Electrical Machines

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Features of

Figures and Tables

The chapters comprise enough illustrations and tables supporting content for students to understand the concepts in a better way.

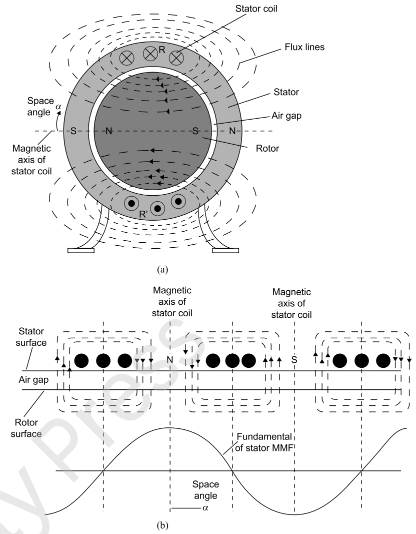

Fig. 8.12 (a) Single phase coil on stator with flux lines (b) Developed view of Figure (a) and MMF distribution along air gap

Table 7.1 Advantages of a bank of three single-phase units over a single three-phase unit

Feature	Single three-phase transformer	Three single-phase transformers bank
Handling, transportation, and installation	A single, three-phase transformer is big and heavy as compared to any of the single-phase transformers. Thus a big three-phase unit is more difficult to handle, transport and install, especially installation in places where space is a constraint, such as in mines.	It is thus recommended to use bank of three single-phase transformers that are easier to transport individually rather than a single big three-phase unit.
Cost of repair and maintenance	Single, three-phase unit is more difficult and costly to repair or even for maintenance than single-phase units. For a single three-phase transformer, the entire unit needs to be taken out of service, entire oil to be drained out, entire heavy core with coils need to be de-tanked before attempting any repair job.	In case of fault in any one of the units of the bank, it can be taken out of service for maintenance without disturbing the other two.
Size and cost of spare unit	If a single three-phase unit is used, then even for a minor fault, the total unit needs to be taken out and replaced by a similar big unit. The size of standby or the spare unit needs to be as big as the original one, thus involving more cost.	In case of fault in any one of the three single-phase transformers, only the faulty unit needs to be replaced by the spare one and service can be restored quickly. It is obviously less costly to keep one standby (spare) when three single-phase units are used in a bank, rather than a single three-phase unit.

Solved Examples

Various solved examples are added to each chapter for readers to have a thorough understanding of the concepts.

Example 11.1 A 60 Hz, star connected synchronous generator gives 12 kV as line voltage on open circuit and has a flux per pole of 15×10^{-2} Wb. The distribution factor of the full pitch coil is 0.96. Find number of armature conductors in series per phase.

Solution: Line voltage at open circuit is the induced EMF

(per phase) $E_f = \frac{12000}{\sqrt{3}} = 6928$ V

Frequency, $f = 60$ Hz

Distribution factor, $k_d = 0.96$

Since the coils are full pitched, pitch factor $k_p = 1$

Thus, winding factor $k_w = k_d k_p = 0.96 \times 1 = 0.96$

Given, flux per pole, $\phi_m = 15 \times 10^{-2}$ Wb

$\therefore$ Induced EMF

$E_f = 4.44 k_d \phi_m f T = 4.44 \times 0.96 \times 15 \times 10^{-2} \times 60 \times T$

Thus, number of turns:

$$T = \frac{6928}{4.44 \times 0.96 \times 15 \times 10^{-2} \times 60} = 180$$

$\therefore$ Number of armature conductor in series per phase = $2 \times T = 2 \times 180 = 360$

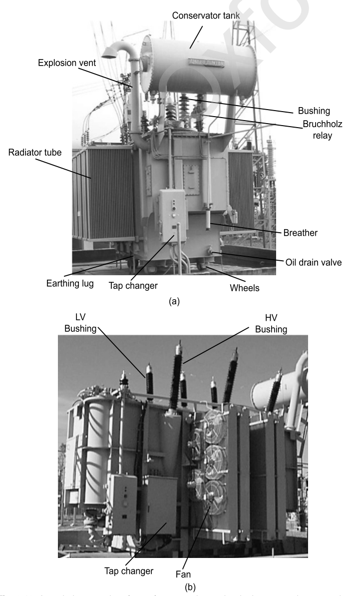

Fig. 6.12 Actual photographs of transformer tank, mechanical parts, and accessories

Photographs of Machines

The book comes with photographs of some important machines segregated across the chapters throughout for students to have an idea of what is going on inside the machine.

the Book

> **Which part of a DC machine has maximum flux density?**
>
> Since the area at the tooth root is very less, flux density at tooth root is very high. In fact, in a DC machine, maximum flux density occurs at the root of the teeth. While designing a DC machine, magnetic circuit calculations are initiated based on the permissible value of flux density at tooth root.

