have some simple relations among the properties that are sufficiently general and accurate. Any equation that relates to the pressure, temperature, and density (or specific volume) of a substance is called an *equation of state*. The simplest and best-known equation of state for substances in the gas phase is the ideal gas equation of state and is expressed as

$$pV_s = RT \text{ or } p = \rho RT \tag{1.8}$$

or

$$\frac{p}{\rho} = RT \tag{1.9}$$

where

p = absolute pressure in N/m^2
V_s = specific volume in m^3/kg
T = absolute temperature in °K (temperature scale in the SI system is the Kelvin scale and the temperature unit on this scale is the kelvin) = 273° + t in °C
R = gas constant

The gas constant R is different for each gas, and is determined from $R = R_u/M$, where R_u is the universal gas constant (8.314 kJ/kmol K) and M is the molecular weight of the gas. The values of R and M for several substances are given in Table 1.9.

Then, mass density is given by

$$\rho = \frac{1}{V_s} = \frac{p}{RT} \text{ kg/m}^3 \tag{1.10}$$

and weight density is given by

$$\gamma = \rho g = \frac{gp}{RT} \text{ N/m}^3 \tag{1.11}$$

As already mentioned, the gas constant R depends on the particular gas. The dimension of R is obtained as follows:

We know the relationship

$$pV_s = RT$$

From that, we get

$$R = \frac{p}{\rho T}$$

In SI units p is expressed in N/m^2, ρ is expressed in kg/m^3, and T is expressed in K. Therefore,

$$R = \frac{\text{N/m}^2}{\text{kg/m}^3 \times \text{K}} = \frac{\text{N-m}}{\text{kg-K}} = \frac{\text{J}}{\text{kg-K}}$$

$$R \text{ in SI} = 287 \frac{\text{J}}{\text{kg-K}}$$

Table 1.9 Molecular weight and gas constant of some substances

Substance	Molecular weight (*M*)	Gas constant (*R*)
Air	28.97	0.2870
Ammonia	17.03	0.4882
Argon	39.95	0.2081
Bromine	159.81	0.05202
Isobutene	58.12	0.1430
n-butane	58.12	0.1430
Carbon dioxide	44.01	0.1889
Carbon monoxide	28.01	0.2968
Chlorine	70.905	0.1173
Ethane	30.070	0.2765
Ethylene	28.054	0.2964
Fluorine	38.0	0.2187
Helium	4.003	2.077
Hydrogen	2.016	4.124
Krypton	83.8	0.09921
Methane	16.04	0.5182
Neon	20.183	0.4119
Nitrogen	28.01	0.2968
Oxygen	32.0	0.2598
Propane	44.097	0.1885
Propylene	42.08	0.1976
Sulfur dioxide	64.06	0.1298
Tetra chloromethane	153.82	0.05405
Xenon	131.3	0.06332

For an ideal gas of volume $\forall$, mass m, and the number of moles $N = m/M$, the ideal gas equation of state can also be written as $p\forall = mRT$ or $p\forall = NR_uT$.

Another fundamental equation of a perfect gas between two state points is as given

$$\frac{p_1\forall_1}{T_1} = \frac{p_2\forall_2}{T_2} \tag{1.12}$$

Pressure Pressure or intensity of pressure is nothing but the compressive stress on a fluid and is given by

$$\text{Pressure,} \quad p = \frac{\text{Force } F}{\text{Area } A} \text{ (for uniform pressure)} \tag{1.13}$$

$$= \frac{dF}{dA} \text{ (for variable pressure)} \tag{1.14}$$

The unit of pressure is N/m^2 = Pa; Pa stands for pascal. Other commonly used units are kPa (kilopascal) = 1000 N/m^2 and bar = 100 kPa = 10^5 N/m^2.

Sometimes, the pressure is expressed in terms of the height h of an equivalent column of fluid of density ρ. Thus,

$$p = \rho g h = \gamma h \tag{1.15}$$

and h (meters of fluid) = p/γ. In such cases, h is called the pressure head.

Note For more details, the readers are advised to refer to Chapter 2.

Example 1.2 Calculate the specific weight, specific mass, specific volume, and specific gravity of a liquid having a volume of 6 m^3 and weight of 44 kN.

Solution Given: Volume of liquid = 6m^3, weight of liquid = 44 kN

$$\gamma = \frac{44}{6} = 7.33 \text{ kN/m}^3 \quad (Ans)$$

$$\rho = \frac{\gamma}{g} = \frac{7.33}{9.81} \times 1000 = 747.19 \text{ kg/m}^3 \quad (Ans)$$

$$V_s = \frac{1}{\rho} = \frac{1}{747.19} = 0.00134 \text{ m}^3/\text{kg} \quad (Ans)$$

$$S = \frac{7.33}{9.81} = 0.747 \quad (Ans)$$

Example 1.3 A volume of 2.5 m^3 of certain liquid weighs 9.81 kN. Determine the specific weight, mass density, and specific gravity of the liquid.

Solution Given: Volume of liquid = 2.5 m^3, weight of liquid = 9.81 kN
Therefore,

$$\gamma = \frac{W}{V} = \frac{9.81}{2.5} = 3.924 \text{ kN/m}^3 \quad (Ans)$$

$$\rho = \frac{m}{V} = \frac{1000}{2.5} = 400 \text{ kg/m}^3 \quad (Ans)$$

$$S = \frac{3924}{9810} = 0.4 \quad (Ans)$$

Example 1.4 Determine the mass density, specific weight, and specific volume of a liquid whose specific gravity is 0.85.

Solution Given: $S = 0.85$; $$S = 0.85 = \frac{\gamma_{\text{liquid}}}{\gamma_{\text{water}}} = \frac{\gamma_{\text{liquid}}}{9.81}$$

Therefore, $$\gamma_{\text{liquid}} = 8.3385 \text{ kN/m}^3 \quad (Ans)$$

But, $$\rho = \frac{\gamma}{g} = \frac{8338.5}{9.810} = 850 \text{ kg/m}^3 \quad (Ans)$$

Specific volume, $$V_s = \frac{1}{\rho} = 0.00117 \text{ m}^3/\text{kg} \quad (Ans)$$

Example 1.5 A mass of liquid weighs 500 N, corresponding to $g = 9.81$ m/s^2. Find (a) its mass and (b) its weight in a planet with the acceleration due to gravity 3.2 m/s^2 and 20.0 m/s^2.

Solution Let W = weight of liquid and m = mass of the same liquid

(a) $W = mg$ or $500 = m \times 9.81$

Therefore, $m = 50.96$ kg (*Ans*)

(b) Mass of the fluid remains constant, regardless of its location. Hence, $m = 50.96$ kg at all locations.

If $g_1 = 3.2$ m/s^2, $W_1 = mg_1 = 50.96 \times 3.2 = 163.072$ N (*Ans*)
If $g_2 = 20.0$ m/s^2, $W_2 = 50.96 \times 20.0 = 1019.2$ N (*Ans*)

Example 1.6 The variation in the density of water ρ with temperature T in the range $20°\text{C} \le T \le 50°\text{C}$ is given in Table 1.10.

Table 1.10

ρ (kg/m^3)	998.2	997.1	995.7	994.1	992.2	990.2	988.1
Temperature (°C)	20	25	30	35	40	45	50

Use these data to determine an empirical equation of the form,

$$\rho = A + BT + CT^2$$

This can be used to predict the density over the range indicated.

Solution Refer to Fig. 1.6.

$$\rho = 1001 - 0.053T - 0.004T^2 \quad (Ans)$$

Example 1.7 A gas weighs 20 N/m^3 at 30°C and at an absolute pressure of 35×10^4 N/m^2. Determine the gas constant and density of the gas.

Solution Given: weight density $\gamma = 20$ N/m^3
temperature $t = 30$°C

Therefore, $T = 273 + 30 = 283$°K

and pressure, $p = 35 \times 10^4$ N/m^2

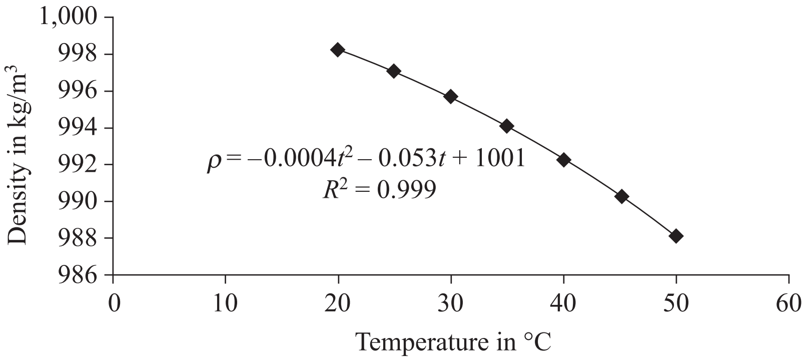

Fig. 1.6 Variation of mass density with respect to temperature

But $\rho = \frac{\gamma}{g} = \frac{20}{9.81} = 2.0387 \text{ kg/m}^3$ (*Ans*)

From the relation $pV_s = RT$, we get

$$R = \frac{p}{\rho T} = \frac{35 \times 10^4}{2.0387 \times 283} = 606.64 \ \frac{\text{J}}{\text{kg-K}} \quad (Ans)$$

1.5 FLUID AS A CONTINUUM

Fluid flows may be modeled either on a macroscopic level or on a microscopic level. The macroscopic model regards the fluid as a continuum and the description is in terms of variations of the macroscopic velocity, density, pressure, and temperature with distance and time. On the other hand, the microscopic or molecular model recognizes the particulate structure of a fluid as a myriad of discrete molecules, and ideally, provides information on the position and velocity of every molecule at all times.

All fluids are composed of molecules in constant motion. However, in most of the engineering applications, we are interested in the average or the mean or the macroscopic effects of many molecules. It is these macroscopic effects that we can perceive and measure. We, thus, treat a fluid as an infinitely divisible substance, a continuum, [continuum means that the distance between fluid particles (or molecules) or the mean free path is small (i.e., small compared to any physical dimensions of the problem)] and do not concern ourselves with the behavior of any individual molecules. The concept of a continuum forms the basis of classical fluid mechanics. The continuum assumption is valid in treating the behavior of fluids under normal conditions.

As a consequence of the continuum assumption, each fluid property is assumed to have a definite value at each point in space. Thus, fluid properties such as density, temperature, velocity, etc. are considered to be continuous functions of position and time. To illustrate the concept of a property at a point, consider the manner in which we determine the density at a point. A region of fluid is shown in Fig. 1.7(a). We are interested in determining the density at the point c, whose coordinates are x_0, y_0, and z_0. The density is defined as mass per unit volume. Thus, the mean or average density within the volume v would be given by $\rho = \frac{m}{v}$.

In general, this will not be equal to the value of density at c. To determine the density at c, we must select a small volume δv, surrounding point c, and then determine the ratio $\delta m/\delta v$. To answer the question, 'How small can we make the volume δv?', let us take the ratio $\delta m/\delta v$ [Fig. 1.7(b)]. Then allow the volume to shrink continuously in size, assuming that the volume δv is relatively large initially. The average density tends to approach an asymptotic value as the volume is shrunk to enclose only homogeneous fluid in the immediate neighborhood of point c. When δv becomes further small that it contains only few number of molecules, it becomes impossible to fix a definite value for $\delta m/\delta v$; then the value will vary erratically as molecules cross into and out of the volume.

Thus, there is a lower limiting value of δv, designated as $\delta v'$ shown in Fig. 1.7(b), which is allowable for use in defining fluid density at a point. The density at a point is defined as

$$\rho = \lim_{\delta v \to \delta v'} \frac{\delta m}{\delta v} \tag{1.15}$$

Since the point c was arbitrary, the density at any point in the fluid could be determined in a similar manner. If density determination were made simultaneously at an infinite number of point in the fluid, we would obtain an expression for the density distribution as a function of the space coordinates $\rho = \rho(x, y, z)$, at the given instant of time. Thus, the complete representation of density is given by $\rho = \rho(x, y, z, t)$. Since the density is a scalar quantity, the field representation is a scalar, representing only a magnitude.

Example 1.8 The mean free path λ of the molecules in air is approximately given by

$$\lambda = 3.8 \times 10^{-5} \frac{T}{p}$$

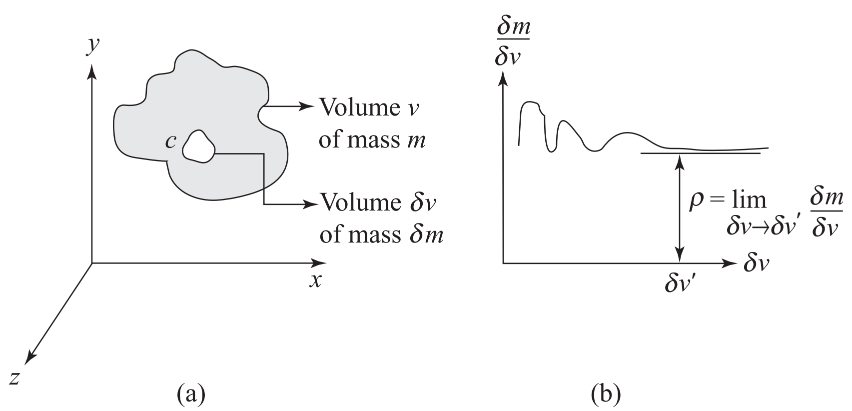

Fig. 1.7 Definition of density at a point

Table 1.11 Atmospheric temperature, pressure, and density values at different altitude

Altitude (m)	Temperature (°C)	Pressure (N/m²)	Density (kg/m³)
1800	3.0	81,000	1.025
4600	−14.7	57,000	0.77
8500	−40.0	33,000	0.493
14,000	−54.0	15,000	0.237
21,000	−54.0	4500	0.0715
31,000	−54.0	1000	0.0171
46,000	45.0	144	0.0016
61,000	71.0	32	0.00032
76,000	−22.0	5.5	0.000077

where T is the temperature in K and p is the pressure in N/m^2. The atmospheric temperature, pressure, and density at different altitudes are given in Table 1.11.

Calculate the mean free path at each altitude. If the flow of air through a 40-mm diameter pipe were to be considered, then state above what altitude the continuum approach will fail (i.e., the mean free path will be of the order of one-hundredth of the pipe diameter).

Solution Use the relation $\lambda = 3.8 \times 10^{-5}\dfrac{T}{p}$.

Table 1.12 shows computed values of mean free path.