Boxed Items

The book contains various boxed items with crisp headings in each chapter, providing additional information related to the ongoing discussion and making the text interesting for the readers.

MATLAB Examples

MATLAB examples are provided at the end of each chapter for students to have a clear understanding of mathematical modeling of Electrical Machines.

MATLAB Example 14.2 A 220 V, four-pole, 50 Hz single-phase induction motor has the following impedances at standstill:

Main winding $Z_m = 30\angle 65°$

Auxiliary winding $Z_a = 40\angle 50°$

1. Calculate input current at starting
2. Calculate value of the capacitor to be connected in series with the auxiliary winding to obtain maximum starting torque
3. Comment on the comparison of starting current, starting power factor and starting torque with and without the additional capacitor

```
%Specifications
V=220;          % Supply voltage
P=4;            % Number of poles
f=50;           % Supply frequency
Zm_m=30; Zm_a=65;   % Main winding impedance magnitude and phase
Za_m=40; Za_a=50;   % Auxiliary winding magnitude and phase

%Part (a)

Zm_a_Rad=Zm_a*pi/180;
Zm=Zm_m*(cos(Zm_a_Rad)+1j*sin(Zm_a_Rad));

Za_a_Rad=Za_a*pi/180;
Za=Za_m*(cos(Za_a_Rad)+1j*sin(Za_a_Rad));

Ims=V/Zm;           % Calculate main winding current at starting
Ias=V/Za;           % Calculate auxiliary winding current at starting
Isl=Ims+Ias;        % Calculate input current at starting

% Part (b)
```

Critical Thinking Questions

1. 'For successful mechanical movement in a singly excited system, it is necessary that air gap between stator and the moving element is non-uniform'. Justify with reason whether the statement is true or false.
2. 'For achieving movement of the rotor of a cylindrical rotor motor, it is necessary that both stator and rotor be excited'. Justify with reason whether the statement is true or false.
3. In a doubly excited system, the salient pole rotor moves in a direction such that the stored energy in magnetic field is reduced, but the co-energy is increased: Justify with reason whether the statement is true or false.
4. 'Direction of rotation of the rotor in a singly excited machine with salient pole structure of rotor can be reversed by reversing the current direction in stator electromagnet'. Justify with reason whether the statement is true or false.
5. Derive the power balance equation in a basic generator from the original energy balance equation

Multiple-choice Questions

1. If the field of a DC shunt motor gets opened while the motor is running, then
   (a) Speed of the motor will reduce
   (b) Motor will attain dangerously high speed
   (c) Armature current will drop
   (d) Armature will oscillate about original speed as the mean speed
2. A cumulatively compound DC generator is supplying 20 A at 200 V. Now, if the series field winding is short circuited, the terminal voltage
   (a) Will remain unaltered at 200 V
   (b) Will rise to 220 V
   (c) Will shoot up to very high value
   (d) Will become less than 200 V

Descriptive Questions

1. Explain the tests to be conducted on three isolated secondary windings of a three-phase transformers for connecting them in (i) star (ii) delta
2. Compare between the use of a bank of three identical single-phase transformers and a single three-phase transformer unit for use as three-phase transformer.
3. What are different phasor groups? Why are pha-

9. How Group-3 and Group-4 transformers can be made to run in parallel?
10. What is open delta connection? Explain its utility.
11. In open-delta transformers, show that the secondary line voltages form a balanced three-phase system of voltages, in case the supply voltages are balanced.

Numerical Questions

1. A three-phase, 3.3 kV, 24-pole, 50 Hz, star connected induction motor has slip ring rotor. The rotor resistance and reactance per phase are 0.03 Ω and 0.5 Ω respectively. Determine the following: (i) The speed at maximum torque and ii) Ratio of full-load torque to maximum torque if full-load torque is obtained at 244 rpm. **[Ans: (i) 235 rpm (ii) 0.69]**
2. A three-phase, star connected, 440 V, 50 Hz, four-pole induction motor has the following per phase constants in ohms referred to stator:
   $r_1 = 0.2$, $x_1 = 0.5$, $r_2 = 0.2$, $x_2 = 0.5$, $X_m = 30$
   Fixed losses (Core, Friction and Windage) are 500 W.
   Calculate the stator current, rotor speed and output torque of the motor when it is operated at rated volt-
3. A three-phase induction motor has starting torque of 100% and a maximum torque of 200% of full-load torque. Find slip at maximum torque. **[Ans: S_{mt} = 0.268]**
4. A three-phase, 3.3 kV, 24-pole, 50 Hz, star connected induction motor has slip ring rotor. The rotor resistance and reactance per phase are 0.03 Ω and 0.5 Ω, respectively. Determine the speed at maximum torque. What resistance is to be added to give 75% of maximum torque at starting? Neglect stator impedance. **[Ans: 235 rpm, Δr_2 = 0.195 Ω]**
5. A three-phase, eight-pole, 60 Hz, I.M is deliberately loaded to a point where pull out will occur. The rotor resistance per phase is 0.4 Ω and motor stalls at 680 rpm. Calculate (i) breakdown slip (ii)

Chapter-end Exercises

Abundant chapter-end exercises including Multiple-choice Questions with answers, Critical Thinking Questions, Descriptive Questions, and Numerical Questions with answers are provided for readers to test their knowledge and their understanding of the concepts.

Appendix

A

A.1 TROUBLESHOOTING AND MAINTENANCE OF TRANSFORMERS

Unlike in rotating machines, transformers do not have any moving or rotating parts, except for tap changers. It is thus seems apparent that transformers are subjected to much less chances of failuires and maintenance requirements are low. This is true to some extent if the transformer is not overloaded. Even then, due to heating, the internal parts of a transformer undergo thermal, chemical, mechanical, and obviously electrical stress the whole time it is under operation. It is thus recommened that transformers should be periodically inspected to ensure their health, especially health of the insulating materials used so that their operating life can be prolonged. Table A.1 summarizes some of the possible abnormal operating states of a power transformer, its possible causes and suggetsed remedied.

Appendix

In order to enhance the knowledge further, an appendix is provided at the end of the book.

Preface

Electrical Machines is one of the more traditional fields of electrical and allied engineering disciplines. It forms a part of the curriculum of most of the core electrical engineering courses as well as that of their allied courses. This book presents a comprehensive treatment of the subject as required for undergraduate students of B. Tech/BE in electrical engineering and other branches. It is also intended to be used as a textbook for the course in Electrical Machines in other branches.

This book *Electrical Machines* is the fruition of our wish to document the knowledge we gathered from our teachers during our undergraduate studies and from industry experience. Moreover, the dearth of electrical machines books to prepare for university examinations and technical job interviews further prompted us to write this book. We have tried to make the language and approach in the book simple and straightforward so that students can have lucid reading.

About the Book

This book offers a balance between theoretical and analytical approach. It includes practical illustrations along with computational approach for solving various critical thinking and numerical problems. Some of the topics dealt with in the book essentially involve mathematics, but throughout the book computations are kept as simple as possible. Photographs of real systems have been used in places where schematic representation needed to be augmented.

The prerequisite for the reader of this book is that he/she should have had introductory courses on basic calculus, vector/phasor analysis, transform theory, circuit analysis, basic electrical measuring instruments and techniques, and elementary mechanics. The text is organized into 15 chapters broadly based under the following heads:

1. Basic electromagnetic concepts and general concepts of electrical machines
2. DC generators and motors
3. Single-phase and three-phase transformers
4. Three-phase and single-phase induction motors
5. Synchronous generators and motors
6. Special machines

Features of the Book

The following are the salient features of the book:

- Lucid and *simple language*
- Additional stress on *constructional features* of machines, thereby enhancing physical understanding
- Important concepts highlighted as *boxed items* in each chapter
- Wealth of solved *numerical examples* to supplement the text
- *Numerous solved MATLAB* examples
- Chapter-end exercises comprising *objective-type questions with answers, critical thinking type questions, broad answer type questions, and numerical exercises with answers*
- *Chapter outlines* and *learning outcomes* listed at the start of each chapter
- Point-wise *summary* at the end of each chapter to enable quick revision
- *Pedagogy includes* as many as 314 solved examples, 303 short and long-answer questions, 159 MCQs, and 137 numerical problems

Organization of the Book

The content of the book is organized into 15 chapters and one appendix:

Chapter 1 provides an introduction to magnets and magnetism. It starts with the basics of electromagnetism and then discusses the basic laws of electromagnetism and electromagnetic induction. Properties of magnetic materials and their types for use in electrical machines are also discussed.