The continuum hypothesis, therefore, holds up to about 61 km (*Ans*)

Note This example is only for illustrative purpose. The value may be sensitive to the relationship used for mean free path.

Table 1.12 Computed values of mean free path

Altitude (m)	λ (m)
1800	1.3×10^{-7}
14,000	5.6×10^{-7}
46,000	8.4×10^{-5}
61,000	4.1×10^{-4}
76,000	1.74×10^{-3}

1.6 VISCOSITY

Viscosity is the most important among all properties, without which the diverse field of fluid mechanics of today might not have come into existence. Viscosity is derived from the word *viscous*, which means sticky, adhesive, or tenacious. We say coconut oil is thin and castor oil is thick; when spilled over inclined surface the so-called thin oil flows down faster compared to the thick oil. Obviously, the terms *thin* and *thick* do not refer to the density of the liquid but to the easiness with which it flows. Similar to solids, fluids also offer resistance to shearing forces/stresses. It is primarily due to cohesion (attraction between similar molecules) and the molecular momentum exchange between fluid layers and as the flow occurs, these effects appear as shearing stresses between the moving layers of a fluid. Hence, viscosity is a property of a fluid that determines the amount of this resistance to shearing stresses.

Viscosity may be defined in different ways. For example, it is a measure of the internal fluid friction that causes resistance to flow or as a property of a fluid that offers resistance to the movement of one layer of fluid over another adjacent layer of the fluid or as a property of a fluid that determines its resistance to shearing stresses.

Newton's law of viscosity states that for a given rate of angular deformation of fluid, the shear stress is directly proportional to the viscosity. The shear stress or shear resistance per unit area to a moving fluid is proportional to the velocity gradient in a direction normal to the area under consideration, in the same way as the stress in elastic solid is related to the strain component.

$$\tau = \frac{F}{A} = \alpha \frac{\partial u}{\partial y} \tag{1.16}$$

or

$$\tau = \mu \frac{\partial u}{\partial y} \tag{1.17}$$

where μ(mu), the constant of proportionality in Eqn (1.17), is called dynamic viscosity or absolute viscosity of the fluid. The relationship given by Eqn (1.17) is called newton's law of viscosity. Any fluid that obeys this law is called Newtonian fluid. The velocity gradient $\frac{\partial u}{\partial y}$ may be visualized as the rate at which one layer moves relative to an adjacent layer. Depending on the sign of velocity gradient, the direction of action of shear force changes. If the shear force acts in the direction of velocity, it is considered positive. It is evident from Eqn (1.17) that $\tau = 0$ when $\frac{\partial u}{\partial y} = 0$. Hence, there would not be any shear force in uniform flow or at the symmetry of a flow. The velocity gradient cannot be infinite as it is not physically possible to have an infinite value for the shear stress. Hence, the value of velocity gradient should change continuously without any jump throughout the flow region including the boundary.

An ideal fluid has no viscosity. There is no fluid that can be classified as a perfectly ideal fluid. However, the fluids with very little viscosity are sometimes considered as 'ideal fluids'. In general, the viscosity of a fluid depends on both the temperature and pressure (Fig. 1.8), although the dependence on pressure is rather weak. For liquids, both dynamic and kinematic viscosities are practically independent of pressure and any small variation with pressure is usually ignored, except at extremely high pressures.

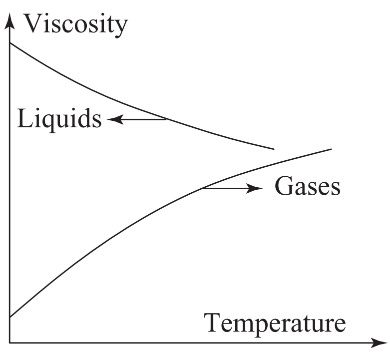

Fig. 1.8 The viscosity of liquids decreases and the viscosity of gases increases with temperature

Viscosity of liquids varies inversely with temperature (because in liquids the shear stress due to intermolecular cohesion decreases with the increase in temperature), while viscosity of gases varies directly with temperature. (In gases, the intermolecular cohesion is negligible and the shear stress is due to the exchange of momentum of the molecules, normal to the direction of motion. The molecular activity increases with temperature and hence the shear stress and also the viscosity of gases will increase with the increase in temperature.)

The kinetic theory of gases predicts the viscosity of gases to be proportional to the square root of the temperature, i.e.,

$$\mu_{\text{gas}} \propto \sqrt{T}$$

This prediction is confirmed from the practical observations; however, some gases need correction factors because of some deviations.

The relation between viscosity and temperature for liquids and gases are as follows.

Sutherland correlation (from the US standard atmosphere)

1. For gases

$$\mu = \frac{\alpha T^{1/2}}{1+\dfrac{\beta}{T}} \tag{1.18}$$

where T is the absolute temperature, and α and β are experimentally determined constants. For air, values of these two constants are

$\alpha = 1.458 \times 10^{-6}$ kg/(ms K$^{1/2}$) and $\beta = 110.4$ K at atmospheric conditions

2. For liquids, the viscosity is approximated as

$$\mu = \alpha 10^{\beta/T-\gamma} \tag{1.19}$$

where again T is absolute temperature and α, β, and γ are experimentally determined constants. For water, $\alpha = 2.414 \times 10^{-5}$ Ns/m^2, $\beta = 247.8$ K, and $\gamma = 140$ K, results in less than 2.5% error in viscosity in the temperature range of 0°C to 370°C (Touloukian et al., 1975)

Other relations are given below.

1. For liquids

$$\mu = \mu_0 \left(\frac{1}{1+\xi t+\zeta t^2} \right) \text{poise} \tag{1.20}$$

where μ = viscosity of liquid at t°C in poise, μ_0 = viscosity of liquid at 0°C in poise, and ξ and ζ are constants for the liquid.

For water, $\mu_0 = 1.79 \times 10^{-3}$ poise, $\xi = 0.03368$, $\zeta = 0.000221$

Once again from the above equation, one can infer that with the increase of temperature the viscosity of liquids decreases.

2. For gases

$$\mu = \mu_0 + \xi t - \zeta t^2 \text{ poise} \tag{1.21}$$

where for air $\psi_0 = 0.000017$, $\xi = 0.000000056$, and $\zeta = 0.1189 \times 10^{-9}$

Once again, from Eqn (1.21), one can infer that with the increase in temperature the viscosity of gases increases.

3. Helmholtz suggested the following expression for water in CGS units:

$$\mu = \frac{0.01776}{1 + 0.033368T + 0.000221T^2} \text{ poise} \tag{1.22}$$

where T is the temperature in °C.

To quantify viscosity for mathematical manipulation, consider a fluid confined between two parallel plates as shown in Fig. 1.9. The upper plate is moving at a velocity u, and the distance between the plates is denoted by y. The layer of fluid in contact with the upper (moving) plate will move with the same velocity as the plate (i.e., u), whereas the layer in contact with the lower (fixed) plate will have a zero velocity.

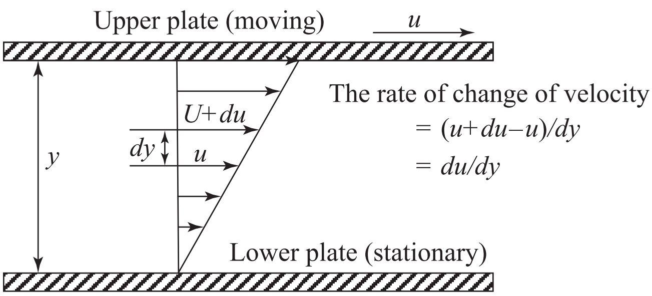

Fig. 1.9

If a linear velocity gradient is assumed, as indicated in Fig. 1.9, and if the shearing stress in the fluid is assumed to be proportional to the rate of change of velocity (newton's law of viscosity), the shearing stress (Fig. 1.10) may be expressed as follows:

$$\tau = \mu \frac{U}{y} \tag{1.23}$$

Dynamic and kinematic viscosity

The proportionality factor for the viscous fluid, as given in Eqn (1.23), is called dynamic or absolute viscosity. Therefore,

$$\mu = \frac{\tau}{\frac{du}{dy}} \frac{\frac{\text{N}}{\text{m}^2}}{\frac{\text{m}}{\text{s}} \times \frac{1}{\text{m}}} = \frac{\text{Ns}}{\text{m}^2} \text{ (ps)} \tag{1.24}$$

Note μ for water $= 1.75 \times 10^{-3}$ Ns/m²

The unit of viscosity in CGS is called poise [one poise = (1/10) Ns/m²].

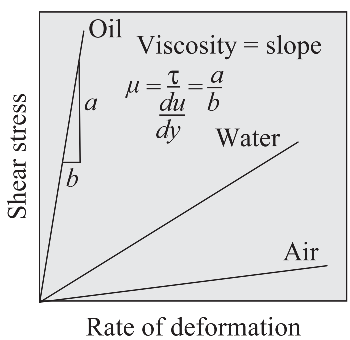

Fig. 1.10 The rate of deformation of a Newtonian fluid is proportional to the shear stress

Kinematic viscosity It is defined as the ratio between the dynamic viscosity and density of fluid.

$$\nu = \frac{\mu}{\rho} \ \frac{\text{m}^2}{s} \tag{1.25a}$$

Note ν for water: 1.75×10^{-6} m^2/s

The unit of kinematic viscosity in CGS is called stoke.

Thus, 1 stoke = 10^{-4} m^2/s

As the water temperature range of the data is considerable, one requires relationship for kinematic viscosity as a function of temperature T. Streeter and Wylie (1979) have given an equation through which one can determine the kinematic viscosity.

$$\nu = 1.792\times10^{-6}\left[1+\left(\frac{T}{25}\right)^{1.165}\right]^{-1} \tag{1.25b}$$

Here ν is in m^2/s and T = water temperature in °C. The maximum percentage of error in using Eqn (1.25b) is 2.2%.

1.6.1 No-Slip Condition of Viscous Fluids

When a viscous fluid flows over a solid surface, the fluid elements adjacent to the surface attain the velocity of the surface. In other words, the relative velocity between the solid surface and adjacent fluid particles is zero. This phenomenon has been established through experimental observations and is known as the *no-slip* conditions. Thus, the fluid elements in contact with a stationary surface have zero velocity. This behavior of no-slip at the solid surface should not be confused with wetting of surfaces by the fluids. For example, mercury flowing in glass tube will not wet the surface, but will have zero velocity at the wall of the tube. The wetting results due to surface tension, whereas, the no-slip condition is a consequence of fluid viscosity. Table 1.13 shows the variation of dynamic viscosity and kinematic

Table 1.13 Variation of dynamic viscosity and kinematic viscosity with respect to temperature

Temperature (°C)	Dynamic viscosity (Ns/m^2)	Kinematic viscosity (m^2/s)
0	1.75×10^{-3}	1.75×10^{-6}
10	1.30×10^{-3}	1.30×10^{-6}
20	1.02×10^{-3}	1.02×10^{-6}
30	8.0×10^{-4}	8.03×10^{-7}
40	6.51×10^{-4}	6.56×10^{-7}
50	5.41×10^{-4}	5.48×10^{-7}
60	4.6×10^{-4}	4.67×10^{-7}
70	4.02×10^{-4}	4.11×10^{-7}
80	3.5×10^{-4}	3.6×10^{-7}
90	3.11×10^{-4}	3.22×10^{-7}
100	2.82×10^{-4}	2.94×10^{-7}

Table 1.14 Values of dynamic viscosity and kinematic viscosity for some common fluids at 20°C and 1 atmospheric pressure

Fluids	Dynamic viscosity (Ns/m^2)	Kinematic viscosity (m^2/s)
	Liquids	
Water	1.00×10^{-3}	1.00×10^{-6}
Sea water	1.07×10^{-3}	1.04×10^{-6}
Gasoline	2.92×10^{-4}	4.29×10^{-7}
Kerosene	1.92×10^{-3}	2.39×10^{-4}
Glycerin	1.49×10^{-3}	1.18×10^{-3}
Mercury	1.56×10^{-3}	1.15×10^{-7}
Castor oil	9.80×10^{-1}	1.02×10^{-3}
	Gases	
Air	1.80×10^{5}	1.494×10^{5}
Carbon dioxide	1.48×10^{5}	0.804×10^{5}
Hydrogen	0.90×10^{5}	10.714×10^{5}
Nitrogen	1.76×10^{5}	1.517×10^{5}
Methane	1.34×10^{5}	2.00×10^{5}
Oxygen	2.00×10^{5}	1.504×10^{5}
Water vapor	1.01×10^{5}	1.352×10^{5}

viscosity with respect to temperature and Table 1.14 gives the values for some common fluids at 20°C.

Specific viscosity It is the ratio of viscosity of fluid to the viscosity of water at 20°C.

Example 1.9 From a table of the properties of liquids it was found that at 20°C carbon tetrachloride had a dynamic viscosity of 9.67×10^{-4} Pas and a kinematic viscosity of 6.08×10^{-7} m^2/s. Calculate its specific gravity and weight density.

Solution Given: $\mu = 9.67 \times 10^{-4}\ \text{Ns/m}^2, \quad \nu = 6.08 \times 10^{-7}\ \text{m}^2/\text{s}$

$$\rho = \frac{\mu}{\nu} = 1590.46\ \text{kg/m}^3$$

$$\gamma = \rho \times g = 1590.46 \times 9.81 = 15.602\ \text{kN/m}^3 \quad (\textit{Ans})$$

$$S = \frac{\gamma_{\text{liquid}}}{\gamma_{\text{water}}} = 1.596 \quad (\textit{Ans})$$

Example 1.10 A volume of 3.2 m^3 of certain oil weighs 27.5 kN. Calculate its mass density, weight density, specific volume, and specific gravity. If kinematic viscosity of the oil is 7×10^{-3} stokes, what would be its dynamic viscosity in centipoises?

Solution Given $V = 3.2\ m^3$, $W = 27.5$ kN, and $\nu = 7\times10^{-3}$ stokes

Note 1 stoke = $10^{-4}\ m^2/s$

$$\gamma = \frac{W}{V} = 8.59\ \text{kN/m}^3 \quad (Ans)$$

$$\rho = \frac{\gamma}{g} = 876.01\ \text{kg/m}^3 \quad (Ans)$$

$$V_s = \frac{1}{\rho} = 1.14\times10^{-3}\ \text{m}^3/\text{kg} \quad (Ans)$$

$$S = \frac{\gamma_{\text{oil}}}{\gamma_{\text{water}}} = 0.87 \quad (Ans)$$

$$\mu = \nu \times \rho = 6.132\,\text{centipoise} \quad (Ans)$$

Example 1.11 Glycerin has a density of 1260 kg/m^3 and a kinematic viscosity of 0.00183 m^2/s. What shear stress is required to deform this fluid at a strain rate of 10^4/s?