Chapter 2 discusses the basic principles of electrical machines and their general concepts including the construction, working principles, and operation. It also lists the different conducting, insulating, and magnetic materials and their properties for use in electrical machines.

Chapter 3 deals with the construction, basic principle, and operation of DC machines. It also discusses the basic guiding equations in DC generators and motors. The chapter also highlights the effects of armature reaction and the methods adopted to compensate armature reaction.

Chapter 4 provides theoretical techniques for predicting the operational characteristics of DC machines. The chapter emphasizes on practical implications of these operational characteristics, as predicted theoretically.

Chapter 5 describes techniques adopted for starting, speed control, and braking of DC motors. It also discusses the various losses that take place in DC machines and the testing techniques followed to estimate their efficiency.

Chapter 6 presents the basic principles, construction, and operation of single-phase transformers. It includes phasor diagram, equivalent circuit, and tests to be performed on two-winding transformers and also discusses the features of auto-transformers. The chapter also details the processes of connecting two or more transformers in parallel, methods of measuring and calculating losses and efficiency, and regulation of single-phase transformers.

Chapter 7 starts with basic constructional features of three-phase transformers and then discusses types of connections including various possible vectors groups. Conditions for parallel operation of three-phase transformers are highlighted. Special three-phase transformer connections such as the open-delta connection, Scott connection, three-phase to six-phase, and 12-phase conversion connections are described. The chapter also discusses the excitation phenomenon, sources of harmonics, and ways of mitigating those in three-phase transformers.

Chapter 8 deals with the construction and operation of three-phase induction motors. It also discusses power flow equations in three-phase induction motors, and also derives the expression for developed torque.

Chapter 9 goes a step forward towards analysis and performance of three-phase induction motors. Phasor diagrams and equivalent circuits are developed. It explains the different parts of the torque–speed characteristics. The chapter details the tests performed on three-phase induction motors and explains ways of developing the circle diagram. It also discusses the different performance characteristics of three-phase induction motors and ends with the effects of abnormal operating conditions.

Chapter 10 introduces the starting, speed control, and braking methods adopted in three-phase induction motors. It also details special constructions available for three-phase induction motors for achieving high-starting torques.

Chapter 11 presents the principle of construction and operation of synchronous generators, that is, alternators. Expression for EMF induced and factors governing its magnitude and frequency are highlighted in the chapter. Phasor diagram and equivalent circuit of alternators are also developed. Tests, performed for identifying equivalent circuit parameters and calculation of voltage regulation, are discussed in detail. Effect of armature reaction on alternator output is also highlighted. This chapter also covers the power equations in both cylindrical rotor type and salient-pole rotor type alternators.

Chapter 12 introduces the necessity, conditions, and procedures for connecting alternators in parallel, either to one another or to an infinite bus. It also discusses the methods of synchronizing, sharing of load, and effects of variations of excitation and mechanical power input on alternator performance. The chapter also discusses transient conditions, hunting, and other stability issues in alternators. The chapter ends with losses and efficiency calculations for alternators.

Chapter 13 discusses the construction and operating principles of synchronous motors. The difficulties and remedies adopted for starting of such motors are discussed in detail. The chapter ends with notes on various applications of synchronous motors.

Chapter 14 discusses the methods of starting a single-phase induction motor and ways of improving the starting performance. It also includes the equivalent circuit of a single-phase induction motor and tests performed for identification of the equivalent circuit parameters. Power flow relations in single-phase induction motors are also included.

Chapter 15 presents the construction, operating principles, performance characteristics, and applications of various special types of machines. These include AC commutator motor, universal motor, Schrage motor, switched reluctance motor, brushless DC motor, hysteresis motor, stepper motor, tacho-generator, synchro, servo motor, linear induction motor, and induction generators.

Appendix A discusses the troubleshooting and maintenance of transformers and national and international standards relevant to electrical machines.

Online resources

To aid teachers and students, the book is accompanied with online resources that are available at https://india.oup.com/orcs/9780199472635. The content for the online resources is as follows:

For Faculty

- Chapter-wise PPTs
- Solutions Manual
- Laboratory Experiments Manual
- Machine Images

For Students

- MCQ Test Generator
- Additional MATLAB codes
- Additional Reading Material
- Laboratory Experiments Manual
- Machine Images

Acknowledgement

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One of us (Prithwiraj Purkait) would not miss putting in record the enlightenment he received from his teachers during undergraduate engineering course whose class notes he still preserves as treasure.

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Last but not the least, we would like to thank the editorial team at Oxford University Press, India for their help and support.

Comments and suggestions for the improvement of the book can be sent to me at praj@ieee.org.

Prithwiraj Purkait

Indrayudh Bandyopadhyay

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6. Single-phase Transformers 278

2 Basic Concepts of Electrical Machines

CHAPTER OUTLINE	• Recognize the fundamentals of electromechanical energy conversion. • Compute force and torque developed in electromechanical systems. • Discuss the general concepts of electric generators and motors. • Describe the constructional details and performance of different types of coils and windings available for rotating electrical machines. • Classify different magnetic, conducting, and insulating materials used in electrical machines.
LEARNING OUTCOME	After going through this chapter, the students will be able to: • Employ the principles of electromechanical energy conversion to describe basic operation of electric generators and motors. • Determine the force and torque developed in electromechanical systems of varying configuration. • Choose appropriate winding scheme for different rotating machines and appraise their performance. • Determine the type of material to be used for construction of a machine following efficiency, performance, and environmental standards.

INTRODUCTION

Michael Faraday's most outstanding discoveries in electromagnetism during 1830s form the very early origins of transformers, alternators, and DC generators. He had a ring made of soft iron over which two copper coils were wound. Upon making and braking current through one of the coils, he observed disturbance of a magnetic needle placed near the second coil. This is considered as the first model of a transformer where interrelationship between electricity and magnetism was observed. In 1831, Faraday performed one more series of experiments in which he passed a coil wound on a hollow paper cylinder swiftly over a cylindrical bar magnet. Ends of the coil were connected to a galvanometer. The galvanometer pointer showed deflection as long as the coil was in motion. This was the first 'alternator' where a relative motion between magnet and conductor produced electricity. In the same year 1831, Faraday discovered a rotating machine that can be thought as the origin of DC generator. He had a revolving copper disc placed in the small gap between a pair of magnetic poles. To make contact with the revolving plate, he amalgamated two metal brushes with mercury, one at the end of the disc and the other on the axle. The two brushes were connected to a galvanometer. The galvanometer showed steady deflection, indicating production of direct current as long as the disc was revolved.

Faraday's achievements in 1831 led to the development of practical electrical machines—dynamos, alternators, and motors that could convert mechanical energy to electrical energy interchangeably in both ways.

In fact, the idea of interaction of electric and magnetic fields as demonstrated by Hans Christian Oersted and André-Marie Ampère back in the 1820s was the basis for development of motors and generators in the years to follow. For over more than 100 years, eminent scientists, physicists and engineers whose long list contain names like Peter Barlow (1822), François Arago (1824), Joseph Henry (1831), William Sturgeon (1833), Joseph Saxton (1833), Thomas Davenport (1837), Werner Siemens (1856), Walter Baily (1879), Galileo Ferraris (1885), Nikola Tesla (1889), Mikhail DolivoDobrovolskyn (1890) and many more contributed to development of the motors and generators that we see today.

In this chapter, the basic principles of electromechanical energy conversion will be described, followed by basic concepts of rotating electrical machines, their basic features and functionalities will be highlighted.

2.1 ELECTROMECHANICAL ENERGY CONVERSION

Different components of a typical electromechanical energy conversion system and their interrelationships are pictorially shown in Figure 2.1. As illustrated in Figure 2.1, an electromechanical system consists of the following three basic components:

- An electrical subsystem (electric circuits such as windings)
- A magnetic subsystem (magnetic field in the magnetic cores and air gap)
- A mechanical subsystem (mechanically movable parts such as a plunger in a linear actuator or as rotor in a rotating electrical machine)

Voltages and currents are used to describe performance of the electrical subsystem and they are governed by the basic circuital laws, namely: Ohm's law, KCL and KVL. State of the mechanical subsystem can be described in terms of torque and speed, and is governed by the Newton's laws. The magnetic subsystem or magnetic field that acts as a *coupling* between the electrical and mechanical subsystems is described by quantities such as magnetic flux, flux density, and field strength. These are governed by the Maxwell's equations.

When coupled with an electric circuit, the magnetic flux interacting with the current in the circuit would produce a force or torque on a mechanically movable part. On the other hand, the movements of the moving part will cause variation of the magnetic flux linking the electric circuit and induce an electromotive force (EMF) in the circuit.