Solution Given

$$\rho = 1260\ \text{kg/m}^3$$

$$\nu = 0.00183\ \text{m}^2/\text{s}$$

$$\frac{du}{dy} = 10^4\,\text{s}^{-1}$$

Therefore, $$\tau = \nu\rho\frac{du}{dy} = 1260\times0.00183\times10^4 = 23.058\ \text{kPa} \quad (Ans)$$

Example 1.12 A liquid has a specific gravity of 1.9 and a kinematic viscosity of 6 stokes. What is its dynamic viscosity?

Solution Given $S = 1.9$, kinematic viscosity = 6 stokes = $6 \times 10^{-4}\ m^2/s$

$$\rho_{\text{liquid}} = S\times\rho_{\text{water}} = 1.9\times1000 = 1900\ \text{kg/m}^3$$

But, $$\nu = \frac{\mu}{\rho}$$

Therefore, $$\mu = 1900\times6\times10^{-4} = 1.14\,\text{Ns/m}^2 \quad (Ans)$$

Example 1.13 The velocity distribution of flow over a plate is parabolic, with vertex 0.3 m from the plate (Fig. 1.11), where the velocity is 1.8 m/s. If the viscosity of the fluid is 0.9 N s/m^2, find the velocity gradients and shear stresses at distances 0 m, 0.15 m, and 0.3 m from the plate.

Solution The equation for velocity profile is given by

$$u = ly^2 + my + n \tag{1.26}$$

where l, m, and n are constants.

Applying the boundary conditions to Eqn (1.26), we get

At $y = 0$; $u = 0$; therefore, Eqn (1.26) becomes, $0 = 0 + 0 + n$ or $n = 0$

At $y = 0.3$ m, $du/dy = 0$; therefore, $du/dy = 2\,ly+m$

$$0 = 2 \times 1 \times 0.3 + m \qquad (1.27)$$

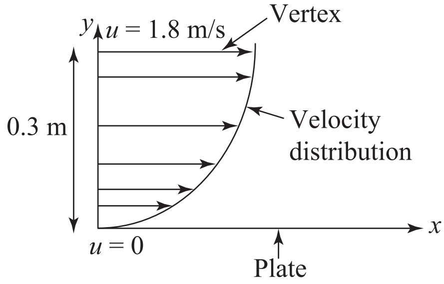

Fig. 1.11 Velocity profile for Example 1.13

At $y = 0.3$ m, $u = 1.8$ m/s; therefore,

$$1.8 = 1 \times 0.3^2 + m \qquad (1.28)$$

Solving Eqns (1.27) and (1.28), we get

$$l = -20,\ m = 12$$

Therefore, the velocity profile will be

$$u = -20y^2 + 12y$$

and

$$\frac{du}{dy} = -40y + 12$$

To find the velocity gradient,

$$\frac{du}{dy}\Big|_{y=0} = 12\ \text{s}^{-1} \quad (Ans)$$

$$\frac{du}{dy}\Big|_{y=0.15} = 6\ \text{s}^{-1} \quad (Ans)$$

$$\frac{du}{dy}\Big|_{y=0.3} = 0\ \text{s}^{-1} \quad (Ans)$$

To find the shearing stress,

$$\tau\big|_{y=0} = \mu \frac{du}{dy}\Big|_{y=0}$$

$$\tau\big|_{y=0} = 0.9 \times 12 = 10.8\ \text{N/m}^2 \quad (Ans)$$

$$\tau\big|_{y=0.15} = 0.9 \times 6 = 5.4\ \text{N/m}^2 \quad (Ans)$$

$$\tau\big|_{y=0.3} = 0.9 \times 0 = 0 \quad (Ans)$$

Example 1.14 The velocity (v) at radius r in a pipe of radius r_0 is given in terms of center line velocity (v_c) for laminar flow as

$$\frac{v}{v_c} = 1 - \left[\frac{r}{r_0}\right]^2$$

If the centerline velocity in a pipe of 1 m diameter is 6 m/s, and the velocity is 0.002 Ns/m^2, draw the velocity and shearing stress profile (Figures 1.12 and 1.13) for a cross section.

Solution Given $$\frac{v}{v_c} = 1 - \left\{\frac{r}{r_0}\right\}^2$$

or $$v = 6\left[1 - \left\{\frac{r}{0.5}\right\}^2\right]$$

Therefore, $$v = 6 - 24r^2, \text{which gives velocity profile.}$$

$$\frac{dv}{dr} = -24 \times 2r = -48r$$

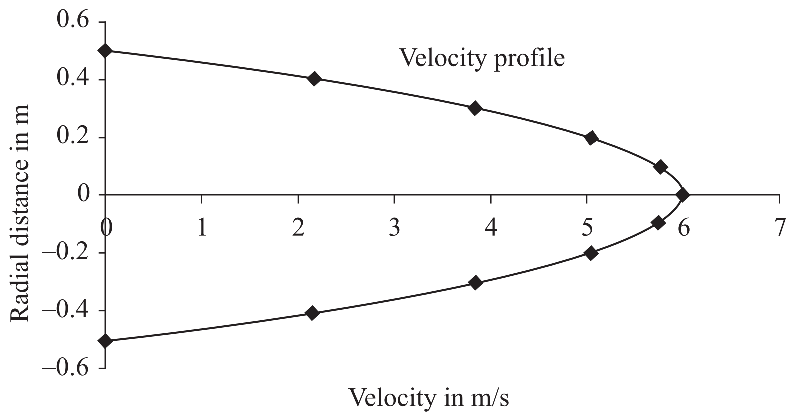

Fig. 1.12 Velocity profile for Example 1.14

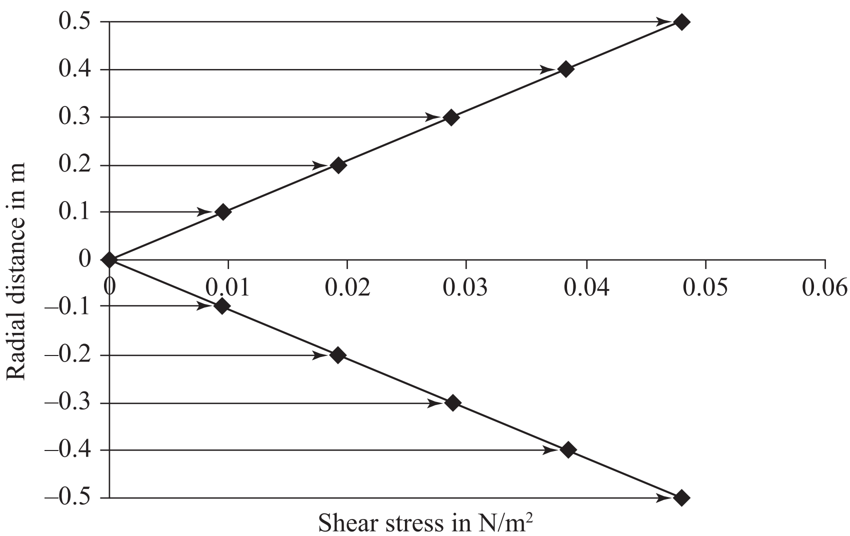

Fig. 1.13 Shear stress distribution for Example 1.14

$$\tau = -\mu \frac{dv}{dr} \quad \text{because } \frac{du}{dy} = -\frac{dv}{dr} = -0.002(-48r)$$
$$= 0.096r, \quad \text{which gives shearing stress profile.}$$

Table 1.15 gives computed values for velocity profile and shear stress profile.

Table 1.15 For Example 1.14

R	*v*	*t*	**Remarks**
0	6	0	Centerline velocity
0.1	5.76	0.0096	
0.2	5.04	0.0192	
0.3	3.84	0.0283	
0.4	2.16	0.0384	
0.5	0	0.048	Boundary

Example 1.15 A fluid of absolute viscosity 8 poise flows past a flat plate and has a velocity 1 m/s at the vertex, which is at 0.2 m from the plate surface. Make calculations for the velocity gradients and shear stress at points 0.05, 0.1, and 0.15 m from the boundary. Assume (a) a straight-line velocity distribution and (b) a parabolic distribution.

Solution (a) For a straight-line velocity distribution, the velocity gradient (Table 1.16 and Fig. 1.14) at the boundary, that is, at $y = 0$, is

$$\frac{du}{dy} = \frac{100-0}{20-0} = 5 \text{ s}^{-1}$$

$$\tau = \mu \frac{du}{dy} = 0.8 \times 5 = 4 \text{ N/m}^2$$

Table 1.16 Velocity gradient and shear stress values

Location	**Velocity gradient**	**Shear stress**
Y = 0	10	8
Y = 0.05	7.5	6
Y = 0.1	5.0	4
Y = 0.15	2.5	2

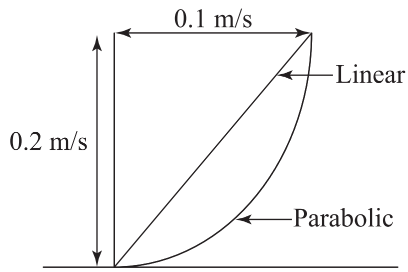

Fig. 1.14 Velocity distribution for Example 1.15

(b) The parabolic velocity distribution can be prescribed by the relation

$$u = ly^2 + my + n$$

and
$$\frac{du}{dy} = 2ly + m$$

Applying boundary conditions, we get

$$u = 0 \text{ at } y = 0, n = 0; \quad u = 1 \text{ m/s at } y = 0.2 \text{ m}; \quad \frac{du}{dy} = 0 \text{ at } y = 0.02 \text{ m}$$

Now, we get $l = -0.25$ and $m = 10$

Therefore, $u = -0.25y^2 + 10y$ and $\dfrac{du}{dy} = -0.5y + 10$

Example 1.16 Air at 20°C forms a boundary layer near a solid wall of sine wave-shaped velocity profile [$v = v_{max} \sin(\pi y/2\delta)$]. The boundary layer thickness is 6 mm and the peak velocity is 10 m/s. Compute the shear stress in the boundary layer at y equal to (a) 0, (b) 3 mm, and (c) 6 mm. Consider the dynamic viscosity of air as 1.81×10^{-5}.

Solution Shear stress is given by $\tau = \mu \dfrac{dv}{dy}$

Given $$v = v_{max} \sin\left[\frac{\pi y}{2\delta}\right]$$

so that $$\frac{dv}{dy} = \left[\frac{\pi v_{max}}{2\delta}\right]\cos\left[\frac{\pi y}{2\delta}\right]$$
$$= 2618\cos(261.8y)$$

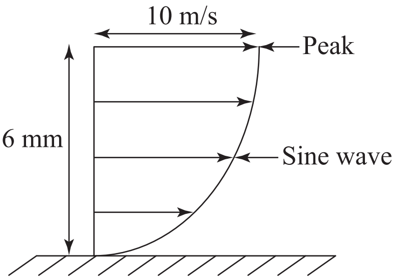

Fig. 1.15 Velocity distribution for Example 1.16

At $y = 0$, $\tau = 0.04739\cos\{261.8(0)\} = 0.0474$ N/m^2 (*Ans*)

At $y = 3$ mm, $\tau = 0.0335$ N/m^2 (*Ans*)

At $y = 6$ mm, $\tau = 0$ (*Ans*)

Example 1.17 A large plate moves with a speed U over a stationary plate on a layer of oil as shown in Fig. 1.15. If the velocity profile is that of a parabola ($u^2 = ay$), with the oil at the plates having the same velocity as the plates, what is the stress on the moving plate from the oil? If a linear profile is assumed, what is the shear stress on the upper plate?

Solution For a parabolic profile, $u^2 = ay$, where $y = d$, $u = U$.

Thus, $$U^2 = ad$$

Therefore, $$a = \frac{U^2}{d}$$

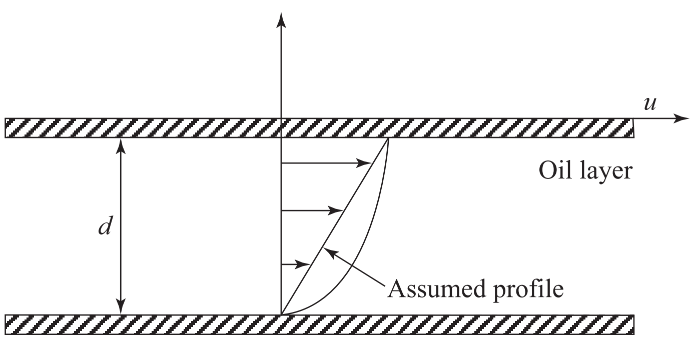

Fig. 1.16 Velocity distribution for Example 1.17

Therefore, $$u^2 = \frac{U^2}{d} \times y = U^2\left[\frac{y}{d}\right]$$

or $$u = U\sqrt{\frac{y}{d}}$$

$$\frac{du}{dy} = \left[U\left\{\frac{1}{\sqrt{d}}\right\}\left\{\frac{1}{2}y^{-1/2}\right\}\right]$$

$$\tau = \mu\frac{du}{dy} = \mu\left[U\left\{\frac{1}{\sqrt{d}}\right\}\left\{\frac{1}{2}y^{-1/2}\right\}\right]$$

For $y = d$, $$\tau = \frac{\mu U}{2d} \quad (Ans)$$

For a linear profile, $$\frac{du}{dy} = \frac{U}{d}$$

Therefore, $$\tau = \mu\frac{U}{d} \quad (Ans)$$

Example 1.18 Water is moving through a pipe. The velocity profile at some section is shown in Fig. 1.17 and is given mathematically as

$$u = \frac{\beta}{4\mu}\left[\frac{d^2}{4} - r^2\right]$$

where u = velocity of water at any position r, β = a constant, μ = viscosity of water, d = pipe diameter, and r = radial distance from centerline. What is the shear stress at the wall of the pipe due to the water? What is the shear stress at a position $r = d/4$? If the given profile persists for a distance L along the pipe, what drag is induced on the pipe by the water in the direction of flow over this distance?

Solution Given, velocity profile as $u = \left[\frac{\beta}{4\mu}\right]\left[\frac{d^2}{4} - r^2\right]$

So, $$\frac{du}{dr} = \left[\frac{\beta}{4\mu}\right](-2r) = \frac{-2\beta r}{4\mu}$$

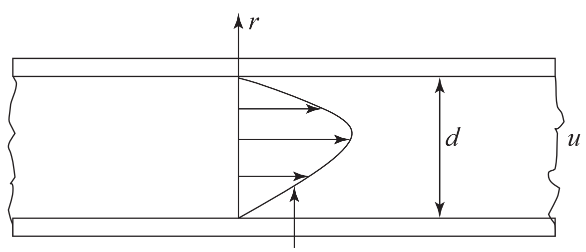

Fig. 1.17 Velocity distribution for Example 1.18

Shear stress is given by $\tau = \mu \dfrac{du}{dr} = \dfrac{-2\beta r}{4}$

At the wall $r = d/2$.

Hence $$\tau_{\text{wall}} = \frac{-2\beta\left(\frac{d}{2}\right)}{4} = -\frac{\beta d}{4} \quad (Ans)$$

At $r = d/4$, $$\tau_{r=d/4} = -\frac{\beta d}{8} \quad (Ans)$$

$$\text{Drag} = (\tau_{\text{wall}})(\text{area}) = (\beta d/4)(\pi dL) = (\beta d^2 \pi L)/4 \quad (Ans)$$

Example 1.19 A plate weighing 150 N and measuring 0.8 m × 0.8 m slides down an inclined plane over an oil film of 1.2 mm thickness. For an inclination of 30° and a velocity of 0.2 m/s, compute viscosity of the fluid.

Solution We have from newton's law of viscosity $\tau = \mu \dfrac{du}{dy}$

But $\tau = \dfrac{\text{force}}{\text{area}} = 150 \sin \dfrac{30^\circ}{0.8 \times 0.8} = 117.19\ \text{N/m}^2$

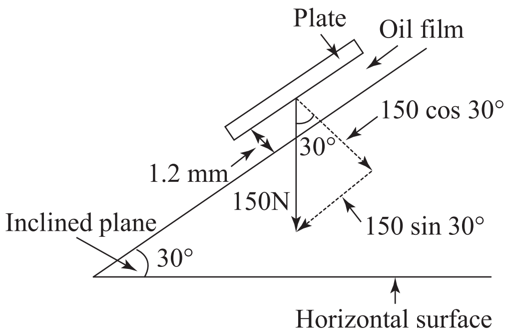

Fig. 1.18

Rate of deformation, $du/dy = (0.2 - 0)/0.12 = 166.67/\text{s}^1$

Therefore, $$\mu = \frac{\tau}{\frac{du}{dy}} = 0.7\ \text{Ns/m}^2 \quad (Ans)$$

Example 1.20 The space between two parallel plates 5 mm apart is filled with crude oil (Fig. 1.19). A force of 2 N is required to drag the upper plate at a constant velocity of 0.8 m/s. The lower plate is stationary. The area of the upper plate is 0.09 m^2. Determine the dynamic viscosity and kinematic viscosity of the oil, if the specific gravity of the oil is 0.9.

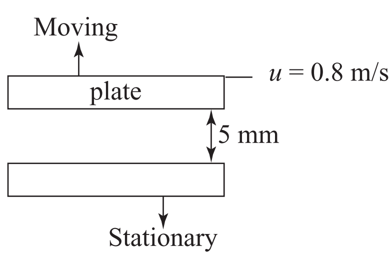

Fig. 1.19

Solution We have from newton's law of viscosity $\tau = \mu \dfrac{du}{dy}$

$$F = \tau A = \left\{\mu \frac{du}{dy}\right\} A$$

$$2 = \mu \times \frac{0.8}{5 \times 10^{-3}} \times 0.09$$

$$\mu = 0.139 \text{ Ns/m}^2 \quad (Ans)$$

$$\nu = \frac{\mu}{\rho} = \frac{0.139}{900} = 1.54 \times 10^{-4} \text{ m}^2\text{/s} \quad (Ans)$$

Example 1.21 A flat plate weighing 0.45 kN has an area of 0.1 m². It slides down an inclined plane at 30° to the horizontal (Fig. 1.20) at a constant speed of 3 m/s. If the inclined plane is lubricated with an oil of viscosity 0.1 Ns/m², find the thickness of the film.

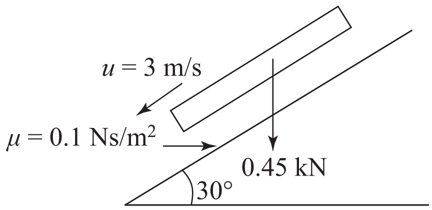

Fig. 1.20

Solution We know from newton's law of viscosity,

$$\tau = \mu \frac{du}{dy}$$

Here, out of 0.45 kN (450 N), only tangential component (450 × sin 30°) is shearing force. Therefore, shear stress will be equal to 450 × sin 30°/0.1.

$$\frac{450 \times \sin 30°}{0.1} = \left(0.1 \times \frac{3}{dy}\right)$$

Therefore,

$$dy = 0.133 \text{ mm} \quad (Ans)$$

Example 1.22 A block weighing 1 kN and having dimensions 200 mm on an edge is allowed to slide down and incline on a film of oil having a thickness of 0.005 mm, as shown in Fig. 1.21. If we use a linear velocity profile in the oil, what is the terminal speed of the block? The viscosity of the oil is 7×10^{-3} Ns/m².

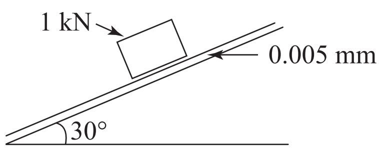

Fig. 1.21

Solution From newton's law of viscosity, we have

$$\tau = \mu \frac{du}{dy} = 7 \times 10^{-3} \left\{\frac{\nu_T}{\frac{0.005}{1000}}\right\} = 1400\nu_T$$

$$F_f = \tau A = \{1400\nu_T\} \left\{\frac{200}{1000}\right\}^2 = 56.0\nu_T$$

At the terminal condition, equilibrium occurs.

Hence, $1000 \times \sin 30° = 56.0\, v_T$

or $v_T = 8.93$ m/s (*Ans*)

Example 1.23 As shown in Fig. 1.21, if the fluid is glycerin at 20°C and the width between the plates is 6 mm, what shear stress is required to move the upper plate at 2.5 m/s? What is the Reynolds number, if d is taken to be the distance between the plates (Fig. 1.22)?

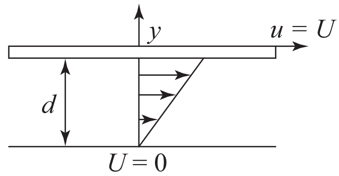

Fig. 1.22

Solution We know from newton's law of viscosity

$$\tau = \mu \frac{du}{dy}$$

At $y = d$,
$$\tau = \mu \frac{du}{d(d)} = 1.49 \frac{2.5}{\frac{6}{1000}} = 621 \text{ N/m}^2 \quad (Ans)$$

$$N_R = \frac{\rho du}{\mu} = 1258 \times \frac{6}{1000} \times \frac{2.5}{1.49} = 12.7 \quad (Ans)$$

Example 1.24 A circular disc of diameter D is rotated in a liquid of viscosity μ at a small distance Δh from a fixed surface (Fig. 1.23). Derive an expression for the torque T, necessary to maintain an angular velocity ω. Neglect the centrifugal effect.

Solution The velocity of the bottom of the disc is a function of the radius and so is the rate of deformation and shear stress.

Shear stress at any radius r is given by

$$\tau = \mu \frac{du}{dy} = \mu \frac{\omega r}{\Delta h}$$

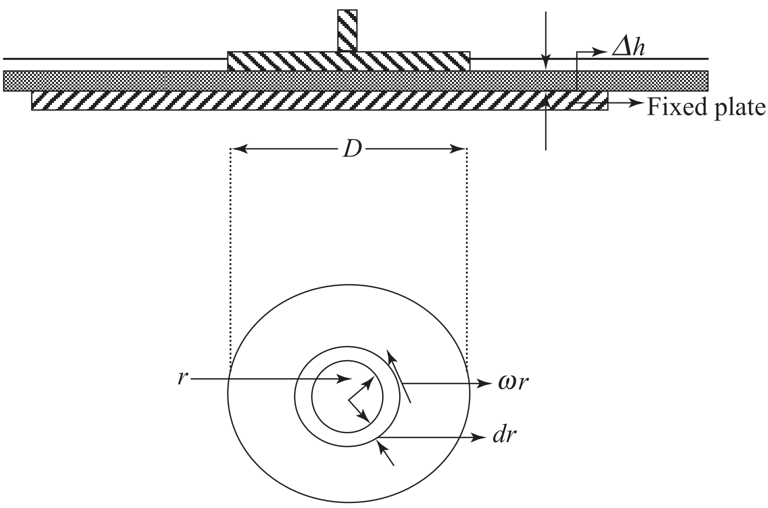

Fig. 1.23

Consider a ring of radius r and width dr. The shear forces in the ring is given by

$$\Delta F = \tau \times 2\pi r \times dr = \mu \frac{\omega r}{\Delta h} \times 2\pi r \times dr = \frac{2\pi\mu\omega}{\Delta h} \times r^2 dr$$

The differential torque, $\Delta T = \Delta F \times r = \dfrac{2\pi\mu\omega}{\Delta h} \times r^2 dr \times r = \dfrac{2\pi\mu\omega}{\Delta h} \times r^3 dr$

Integrating, we get the total torque.

$$T = \int_{r=0}^{r=\theta/2} \frac{2\pi\mu\omega}{\Delta h} \times r^3 dr = \frac{\pi\mu\omega D^4}{32\Delta h} \text{ Nm} \quad (Ans)$$

Example 1.25 A solid cone of radius r_0 and vertex angle 2θ is to rotate at an angular velocity ω (Fig. 1.24). An oil of viscosity μ and thickness h fills the gap between the cone and the housing. Determine the torque T to rotate the cone.

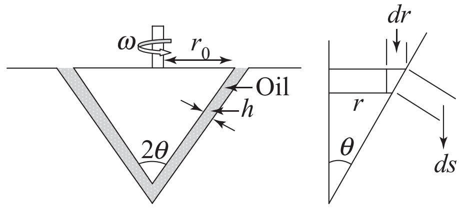

Fig. 1.24

Solution Shear stress on the inclined wall, $\tau = \mu \times \dfrac{du}{dy} = \mu \times \dfrac{V}{h} = \mu \dfrac{\omega r}{h}$

Considering an elemental area, $2\pi r \times \dfrac{dr}{\sin\theta} = dA$

Differential torque, $dT = rdF = r(\tau 2\pi r)\dfrac{dr}{\sin\theta}$

$$= r\left[\mu \frac{\omega r}{h} \times 2\pi r \times \frac{dr}{\sin\theta}\right] = \mu \frac{2\pi\omega}{h} \frac{1}{\sin\theta} r^3 dr$$

Torque T is given by $T = \displaystyle\int_0^{r_0} dT = \frac{2\pi\omega\mu}{h\sin\theta} \times \int_0^{r_0} r^3 dr$

$$T = \frac{\pi\omega\mu}{2h\sin\theta} r_0^4 \quad (Ans)$$

Example 1.26 Inside a 60-mm diameter cylinder a piston of 59 mm diameter rotates concentrically. Both the cylinder and piston are 80 mm long (Fig. 1.25). If the

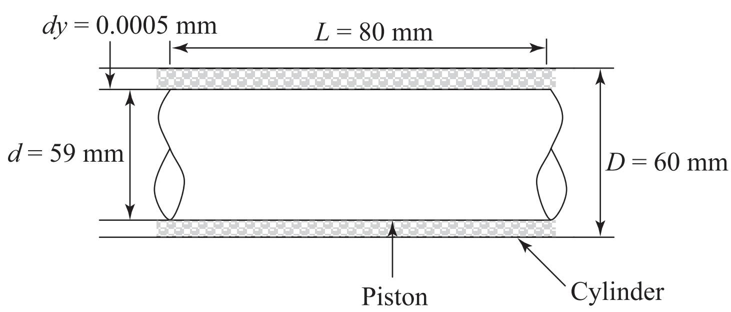

Fig. 1.25 Longitudinal section of piston and cylinder

space between the cylinder and piston is filled with oil of viscosity of 0.3 Ns/m^2 and a torque of 1.5 Nm is applied, find the rpm of the piston and the power required.

Solution Given: D = 60 mm, d = 59 mm, L = 80 mm, dy = 0.0005 m, and torque T = 1.5 Nm

We know that

Torque = shear force × d/2

or 1.5 = F × 0.059/2

Therefore, F = 50.85 N

But $$F = \tau \times \text{area} = \tau \times \pi d L$$

whereas $$\tau = \mu \frac{du}{dy}$$

Therefore, $$\tau = 0.3 \times \frac{u}{0.0005}$$

or $$F = \left[0.3 \times \frac{u}{0.0005}\right] \times \pi \times 0.059 \times 0.08 \times \frac{0.059}{2}$$

or $$u = 5.72 \text{ m/s}$$

But $$u = \frac{\pi d N}{60}$$

From this, we get N = 1849.5 rpm

But

$$p = T \times \frac{2\pi \text{N}}{60} = 1.5 \times 2 \times \pi \times \frac{1849.5}{60} = 290.5 \text{ W}$$

Example 1.27 A cylinder of 0.12 m radius rotates concentrically inside a fixed hollow cylinder of 0.13 m radius. Both the cylinders are 0.3 m long. Determine the viscosity of the fluid that fills the space between the cylinders if a torque of 0.88 Nm is required to maintain an angular velocity of 2π rad/s.