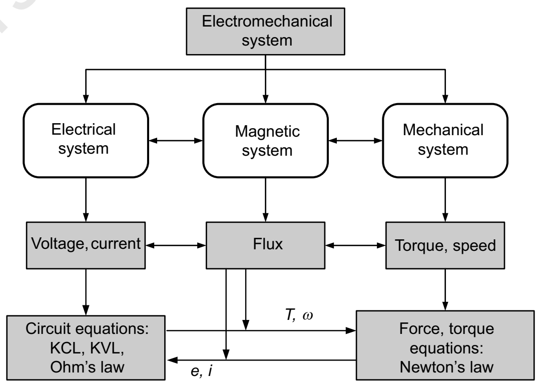


Fig. 2.1 Mapping of electromechanical energy conversion components

Basic operating principle of all rotating electrical machines is based on conversion of electrical energy to mechanical energy and vice versa. This conversion of one form of energy to the other form generally takes place through a coupling medium, either a magnetic field or an electric field. In a conventional motor, electrical energy is drawn from the supply which is converted first to magnetic energy in the coupling medium and then converted back to mechanical energy in the rotating part of the machine to drive mechanical loads such as line shafts or machine tools. On the other hand, when the machine is driven mechanically by a prime mover such as engine or turbine, the mechanical energy is transmitted through a magnetic field to be converted to electrical energy from the generator output terminals. The electromechanical energy conversion process is reversible, except for the losses in the system that may take place during the conversion process. The term 'reversible' implies that the energy can be transferred back and forth between the electrical and the mechanical systems.

2.2 ENERGY BALANCE DURING ELECTROMECHANICAL ENERGY CONVERSION

The *principle of conservation* of energy holds good for electromechanical energy conversion systems, as is expected for any energy conversion process. Since the mass of electrical machine does not change during its operation, the input energy must equal the output energy; energy can neither be created nor be destroyed.

During any such energy conversion process, in addition to the energy stored in coupling magnetic field, various losses also take place as power flows from input source to the output load. For example, in a motor as input power flows through the motor coils, then through to the coupling magnetic field, and finally out of the motor shaft, there will be ohmic losses (I^2R) in coils, hysteresis, and eddy current loss in magnetic core, and finally frictional losses due to moving parts. All of these losses will produce heat, raise temperature of the machine and reduce its useful output, thereby reducing efficiency of the energy conversion system.

The energy balance equation during such a conversion process in a motor can be developed keeping in mind the principle of conservation of energy as:

$$\begin{bmatrix}\text{Electrical Energy}\\ \text{Input}\end{bmatrix} = \begin{bmatrix}\text{Mechanical Energy}\\ \text{Output}\end{bmatrix} + \begin{bmatrix}\text{Energy Stored in}\\ \text{Magnetic Field}\end{bmatrix} + \begin{bmatrix}\text{Energy}\\ \text{Losses}\end{bmatrix} \tag{2.1}$$

These same losses will take place in a generator operation as well, only difference being the direction of flow of energy. In a generator, mechanical energy is supplied as input, that flows through the coupling magnetic field to be finally converted to electrical energy as output. Energy balance equation for a generator can hence be written as:

$$\begin{bmatrix}\text{Mechanical Energy}\\ \text{Input}\end{bmatrix} = \begin{bmatrix}\text{Electrical Energy}\\ \text{Output}\end{bmatrix} + \begin{bmatrix}\text{Energy Stored in}\\ \text{Magnetic Field}\end{bmatrix} + \begin{bmatrix}\text{Energy}\\ \text{Losses}\end{bmatrix} \tag{2.2}$$

Note the similarity between the energy conversion equations for motor Eq. (2.1) and generator Eq. (2.2). In fact, one single equation, say Eq. (2.1) can be used for both motors as generators if we consider that electrical and mechanical energy terms have positive values for motor action, while they have negative values for generator actions.

If the total losses be divided among three groups, (a) ohmic losses as electrical loss, (b) core losses as magnetic loss, and (c) friction losses as mechanical loss, then the energy balance equation of Eq. (2.1) can be rewritten as:

$$\begin{bmatrix}\text{Electrical Energy Input}\\ -I^2R\ \text{Losses}\end{bmatrix} = \begin{bmatrix}\text{Mechanical Energy Output}\\ +\ \text{Friction Losses}\end{bmatrix} + \begin{bmatrix}\text{Energy Stored in Magnetic Field}\\ +\ \text{Core Losses}\end{bmatrix} \tag{2.3}$$

The term on left hand side of Eq. (2.3) indicates the useful electrical energy available after ohmic losses that is being passed on to the coupling field for onward conversion. If the supply voltage is v and input current is i, passing through winding resistance of r, then the incremental amount of useful electrical energy reaching the coupling field within a small time dt is given by:

$$dE_e = vidt - i^2rdt = (v - ir)idt \tag{2.4}$$

This energy when reaches the coupling magnetic field, will induce a voltage such as the supply voltage is opposed. This induced EMF $\left(e = -\dfrac{d\psi}{dt}\right)$ will have instantaneous magnitude of $e = (v - ir)$ in order to balance the voltage properly.

The first term on right hand side of Eq. (2.3) indicates the useful mechanical energy output plus the mechanical losses. These two together can be termed as the total energy converted to mechanical form, or in other words the total mechanical energy *developed*. In the small time dt, the incremental amount of energy converted to mechanical energy is denoted by the symbol dE_m.

The second term on right hand side of Eq. (2.3) is the total energy absorbed by the coupling magnetic field that includes energy stored in it as well as the magnetic losses. This incremental energy is denoted by dE_f.

Thus, Eq. (2.3) can be rewritten in differential form as:

$$dE_e = eidt = dE_m + dE_f \tag{2.5}$$

2.3 FORCE AND TORQUE IN ELECTROMECHANICAL SYSTEMS

All electrical equipments that operate on the principles of electromechanical energy conversion, be it rotating or not, produce some kind of force or torque due to interaction of magnetic field with current carrying conductors. In devices such as relays, electromagnets, moving-iron instruments etc., there is only one excitation winding that creates the operating magnetic field and hence are called *singly-excited* devices. These devices have limited range of movement or rotation. In rotating machines such as DC machines, synchronous machines, or even in energy meters and wattmeters, there are more than one independent excitation coils that make them *doubly-excited* or even sometimes *multiply-excited* magnetic systems.

2.3.1 Singly Excited Magnetic System

Consider the magnetic coupling of Figure 2.1 to be composed of a loss-less magnetic system as shown in Figure 2.2 with a single exciting winding placed on core of the magnetic structure. A movable plunger fixed at one end is placed in the air gap so that it can move in or out of the air gap linearly under magnetic force exerted by the electromagnet.

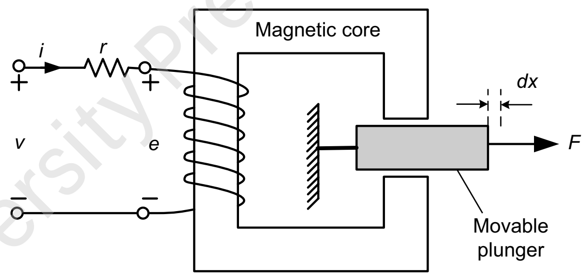


Fig. 2.2 Singly-excited linear electromechanical energy conversion system

Let a supply voltage v be supplied to the exciting winding that has a lumped resistance r as shown in Figure 2.2. After drop in the resistance, a voltage of e is available for conversion of electrical energy to magnetic energy, i.e., $e = (v - ir)$. This voltage available after drop in the resistance is responsible for producing magnetism in the magnetic core and is balanced by the opposing EMF induced in the coil $\left(e = -\frac{d\psi}{dt}\right)$ by the electromagnetic field. Let, under action of this magnet, the plunger moves by a small distance dx within a small time dt in the direction shown under action of the force F.

Thus, mechanical work done by the force acting on the plunger during this time is

$$dE_m = Fdx \tag{2.6}$$

From Eq. (2.4), electrical energy supplied after loss in the winding resistance is

$$dE_e = vidt - i^2 rdt = (v - ir)idt \tag{2.7}$$

From the relation: $|e| = |(v - ir)| = \left|\frac{d\psi}{dt}\right|$

we can obtain an alternate expression for the electrical energy supplied as:

$$dE_e = (v - ir)idt = eidt = id\psi \tag{2.8}$$

Now go back to the energy balance equation of an electromechanical system derived in Eq. (2.5), so that total incremental energy stored in the magnetic field can be expressed as:

$$dE_f = dE_e - dE_m = id\psi - Fdx \tag{2.9}$$

Equation (2.9) indicates that energy stored in the magnetic field is a function of flux linkage ψ of the exciting winding and displacement x of the plunger. Mathematically, such dependence of the magnetic energy stored on ψ and x can be written in partial differential form as:

$$dE_f(\psi,x) = \frac{\partial E_f(\psi,x)}{\partial \psi} d\psi + \frac{\partial E_f(\psi,x)}{\partial x} dx \tag{2.10}$$