Solution The torque applied = the resisting torque by the fluid
= shear stress × surface area × torque

Hence, at any radial location r from the axis of rotation

$$0.88 = \tau \times (2\pi r \times 0.3) r$$

or $$\tau = \frac{0.467}{r^2}$$

We have $$\tau = \mu \frac{dv}{dy}$$

Therefore, $$\frac{dv}{dy} = \frac{\tau}{\mu} = \frac{0.467}{\mu r^2}$$

Rearranging the above expression and substituting ($-dr$) in place of dy (the minus sign indicates that r, the radial distance, decreases as v increases), we obtain

$$\int_{v_{\text{outer}}}^{v_{\text{inner}}} dv \frac{0.467}{\mu}\int_{0.13}^{0.12} -\frac{dr}{r^2}$$

Hence,

$$(v_{\text{inner}} - v_{\text{outer}}) = \frac{0.467}{\mu}\left\{\frac{1}{r}\right\}_{0.13}^{0.12}$$

But $v_{\text{inner}} = 0.754$ m/s $(2 \times \pi \times 0.12)$

$v_{\text{outer}} = 0$ m/s (fixed)

Therefore, substituting the above values, we get

$$(0.754 - 0) = \frac{0.467}{\mu}\left[\frac{1}{0.12} - \frac{1}{0.13}\right]$$

or $\mu = 0.397\,\text{Pas}$ (*Ans*)

Example 1.28 A dash pot 12 cm in diameter and 15 cm long slides vertically down into an annulus of 12.05 cm diameter cylinder (Fig. 1.26). The oil that fills the annular space has a viscosity of 1 poise. Find the speed with which the piston slides down if load of the piston is 15 N.

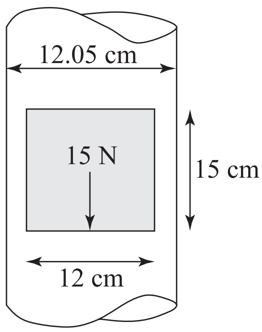

Fig. 1.26

Solution Since the space between the dash pot and the cylinder is very small, that is, the oil film is thin, we can assume that $du/dy = u/t$, where u is the velocity of piston and t is the oil film thickness.

Shear stress, $\tau = \mu \frac{du}{dy} = \mu\left\{\frac{u}{t}\right\}$

Shear or viscous force = $\tau \times \text{area} = \mu \frac{u}{t}(2\pi r l)$

Here, $r = 6$ cm $= 0.06$ m, $\mu = 0.1$ Ns/m^2, $t = 0.00025$ m, viscous force =15 N.

Therefore, $u = 0.663$ m/s (*Ans*)

Example 1.29 A 1.5 cm wide gap between two vertical plane surfaces is filled with an oil of specific gravity 0.9 and dynamic viscosity 2.0 Ns/m^2. A metal plate 1.0 m $\times$ 1.0 m $\times$ 0.1 cm thick and weighing 20 N is placed midway in the gap (Fig. 1.27). Find the force required if the plate is to be lifted up with a constant velocity of 0.1 m/s.

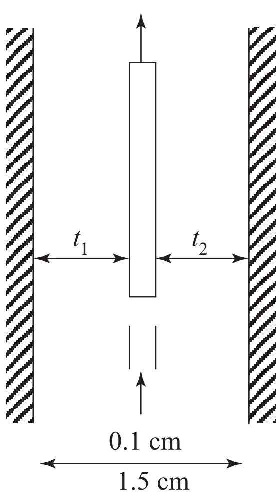

Fig. 1.27

Solution The shear stresses on two sides of the plate are as given

$$\tau_1 = \mu\frac{du}{dy} = \mu\frac{v}{t_1} \text{ and } \tau_2 = \mu\frac{v}{t_2}$$

Drag force or viscous resistance against the motion of the plate is given by

$$F = \left[\mu\frac{v}{t_1} + \mu\frac{v}{t_2}\right]A$$

$$= \mu A v\left[\frac{1}{t_1} + \frac{1}{t_2}\right]$$

Since the plate is midway in the gap, $t_1 = t_2$.

Therefore, $F = 2\frac{\mu A v}{t}$

But $t = \frac{1.5 - 0.1}{2} = 0.7\,\text{cm} \text{ or } 0.007\,\text{m}$

Therefore, $F = \frac{2 \times 2.0 \times 1.0 \times 1.0 \times 0.1}{0.007} = 57.14\,\text{N}$

Upthrust or buoyant force on the plate = specific weight × volume of oil displaced

$$= 0.9 \times 9810 \times 1.0 \times 1.0 \times 0.001 = 8.829\,\text{N}$$

Effective weight of the plate = 20 – 8.829 = 11.171 N

Therefore, the total force required to lift the plate at the given velocity

$$= 57.14 + 8.829 = 65.97\,\text{N} \quad (Ans)$$

Example 1.30 Two large fixed parallel planes are 12 mm apart. The space between the surfaces is filled with an oil of viscosity 0.9 N s/m^2. A flat thin plate 0.2 m^2 area moves through the oil at a velocity of 0.25 m/s.

Calculate the drag force

1. when the plate is equidistant from both the planes (Fig. 1.28)
2. when the thin plate is at a distance of 3.5 mm from one of the plane surfaces (Fig. 1.29).

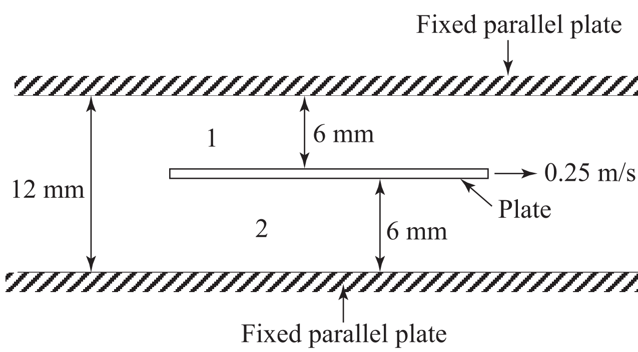

Fig. 1.28

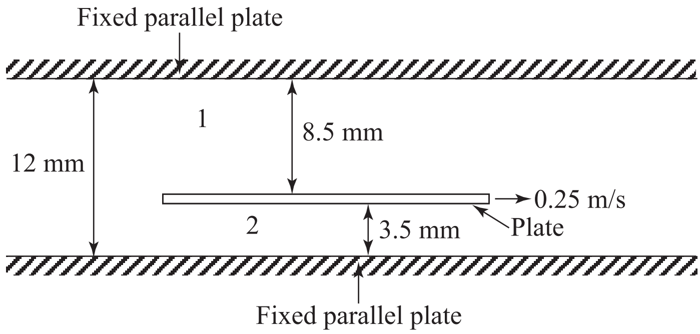

Fig. 1.29

Solution Distance between the fixed parallel planes = 12 mm = 0.012 m

Area of thin plate A = 0.2 m^2, velocity of plate = 0.25 m/s, viscosity of oil μ = 0.9 Ns/m^2, and drag force F = ?

1. When the plate is equidistance from both the planes

$$\tau_1 = \mu \frac{du}{dy}\Big|_1 = 0.9 \times \frac{0.25}{0.006} = 37.5\,\text{N/m}^2$$

$$F_1 = \tau_1 \times A = 37.5 \times 0.2 = 7.5\,\text{N}$$

$$\tau_2 = \mu \frac{du}{dy}\Big|_2 = 37.5\,\text{N/m}^2 \text{ and } F_2 = 7.5\,\text{N}$$

$$F = 7.5 + 7.5 = 15\,\text{N} \quad (Ans)$$

2. When the plate is at 3.5 mm from one of the fixed plates

$$F_1 = \tau_1 A = \mu \frac{du}{dy}\Big|_1 A = 0.9 \times \frac{0.25}{0.0085} \times 0.2 = 5.29\,\text{N}$$

$$F_2 = \tau_2 A = \mu \frac{du}{dy}\Big|_2 A = 0.9 \times \frac{0.25}{0.0035} \times 0.2 = 12.857\,\text{N}$$

Total force, $F = F_1 + F_2 = 5.29 + 12.857 = 18.147\,\text{N} \quad (Ans)$

1.7 VAPOR PRESSURE OF LIQUIDS (P_V) AND CAVITATIONS

We will first explain vapor pressure.

1.7.1 Vapor Pressure

All liquids and solids have a tendency to evaporate to a gaseous form, and all gases have a tendency to condense back into their original form (i.e., either liquid or solid). Liquids have a property of releasing their molecules into the space above their surface. The liquid is then said to be vaporized or evaporated. Evaporation occurs at the surface of the liquid. If the surface is exposed to the atmosphere, evaporation generally occurs continuously. If, however, the surface is within an enclosed space, evaporation will occur only until the air within the enclosed space becomes saturated with vapor. Pressure caused by vapor molecules within such a closed space is called vapor pressure. Thus, vapor pressure is the pressure of a vapor in equilibrium with its non-vapor phase.

Vapor pressure increases with increase in temperature. At any given temperature, for a particular substance, there is a pressure at which the gas of that substance is in dynamic equilibrium with its liquid or solid forms. This is known as vapor pressure of the substance at that temperature.

The vapor pressures of some liquids at 20°C are given in Table 1.17 and the variation of water with respect to temperature is given in Table 1.18.

Equilibrium vapor pressure is an indication of a liquid's evaporation rate. It relates the tendency of molecules and atoms to escape from a liquid or a solid. A substance with a high vapor pressure at normal temperature is often referred to as a volatile substance. According to the Clausius–Clapeyron relation, the vapor pressure of any substance increases non-linearly with temperature (Fig. 1.30).

1.7.2 Boiling Point

The boiling point of a liquid is the temperature at which the vapor pressure of the liquid equals the environmental pressure surrounding the liquid.

A liquid in vacuum environment has a lower boiling point than when it is at atmospheric pressure and a liquid in a high-pressure environment has a higher boiling point than when the liquid is at atmospheric pressure. In other words, all liquids may have many number of boiling points.

The normal boiling point (also called the atmospheric boiling point or the atmospheric pressure boiling point) of a liquid is the special case at which the vapor pressure of the liquid equals the ambient atmospheric pressure. At that temperature,

Table 1.17 Vapor pressures of some liquid at 20°C

Liquids	Vapor pressure (kN/m^2)
Water	2.34
Sea water	2.34
Carbon tetrachloride	12.1
Benzene	10.0
Mercury	0.0000017
Gasoline	55.0
Kerosene	3.11
Ammonia	910
Glycerin	0.000014

Table 1.18 Variation of vapor pressure with respect to temperature

Temperature (°C)	Vapor pressure (kPa)
0	0.611
10	1.23
20	2.34
30	4.24
40	7.38
50	12.3
60	19.9
70	31.2
80	47.4
90	70.1
100	101.3

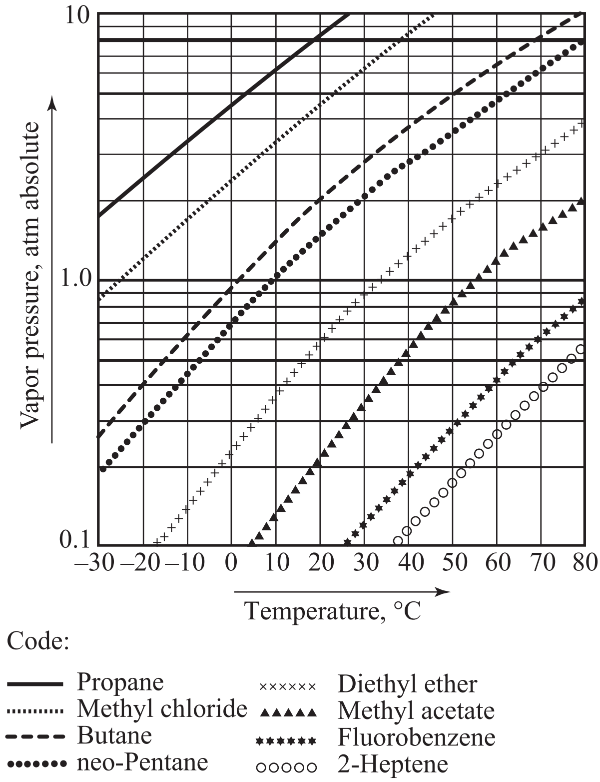

Fig. 1.30 Vapor pressure chart

the vapor pressure of the liquid becomes sufficient to overcome the atmospheric pressure and lift of the liquid to form bubbles inside the bulk of the liquid.

The heat of vaporization is the amount of heat required to convert or vaporize a saturated liquid (i.e., a liquid at its boiling point) into a vapor. Liquids may change to vapor at room temperatures below their boiling points through the process of evaporation.

Let the molecules impinging on the surface exert a partial pressure called vapor pressure (p_v). And let this pressure of the liquid vapor combined with the pressure of other gases in the atmosphere make up the total atmospheric pressure (p_a).

If $p_v < p_a$, then

Number of molecules leaving the surface > number of molecules re-entering the surface

That is, evaporation is taking place.

If $p_v > p_a$, then

Number of molecules leaving the surface < number of molecules re-entering the surface

That is, condensation is taking place.

If $p_v = p_a$, then

Number of molecules leaving the surface = number of molecules re-entering the surface

That is, boiling takes place, and for this equilibrium condition, p_v is called the saturation vapor pressure (SVP).

Thus, when the vapor pressure is equal to the atmospheric pressure or ambient pressure (in a closed vessel), boiling takes place. This pressure is a function of temperature. As the temperature increases, the vapor pressure also increases until the boiling point is reached for the ambient pressure. At sea level, water boils at 100°C and at high altitude (mountain peaks), where the atmospheric pressure is less, water boils at a temperature less than 100°C. When a liquid is confined in an enclosed vessel it may boil even at room temperature, if the ambient pressure is decreased to the magnitude of the vapor pressure of the liquid at that temperature.

Example 1.31 At what pressure in millibars will 40°C water boil?

Solution Vapor pressure at 40°C is 7.38 kN/m^2.
Hence, water will boil at 7.38 kN/m^2 = 7380 N/m^2 = 73.8 mbar (*Ans*)
(since 1 mbar = 100 N/m^2)

1.7.3 Cavitation

The SVP is of great practical use in fluid problems. If the pressure at any point in a fluid phenomenon approaches the vapor pressure, the liquid starts vaporizing. Vapor bubbles that are created in the region of low pressure are carried with the liquid to the region of high pressure. These bubbles collapse in the region of high pressure and explosion of bubbles takes place. This explosion causes damage to the walls of the conduit and also creates air pockets in the flow. The phenomenon is known as cavitation. Because of the destructive nature of cavitation, its occurrence in flow problems should be avoided. This is possible if the pressure at any point in the fluid phenomenon is not permitted to fall below the SVP. To avoid cavitations (cavity formation) in problems related to flow of water, the pressure is not permitted to fall below 2.5 m of water.

Example 1.32 At what pressure can cavitation be expected at the inlet of a pump that is handling water at 20°C?

Solution Cavitation occurs when the internal pressure drops to the vapor pressure. Vapor pressure of water at 20°C is 2.34 kN/m^2 and hence cavitation can be expected at that pressure.

1.8 BULK MODULUS (K) AND COMPRESSIBILITY (β)

We will first define bulk modulus of elasticity.

1.8.1 Bulk Modulus

Elasticity of fluids is measured in terms of bulk modulus of elasticity (K), which may be defined as the ratio of compressive stress to volumetric strain. This bulk modulus is analogous to the modulus of elasticity for solids. However, for fluids, it is defined on a volume basis rather than in terms of familiar one-dimensional stress–strain relation for solid bodies.

Consider a cylinder fitted with piston as shown in Fig. 1.31.

Let ∀ = volume of gas enclosed in the cylinder, p = pressure of gas when volume is ∀, which is also equal to P/A, where, A is the area of cross section of the cylinder.

Let the pressure be increased to $p + dp$, then the volume of gas decreases from ∀ to ∀ – d∀.

Therefore, Increase in pressure = dp

Decrease in volume = $d\forall$

$$\text{Volumetric strain} = -\frac{d\forall}{\forall}$$

Negative sign indicates decrease in volume with increase in pressure.

Therefore, bulk modulus K is given by

$$K = \frac{dp}{-\dfrac{d\forall}{\forall}} \tag{1.29}$$

Steepening of the curve with increasing pressure shows that as fluids are compressed, it becomes increasingly difficult to compress further. In other words, the value of K increases with increase in pressure. The bulk modulus of elasticity K is not constant, but it increases with increase in pressure and further it decreases with increase in temperature.

At NTP (normal temperature and pressure),

$$K_{\text{water}} = 2.07 \times 10^6 \text{ kN/m}^2 \quad \text{and} \quad K_{\text{air}} = 101.3 \text{ kN/m}^2$$

This indicates that air is about 20,000 times more compressible than water.With a decrease in the volume of a given mass, $m = \rho\forall$, will result in an increase in density. Equation (1.29) can also be expressed as

$$K = \frac{dp}{d\rho/\rho} \tag{1.30}$$

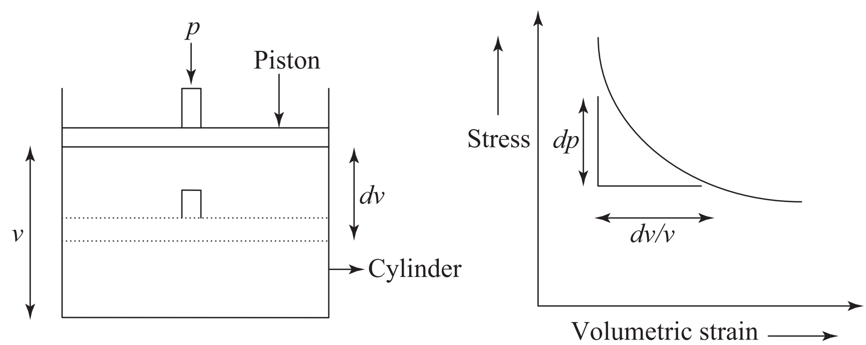

Fig. 1.31 Piston and cylinder experiment

When gases are compressed or expanded, the relationship between pressure and density depends on the nature of the process. If the compression or expansion takes place under constant temperature conditions (isothermal process), then

$$\frac{p}{\rho} = \text{constant} \tag{1.31}$$

If the compression or expansion is frictionless and no heat is exchanged with the surroundings (isentropic process), then

$$\frac{p}{\rho^k} = \text{constant} \tag{1.32}$$

where k is the ratio of the specific heat at constant pressure c_p to the specific heat at constant volume c_v (that is, $k = c_p/c_v$). The two specific heats are related to the gas constant R through the equation $R = c_p - c_v$. Here p is the absolute pressure and the value of k for air =1.4.

1.8.2 Compressibility

Compressibility is nothing but reciprocal of modulus of elasticity k. That is,

$$\beta = \frac{1}{K} \tag{1.33}$$

The property by which fluids undergo a change in volume under the action of external pressure is known as compressibility. It decreases with an increase in pressure of fluid, as the volume modulus increases with the increase of pressure. The variation in the volume of water with the variation of pressure is so small that for all practical purposes it is neglected. Thus, the water is considered to be an incompressible liquid. However, in case of water flowing through pipes, when sudden or large change in pressure (e.g., water hammer) takes place, then the compressibility must be taken into account.

1.8.3 Speed of Sound

Another important consequence of the compressibility of fluids is that disturbances introduced at some point in the fluid propagate at a finite velocity. In certain situation, these small disturbances can propagate at a rate equal to the speed of sound c. The speed of sound is related to changes in pressure and density of the fluid medium through Eqn (1.34).

$$c = \sqrt{\frac{dp}{d\rho}} \tag{1.34}$$

or in terms of bulk modulus

$$c = \sqrt{\frac{K}{\rho}} \tag{1.35}$$

Since the disturbance is small, one can assume the process to be isentropic. For gases under isentropic process,

$$K = kp$$

Therefore $$c = \sqrt{\frac{kp}{\rho}} \tag{1.36}$$

or $$c = \sqrt{kRT} \tag{1.37}$$

Thus, for ideal gases the speed of sound is proportional to the square root of the absolute temperature.

Example 1.33 A liquid compressed in a cylinder has a volume of 1000 cm^3 at 1 MN/m^2 and a volume of 995 cm^3 at 2 MN/m^3. What is the bulk modulus of elasticity?

Solution We have $K = -\dfrac{\Delta p}{\dfrac{\Delta\forall}{\forall}} = -\dfrac{2-1}{(995-1000)/1000} = 200 \text{ MPa}$ (*Ans*)

Example 1.34 For K = 2.2 GPa for the bulk modulus of elasticity for water, what pressure is required in reducing its volume by 0.5%?

Solution

$$K = -\frac{\Delta p}{\Delta\forall/\forall},\ 2.2 = -\frac{p_2 - 0}{-0.005}$$

$$p_2 = 0.0110\,\text{GPa} = 11.0 \text{ MPa} \quad (Ans)$$

Example 1.35 When the pressure of liquid is increased from 3 MN/m^2 to 6.0 MN/m^2, its volume is decreased by 0.1%. What is the bulk modulus of elasticity of the liquid?

Solution Given: Initial pressure = 3.0 MN/m^2
Final pressure = 6.0 MN/m^2
Increase in pressure = 6 – 3 = 3 MN/m^2
Decrease in volume = 0.1%

Therefore,

$$K = \frac{dp}{-d\forall/\forall} = \frac{3.0}{\dfrac{0.1}{100}} = 3.0 \times 10^9 \text{ N/m}^2 \quad (Ans)$$

Example 1.36 A high pressure steel container is filled with liquid at a pressure of 10 atm. The volume of the liquid is 1.23200 l. At a pressure of 25 atm, the volume of the liquid equals 1.23100 l. What is the average bulk modulus of the elasticity of the liquid over the given range of pressure, if the temperature after compression is allowed to return to the original temperature? What is the coefficient of compressibility?

Solution $$K = \frac{dp}{-d\forall/\forall} = -\frac{(25-10)101.3}{(1.23100-1.23200)/1.23200} = 1872 \text{ MN/m}^2 \quad (Ans)$$

$$\beta = \frac{1}{K} = \frac{1}{1872} = 0.000534 \text{ m}^2\text{/MN} \quad (Ans)$$

1.9 CAPILLARITY OR MENISCUS EFFECT

Capillarity is a phenomenon by which a liquid (depending upon its specific gravity) rises into a thin glass tube or below its general level, because of the combined effect of cohesion and adhesion. (Adhesion means an attraction between the molecules of a liquid and the molecules of a solid boundary in contact with the liquid. This property enables a liquid to stick to another body.)

Figure 1.32 shows the phenomenon of rising water in the tube of a smaller diameter.

Let d = diameter of the capillary tube, θ = angle of contact of the water surface, h = height of capillary rise, σ = surface tension force/unit length, and γ = weight density (ρg). For a length of πd, surface tension force = $\sigma\pi d$.

Equating the vertical component of surface tension force and weight of water, we get $\sigma(\pi \times d \times \cos\theta) = \frac{\pi}{4} \times d^2 \times h \times \gamma$

or
$$h = \frac{4\sigma\cos\theta}{\gamma d} \tag{1.38}$$

For water and glass combination, $\theta = 0$. Therefore,

$$h = \frac{4\sigma}{\gamma d} \tag{1.39}$$

Note The smaller the diameter of the capillary tube, the greater is the capillary rise or depression. At the same time, it should not be smaller than 8 mm and further it should not be more than 12 mm. Also, the capillary rise is usually measured to the bottom of the meniscus.

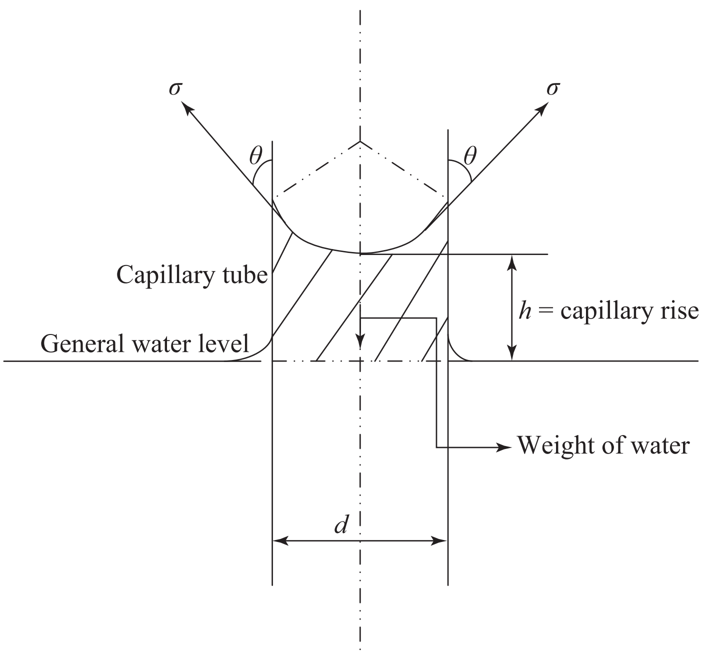

Fig. 1.32 Effect of capillarity in case of water

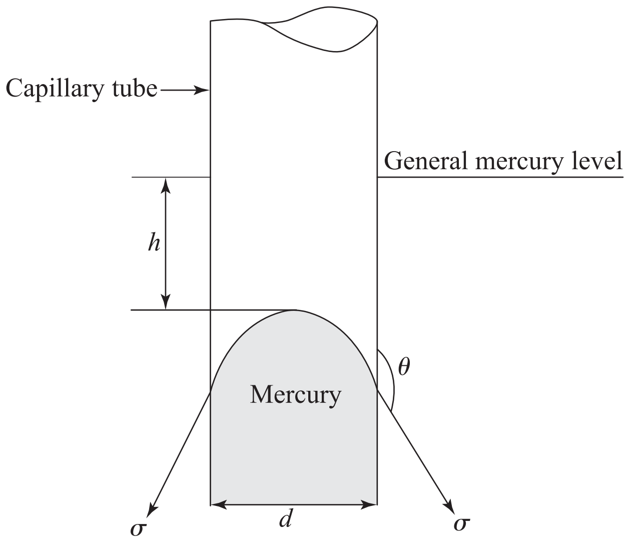

Fig. 1.33 Effect of capillarity in case of mercury

In case of mercury, there is a capillary depression as shown in Fig. 1.33.

Notes

1. For wetting liquid (water) $\theta < \pi/2$; for pure water $\theta = 0$ (pure water in contact with clean glass); otherwise, $\theta = 25°$ (slightly contaminated water)
2. For non-wetting liquid (mercury) $\theta > \pi/2$; for mercury θ varies between 130° and 150°

Example 1.37 Two parallel wide, clean, glass plates separated by a distance d of 1 mm are placed in water, as shown in Fig. 1.34. How far does the water rise due to the capillary action away from the ends of the plate? Take surface tension = 0.0730 N/m.

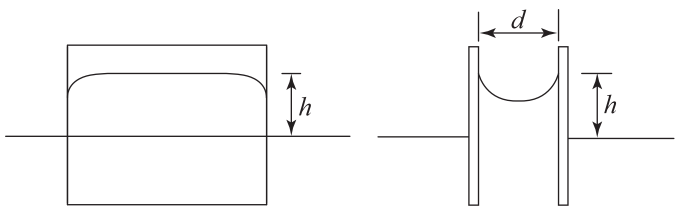

Fig. 1.34

Solution Because the plates are clean, the angle of contact between water and glass is taken as zero, considering the free body diagram of unit width of the raised water, away from the ends.

Summing forces in the vertical directions gives

$$2 \times \sigma \times \frac{1}{1000} - \left(\frac{1}{1000}\right)^2 \times h \times \gamma = 0$$

or $h = 0.0143$ m or 14.3 mm

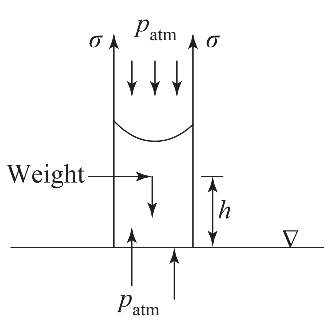

Fig. 1.35

Example 1.38 Calculate the capillary effect in millimeters in a glass tube of 3 mm in diameter, when immersed in water and mercury. The temperature of the liquid is 20°C and the values of surface tension of water and mercury at 20°C in contact with air are 0.0735 N/m and 0.51 N/m, respectively. The contact angle for water $\theta = 0°$ and for mercury $\theta = 130°$. Take specific weight of water at 20°C as equal to 9810 N/m^3.

Solution We have capillary effect for water,

$$h = \frac{4 \times 0.0735 \times \cos(0)}{9810 \times 0.003} = 9.98 \text{ mm} \quad (Ans)$$

Capillary effect for mercury,

$$h = \frac{4 \times 0.051 \times \cos 130}{9810 \times 0.003} = -4.45 \text{ mm} \quad (Ans)$$

Example 1.39 A U-tube is made up of two capillaries of bores 1.0 mm and 2.2 mm, respectively. The tube is held vertically with zero contact angles. It is partially filled with liquid of surface tension 0.06 N/m. If the estimated difference in the level of two menisci is 15 mm, determine the mass density of the liquid.

Solution Given bores of capillaries: $d_1 = 1.0 \text{ mm} = 0.001 \text{ m}$
$d_2 = 2.2 \text{ mm} = 0.0022 \text{ m}$

Difference of level, $h_1 - h_2 = 15 \text{ mm} = 0.015 \text{ m}$

So, we have $$h_1 = \frac{4\sigma\cos\theta}{\gamma d_1} \quad \text{and} \quad h_2 = \frac{4\sigma\cos\theta}{\gamma d_2}$$

Now, $$h_1 - h_2 = \frac{4\sigma}{\gamma}\left[\frac{1}{d_1} - \frac{1}{d_2}\right]$$

or $$0.015 = \frac{4 \times 0.06}{\rho \times 9.81}\left[\frac{1}{0.001} - \frac{1}{0.0022}\right]$$

So, $$\rho = 889.63 \text{ kg/m}^3 \quad (Ans)$$

Example 1.40 Develop a formula for the capillary rise of a fluid having surface tension σ and a contact angle θ between

1. two concentric glass tubes of radii r_o and r_i (Fig. 1.36)
2. two vertical glass plates set parallel to each other and having a gap t between them (Fig. 1.37)

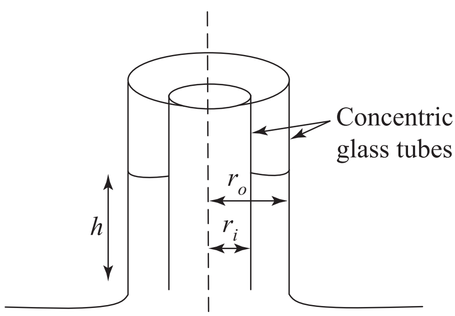

Fig. 1.36

Solution

1. At equilibrium,

$$\gamma h \pi (r_o^2 - r_i^2) = \sigma \times \cos\theta \times 2\pi(r_o + r_i)$$

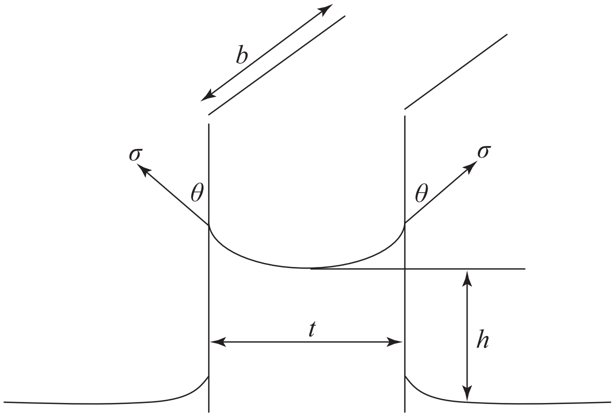

Fig. 1.37

or $$h = \frac{2\sigma\cos\theta}{\gamma(r_o - r_i)} \quad (Ans)$$

2. We have $$\gamma hbt = \sigma\cos\theta \times 2b$$

or $$h = \frac{2\sigma\cos\theta}{\gamma t} \quad (Ans)$$

Example 1.41 The glass tube in Fig. 1. 38 is used to measure pressure p in the water tank. The diameter of the tube is 0.9 mm and water is at 30°C. After correcting for the surface tension, what is the true water height in the tube? What percentage of error is made at equilibrium, if no correction is computed?

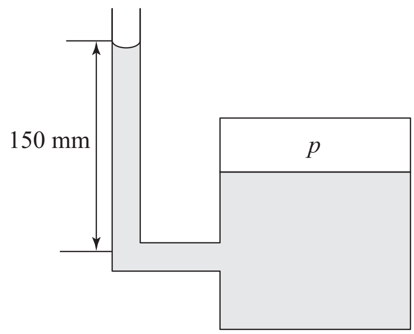

Fig. 1.38 Water tank

Solution Height of water in the glass tube (capillary correction) is given by

$$h = \frac{2\sigma\cos\theta}{\rho gr} = \frac{2\times 0.0712\times\cos\theta}{1000\times 9.81\times 0.45} = 0.0322\,\text{m or } 3.22\,\text{cm}$$

Therefore, true height of water in the tube = 15 – 3.22 = 11.78 cm (*Ans*)

Neglecting capillary correction causes

$$\frac{3.22}{15} = 0.2146 \text{ or } 21.46\%\,\text{error} \quad (Ans)$$

1.10 SURFACE TENSION (σ_s)

Many natural phenomena are associated with surface tension. Some of them are listed below:

1. A small quantity of liquid assuming the shape of globules and becoming spherical when made smaller
2. Rain drop falling over lotus leaves
3. Mercury spilling over the floor
4. Walking of some insects over water
5. Floating of a carefully placed needle on a water surface

These observations tell us that liquids behave as if their surfaces were stretched like membranes under tension. Actually, there is no membrane, but a membrane-like situation is obtained by the property of cohesion (cohesion means intermolecular attraction between molecules of same liquid).

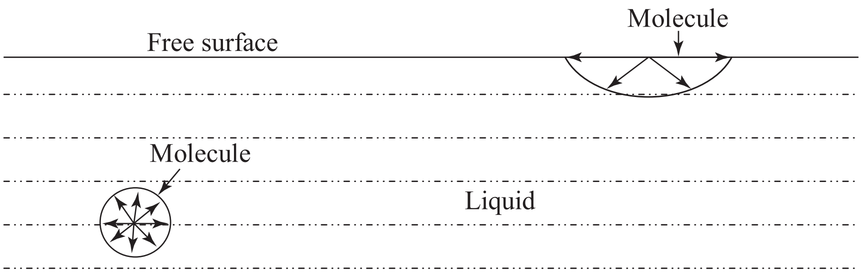

Fig. 1.39 Surface tension

Consider a free surface as shown in Fig. 1.39. All molecules inside the medium are attracted equally in all directions by the surrounding molecules, but the one on the surface does not have a molecule above to pull it upwards, and it is therefore attracted inwards. This results in an inward attraction on particles in and near the surface and tends to make the surface area as small as possible. Consequently, the surface film is under a tension equal to its length.

The tensile strength of the surface film computed per unit length is termed as *surface tension*. Since the magnitude is small compared to gravitational forces and pressure, the surface tension is usually neglected, but becomes quite significant when there is a free surface and the boundary conditions are small as in the case of small-scale models of hydraulic engineering structures. The surface tension is expressed in N/m. The values of surface tension (Table 1.19) depend on the following factors:

1. Nature of liquid
2. Nature of surrounding matter (e.g., solid, liquid, or gas)
3. Kinetic energy (and hence, the temperature of the liquid molecules)

Table 1.19 Variation of surface tension with respect to temperature

Temperature (°C)	**σ (N/m) (water–air)**
0	0.0756
10	0.0742
20	0.0728
3	0.0712
40	0.0696
50	0.0679
60	0.0662
70	0.0644
80	0.0626
90	0.0608
100	0.0589

As the water temperature range of the data is considerable, one requires relationships for σ as functions of temperature T. Streeter and Wylie (1979) have given the variation of σ for water with T ranging from 0°C to 100°C in a tabular form. Using these data, the following best-fit equations were obtained in SI units. The maximum percentage error in the use of Eqn (1.40) is 1.0%, which occurs in a very narrow band of temperature.

$$\sigma = 0.0762\exp(-0.00233T) \tag{1.40}$$

1.10.1 Pressure Inside a Water Droplet

To obtain pressure inside a water droplet, use of surface tension is needed, which is explained further.

Let p be the pressure inside the droplet above the outside pressure where d is the diameter of the droplet and σ is the surface tension of the liquid (Fig. 1.40). From the free body diagram, we have

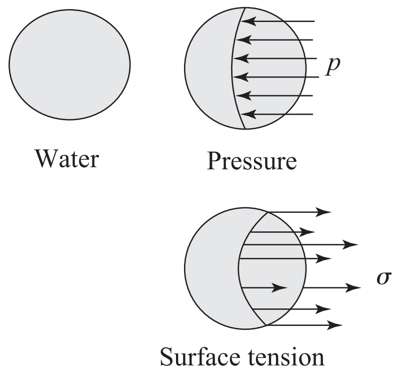

Fig. 1.40 Free body diagram of water droplet

$$\text{Pressure force} = p \times \frac{\pi}{4} d^2$$

Surface tension force acting around the circumference = $\sigma \times \pi d$

Under the equilibrium conditions, these two forces will be equal and opposite, i.e.,

$$p \times \frac{\pi}{4} d^2 = \sigma \times \pi \times d$$

Therefore,
$$p = \frac{4\sigma}{d} \tag{1.41}$$

From Eqn (1.41), it is seen that pressure intensity decreases with the increase in the diameter of the droplet.

1. Pressure inside a soap bubble (Fig. 1.41)

Soap bubbles have two surfaces on which surface tension σ acts.

From the free body diagram, we have

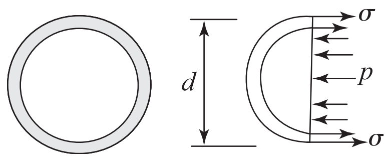

Fig. 1.41 Free body diagram of soap bubble

$$p \times \frac{\pi}{4} d^2 = 2 \times \sigma \times \pi \times d$$

or
$$p = \frac{8\sigma}{d} \tag{1.42}$$

Since the soap solution has a high value of surface tension σ, even with small pressure of blowing a soap bubble will tend to grow larger in diameter (hence, formation of large soap bubbles).

2. A liquid jet

Let us consider a cylindrical liquid jet of diameter d and length l, as shown in Fig. 1.42.

Pressure force = $p \times l \times d$

Surface tension force = $\sigma \times 2 \times l$

Equating the two forces, we have

$$p = \frac{2\sigma}{d} \qquad (1.43)$$

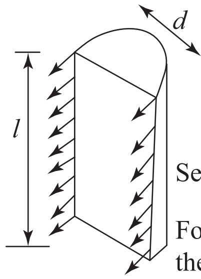

Fig. 1.42 Free body diagram of a liquid jet

Example 1.42 A soap bubble 60.0 mm in diameter has an internal pressure in excess of the outside pressure of 25 N/m^2. What is the tension in the soap film?

Solution Given, diameter of the soap bubble = 60×10^{-3} m and p = 25 N/m^2

We have $$p = \frac{8\sigma}{d}$$

So, $$25 = \frac{8 \times \sigma}{60.0 \times 10^{-3}}$$

or $$\sigma = 0.1875 \text{ N/m} \quad (Ans)$$

Example 1.43 To form a stream of bubbles, air is introduced through a nozzle into a tank of water (at 20°C). If the process requires 2.0 mm diameter bubbles to be formed, by how much should the air pressure at the nozzle must exceed that of the surrounding water. Take surface tension at 20°C = 0.0735 N/m.

Solution Given, diameter of the bubbles to form = 2.0 mm =2×10^{-3} m

Surface tension = 0.0735 N/m

We have $$p = \frac{4\sigma}{d}$$

$$= \frac{4 \times 0.0735}{2 \times 10^{-3}}$$

or $$p = 147 \text{ N/m}^2 \quad (Ans)$$

Example 1.44 What force is necessary to lift a thin platinum wire ring of 4.0 cm in diameter from a water surface? Assume the surface tension of water as 0.0728 N/m and neglect the weight of the wire (Fig. 1.43).

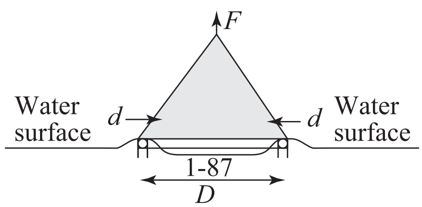

Fig. 1.43

Solution Given, diameter of the wire = 4.0 cm = 0.04 m

Surface tension = 0.0728 N/m

Assuming $d << D$,

$$F = 2(\pi D \sigma) = 2 \times \pi \times 0.04 \times 0.0728$$

$$F = 0.01829 \text{ N} \quad (Ans)$$

Example 1.45 A spherical water droplet of 1.2 mm in diameter splits up in air into 60 smaller droplets of equal size. Find the work required in splitting up the droplet. The surface tension coefficient of water in air is 0.073 N/m.

Solution

Note An increase in the surface area out of a given mass takes place when a bigger droplet splits up into a number of smaller ones. So, the work required is given by the product of surface tension coefficient and the increase in surface area.

Let, d be the diameter of the smaller droplets.

From conservation of mass, $60\times\pi\times\dfrac{d^3}{6}=\dfrac{\pi\times 0.0012^3}{6}$, $d=0.31\times10^{-3}\,\text{m}$

Initial surface area (due to single droplet) $=\pi\times(0.012)^2=4.523\times10^{-6}\ \text{m}^2$

Final surface area (due to 60 smaller droplets) $=60\times\pi\times(0.31\times10^{-3})=0.0584\,\text{m}^2$

Hence, the increase in surface area $=0.0584-4.523\times10^{-6}=0.0584\,\text{m}^2$

The required work = 0.073 × 0.584 = 4.26 × 10^{-3} J (*Ans*)

Example 1.46 Calculate the work done in blowing a soap bubble of diameter 15 cm. Assume the surface tension of soap solution = 0.04 N/m.

Solution We know that the soap has two interfaces.
So,
Work done = surface tension × total surface area

$$=0.04\times4\times\pi\times\left(\frac{15}{2}\times10^{-2}\right)^2\times2=5.65\times10^{-3}\ \text{Nm}\quad(\textit{Ans})$$

Example 1.47 If the surface tension at air-water interface is 0.073 N/m, what is the pressure difference between the inside and outside of an air bubble of diameter 0.02 mm?

Solution An air bubble has only one surface. Therefore,

$$\Delta p=\frac{4\sigma}{d}=\frac{4\times0.073}{0.02\times10^{-3}}=14.6\ \text{kPa}\quad(\textit{Ans})$$

SUMMARY

- Fluid mechanics is a branch of mechanics that deals with the static, kinematic, and dynamic aspects of fluids. Fluids are at rest when there is no external unbalanced force and this aspect of the study of fluids is called fluid statics. Kinematics refers to the study of fluids in motion where pressure forces are not considered, and if the pressure forces are also considered for the fluid in motion, it is called fluid dynamics.
- The mass density or specific mass of a liquid is equal to mass per unit volume, i.e., $\rho=\dfrac{m}{V}$.
- The weight density or specific weight of a fluid is equal to weight per unit volume, i.e., $\gamma=\dfrac{W}{V}=\rho g$.

- Specific volume is a reciprocal of mass density, i.e., $V_s = \frac{1}{\rho}$.
- Specific gravity is defined as the ratio of the specific weight of the liquid to the specific weight of a standard liquid, i.e., $S = \frac{\gamma_{\text{liquid}}}{\gamma_{\text{water}}}$.
- Relative density is a dimensionless ratio of the densities of two material, i.e., $G = \frac{\rho_{\text{obj}}}{\rho_{\text{reference}}}$.
- Gases are highly compressible in comparison to liquids, with changes in gas density directly related to changes in pressure and temperature through the equation $p = \rho RT$. This equation is known as ideal or perfect gas law.
- The shear stress is proportional to the velocity gradient, i.e., $\tau = \mu \frac{du}{dy}$.
- Kinematic viscosity is given by $\nu = \frac{\mu}{\rho}$.
- Poise and stokes are the units of dynamic viscosity and kinematic viscosity, respectively, in CGS units.
- Bulk density of elasticity is given by $K = \frac{-dp}{\left(\frac{d\forall}{\forall}\right)}$.
- Compressibility is the reciprocal of bulk modulus of elasticity, i.e., $\beta = \frac{1}{K}$.
- A liquid forms an interface with a second liquid or gas. The surface energy per unit area of interface is known as surface tension or coefficient of surface tension.

  (a) Surface tension is expressed in N/m.

  (b) For liquid drop, $p = \frac{4\sigma}{d}$.

  (c) For soap bubble, $p = \frac{8\sigma}{d}$.

  (d) For liquid jet, $p = \frac{2\sigma}{d}$.
- Liquids have both cohesion and adhesion, which are forms of molecular attraction. The rise or fall of liquid in small diameter tubes is due to capillarity. Liquids such as water, which wet a surface, cause capillary rise. In non-wetting liquids (e.g., mercury), capillary depression is caused.
- Capillary rise or fall of a liquid is given by $h = \frac{4\sigma \cos\theta}{\gamma d}$.
- The value of θ for water is considered equal to zero and for mercury equal to 128°.
- All liquids exposed to a gaseous environment have a tendency to evaporate. Evaporation is a process in which the liquid loses its molecules to the gas surrounding it. The rate of evaporation depends on the difference in molecular energy levels between the liquid and the gas. The pressure at which the liquid begins to boil is called vapor pressure of the liquid at that temperature.

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Multiple Choice Questions

1. The mass per unit volume of a liquid at a standard temperature and pressure is called
   (a) specific weight (b) mass density
   (c) specific gravity (d) none of the above
2. The weight per unit volume of a liquid at a standard temperature and pressure is called
   (a) mass density (b) specific gravity
   (c) specific weight (d) none of the above
3. Which of the following is the specific weight of water in SI units?
   (a) 9.81 kN/m^3 (b) 9.81×10^6 kN/m^3
   (c) 9.81 N/m^2 (d) none of the above
4. The specific gravity of water is taken as
   (a) 0.001 (b) 0.01
   (c) 0.1 (d) 1
5. The specific gravity of sea water is _______that of pure water.
   (a) Same as (b) Less than
   (c) More than
6. The density of liquid in gm/cm^3 is numerically equal to its specific gravity.
   (a) True (b) False
7. When a shear stress is applied to a substance it is found to resist it by static deformation. The substance is a
   (a) liquid (b) solid
   (c) gas (d) fluid

8. The condition of no-slip at rigid boundaries is applicable to
   (a) flow of Newtonian fluid (b) flow of ideal fluids only
   (c) flow of all real fluids (d) flow of all non-Newtonian fluids
9. The variation in the volume of a liquid with the variation of pressure is called its
   (a) surface tension (b) compressibility
   (c) capillarity (d) viscosity
10. When a tube of smaller diameter is dipped in water, the water rises in the tube with an upward ______ surface.
   (a) Concave (b) Convex
11. Newton's law of viscosity relates to which of the following?
   (a) Pressure, velocity, and viscosity
   (b) Shear stress and rate of angular deformation in a fluid
   (c) Shear stress, temperature, viscosity, and velocity
   (d) None of the above
12. With an increase in size of tube, the rise or depression of liquid in the tube due to surface tension will
   (a) decrease
   (b) increase
   (c) remain unchanged
   (d) depend upon the characteristics of liquid
13. In the manufacture of lead shots, the property of surface tension is utilized.
   (a) Agree (b) Disagree
14. Newton's law of viscosity states that
   (a) shear stress is directly proportional to the velocity
   (b) shear stress is directly proportional to the velocity gradient
   (c) shear stress is directly proportional to shear strain
   (d) shear stress is directly proportional to the viscosity
15. Kinematic viscosity is defined as equal to
   (a) dynamic viscosity/density (b) dynamic viscosity × density
   (c) dynamic viscosity × pressure (d) pressure × density
16. Poise is the unit of
   (a) mass density (b) kinematic viscosity
   (c) viscosity (d) velocity gradient
17. Stoke is the unit of
   (a) surface tension (b) viscosity
   (c) kinematic viscosity (d) none of the above
18. Surface tension is the unit of
   (a) force per unit area (b) force per unit length
   (c) force per unit volume (d) none of the above
19. The viscosity of
   (a) liquids increases with temperature
   (b) gases increases with temperature

(c) fluids decreases with temperature
(d) fluids increases with temperature

20. The gases are considered incompressible when Mach number
(a) is equal to 1.0 (b) is equal to 0.5
(c) is more than 0.3 (d) is less than 0.2

21. Which of the following property do practical fluids possess?
(a) Viscosity (b) Surface tension
(c) Compressibility (d) All of the above

22. To which of the following does water belong to?
(a) Newtonian fluids (b) Non-Newtonian fluids
(c) Compressible fluids (d) None of the above

23. A fluid is a substance that
(a) always expands until it fills any container
(b) is practically incompressible
(c) cannot withstand any shear force
(d) obeys the newton's law of viscosity

24. The property of fluids by which their molecules get attracted to another body is known as
(a) capillary action (b) surface tension
(c) adhesion (d) cohesion

25. The bulk modulus of elasticity
(a) is independent of temperature
(b) increases with the pressure
(c) has the dimensions of 1/P
(d) is larger when the fluid is more compressible

26. Falling drops of water become spheres due to
(a) adhesion (b) cohesion
(c) surface tension (d) viscosity

27. The gases are considered incompressible when Mach number
(a) is equal to 1.0 (b) is equal to 0.5
(c) is more than 0.3 (d) is less than 0.2

28. An ideal fluid is defined as the fluid which
(a) is incompressible
(b) is compressible
(c) has negligible surface tension
(d) is incompressible and non-viscous (inviscid)

29. A 40 cm cubical block slides on oil (viscosity = 0.8 Pas), over a large plane horizontal surface. If the oil film between the block and the surface has a uniform thickness of 0.4 mm, what will be the force required to drag the block at 4 m/s? Ignore the end effects and treat the flow as two dimensional.
(a) 1280 N (b) 1640 N
(c) 1920 N (d) 2560 N

Review Questions

1. Define specific weight, mass density, specific volume, and specific gravity.
2. What are the different properties of liquid?
3. Define a fluid. What is the difference between an ideal fluid and a real fluid?
4. What is the difference between a fluid and a solid? Differentiate between compressible fluids and incompressible fluids.
5. Define Newtonian and non-Newtonian fluids.
6. Why is the specific weight of sea water more than that of pure water? Give their numerical values.
7. Define the terms cohesion and adhesion.
8. What kind of rheological materials are paint and grease?
9. Distinguish between Newtonian and non-Newtonian fluids.
10. What is vapor pressure? What is its significance in flow problems? What do you understand by the term cavitation?
11. Why do the different liquids exert different vapor pressures?
12. Write a short note on surface tension.
13. Define surface tension. Derive expressions for the pressure (a) within a droplet of water and (b) inside a soap bubble.
14. Define the term viscosity and give the units in which it is expressed.
15. On what factors does the viscosity depend?
16. What is the difference between dynamic viscosity and kinematic viscosity? State their units of measurements.
17. State the Newton's law of viscosity and give examples of its application.
18. How does viscosity of a fluid vary with temperature?
19. Explain the phenomenon of capillarity. Obtain an expression for capillary rise of a liquid.
20. Define compressibility. How is it related to bulk modulus of elasticity?
21. Mention some examples where compressibility of water is taken into account.

Problems

1. If the specific weight of a liquid is 8 kN/m^3, what is its mass density?
   (**Ans:** 815 kg/m^3)
2. If specific gravity of a liquid is 0.8, make calculations for its mass density, specific volume, and specific weight.
   (**Ans:** 800 kg/m^3, 1.25×10^{-3} m^3/kg, 7848 N/m^3)
3. Calculate the specific weight, specific mass, and specific gravity of a liquid having a volume as 4 m^3 and weighing 30 kN.
   (**Ans:** 7500 N/m^3, 764.53 kg/m^3, 0.76)
4. One liter of petrol weighs 7.02 N. Calculate the specific weight, density, specific volume, and relative density.
   (**Ans:** 7.02 kN/m^3, 716 kg/m^3, 1.395×10^{-3} m^3/kg, 0.716)
5. Air is kept at a pressure of 200 kPa and a temperature of 30°C in a 500 L container. What is the mass of the air? (**Ans:** 1.15 kg)

6. Calculate the gas constant and density of certain gas weighing 14.7 N/m^3 at 30°C and at an absolute pressure of 196.2 kN/m^2. (**Ans:** 430 J/kgK)

7. Carbon tetrachloride at 20°C has a viscosity of 0.000967 Ns/m^2. What shear stress is required to deform this fluid at a strain rate of 5000 s^{-1}? (**Ans:** 4.84 Pa)

8. A plate 0.5 mm distant from a fixed plate moves at 0.25 m/s and requires a force per unit area of 2.0 Pa to maintain this speed. Determine the viscosity of the fluid between the plates. (**Ans:** 0.00400 Ns/m^2)

9. Two horizontal flat plates are placed 0.15 mm apart and the space between them is filled with an oil of viscosity 1 poise. The upper plate of area 1.5 m^2 is required to move with a speed of 0.5 m/s relative to the lower plate. Determine the necessary force and power required to maintain this speed. (**Ans:** 500 N, 0.25 kW)

10. The velocity distribution over a plate is given by, $u = \frac{3}{4}y - y^2$, where u is the velocity in meters per second at a distance y meter above the plate. Determine the shear stress at $y = 0$ and $y = 0.2$ m. Take $\mu = 8.4$ poise. (**Ans:** 0.63 N/m^2, 0.294 N/m^2)

11. The specific gravity of water at 20°C is 0.998 and its viscosity is 0.001008 Ns/m^2. Find its kinematic viscosity. (**Ans:** 1.009×10^{-6} m^2/s)

12. A piston of 69 mm diameter rotates concentrically inside a cylinder 70 mm diameter. Both the piston and the cylinder are 80 mm long. Find the tangential velocity of the piston if the space between the cylinder and the piston is filled with oil of viscosity 0.235 Ns/m^2 and the torque of 0.0143 N m is applied. (**Ans:** 4.87 m/s)

13. An increase in pressure of a liquid from 7.5 MPa to 15 MPa- results into 0.2% decrease in its volume. Determine the bulk modulus of elasticity and coefficient of compressibility of a liquid. (**Ans:** 3.75×10^9 N/m^2, 0.267×10^{-9} m^2/N)

14. A 20 mm wide gap between two vertical plane surfaces is filled with an oil of specific gravity 0.85 and dynamic viscosity 2.5 Ns/m^2. A metal plate 1.25 m × 1.25 m × 0.2 cm thick and weighing 30 N is placed mid-way in the gap. Find the force if the plate is to be lifted up with a constant velocity of 0.12 m/s. (**Ans:** 108.11 N)

15. A square plate of size 1 m × 1 m and weighing 392.4 N slides sown an inclined plane with a uniform velocity of 0.2 m/s as shown in Fig. 1.44. The inclined plane is laid on a slope of 5 vertical to 12 horizontal and has an oil film of 1 mm thickness. Calculate the dynamic viscosity of oil. (**Ans:** 0.755 N s/m^2)

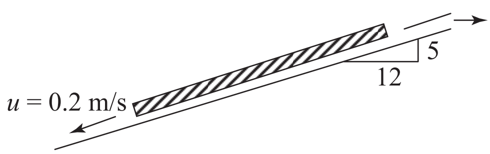

Fig. 1.44

16. A liquid has a viscosity of 0.005 N s/m^2 and density of 850 kg/m^3. Calculate the kinematic viscosity. (**Ans:** 5.882×10^{-6} m^2/s)

17. Calculate the work done in blowing a soap bubble of diameter 12 cm. Assume the surface tension of soap solution = 0.04 N/m. (**Ans:** 36.2×10^{-4} N m)

18. Neglecting the weight of the wire, what force is required to lift a thin wire ring 40 mm in diameter from a water surface at 20°C? (**Ans:** 0.0183 N)

19. What is the pressure within a 1 mm diameter spherical droplet of water relative to the atmospheric pressure outside? Assume σ for pure water to be 0.073 N/m. (**Ans:** 292 N/m^2)

20. A capillary tube having an inside diameter 5 mm is dipped in water at 20°C. Determine the height of water which will rise in the tube. Take $\sigma = 0.075$ N/m and $\alpha = 60°$. (**Ans:** 5.2 mm)

21. Find the capillary rise in a 3 mm glass tube when immersed vertically in water. Assume $\sigma = 0.071$ N/m. (**Ans:** 9.69 mm)

22. Distilled water at 10°C stands in a glass tube of 8 mm diameter at a height of 25 mm. What is the true static height? (**Ans:** 21.2 mm)

23. At 30°C what diameter glass tube is necessary to keep the capillary height change of water less than 1mm? (**Ans:** ≥29.2 mm)

24. Derive an expression for pressure difference across a spherical droplet. Using the result, find the surface tension in a soap bubble of 50 mm diameter when the inside pressure is 1.96 N/m^2 above the atmosphere. (**Ans:** 0.0125 N/m)

Answers to Multiple Choice Questions

1. (b), 2. (c), 3. (a), 4. (d), 5. (c), 6. (a), 7. (b), 8. (c), 9. (b), 10. (a), 11. (b), 12. (a), 13. (a), 14. (b), 15. (a), 16. (c), 17. (c), 18. (b), 19. (b), 20. (d), 21. (d), 22. (a), 23. (c), 24. (c), 25. (c), 26. (c), 27. (d), 28. (d), 29. (a)