Comparing Eqs. (2.9) and (2.10), we express current and force in terms of the magnetic energy stored as:

$$i = \frac{\partial E_f(\psi,x)}{\partial \psi} \quad \text{and} \quad F = -\frac{\partial E_f(\psi,x)}{\partial x} \tag{2.11}$$

From basic electromagnetic principles (Section 1.8.2), energy stored in an electromagnet can be expressed as:

$$E_f = \int dE_f = \int_0^{\psi} i\,d\psi \tag{2.12}$$

When the energy stored in the magnetic field is a function of flux linkage ψ of the exciting winding and displacement x of the plunger, then Eq. (2.12) is re-written as:

$$E_f(\psi,x) = \int_0^{\psi} i(\psi,x)\,d\psi \tag{2.13}$$

In the configuration shown in Figure 2.2, reluctance of the air gap is much higher as compared to reluctance of the flux path through magnetic core. Thus, for all practical purposes, the magnetic circuit behaves predominantly like that of air, i.e., it obeys linear magnetization characteristics with most of the magnetic energy being stored in the air gap. For such a linear magnetic path and for reasonably limited range of movement of the plunger such that reluctance to the flux path does not change, the flux linkage ψ and exciting current i are linearly proportional. The inductance value is thus independent of exciting current and hence independent of flux linkage ψ, but will, however, depend on displacement x of the plunger. Current I can then be expressed as a function of flux linkage and self-inductance of the coil:

$$L(x) = \frac{\psi}{i(\psi,x)} \quad \text{or} \quad i(\psi,x) = \frac{\psi}{L(x)} \tag{2.14}$$

Thus, magnetic energy stored from Eq. (2.13) for a linear magnetic material can be written as:

$$E_f(\psi,x) = \int_0^{\psi} \frac{\psi}{L(x)}\,d\psi \quad \text{or,} \quad E_f(\psi,x) = \frac{1}{2}\frac{\psi^2}{L(x)} \tag{2.15}$$

Electromechanical force Eq. (2.11) acting on the movable plunger thus can be expressed as:

$$F = -\frac{\partial E_f(\psi,x)}{\partial x} = -\frac{d}{dx}\left[\frac{1}{2}\frac{\psi^2}{L(x)}\right]$$

or, $$F = -\frac{1}{2}\psi^2 \frac{d}{dL}\left[\frac{1}{L(x)}\right]\frac{dL(x)}{dx} \quad \text{or,} \quad F = \frac{1}{2}\frac{\psi^2}{L(x)^2}\frac{dL(x)}{dx}$$

or, $$F = \frac{1}{2}\left[\frac{\psi}{L(x)}\right]^2 \frac{dL(x)}{dx} \quad \text{or,} \quad F = \frac{1}{2} i^2 \frac{dL(x)}{dx} \tag{2.16}$$

An alternate method for finding out expression for electromechanical force on the plunger is available using the concept of energy and co-energy in a magnetic field as discussed in Section 1.8.2.

Recall the ψ versus i curve of the magnetic material (reproduced in Figure 2.3), where area under the curve adjacent to ψ axis is the magnetic energy stored E_f, while area under the curve adjacent to i axis E'_f was termed as the co-energy.

At any operating point (ψ, i) as indicated in Figure 2.3, we have:

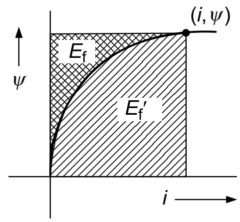


Fig. 2.3 Energy stored and co-energy

$$E_f + E'_f = i\psi \tag{2.17}$$

Note that E_f is the area with the ψ axis, and E'_f is the area with i axis. Thus, E_f can be written as a function of ψ, while E'_f as a function of i; both being functions of displacement x of the plunger as well. Thus, Eq. (2.17) is re-written is the form: $E_f(\psi, x) + E'_f(i, x) = i\psi$ (2.18)

In differential form: $dE_f(\psi, x) + dE'_f(i, x) = \psi di + id\psi$ (2.19)

Incremental co-energy is thus: $dE'_f(i, x) = \psi di + id\psi - dE_f(\psi, x)$ (2.20)

Recall from Eq. (2.8) that $dE_f = id\psi - Fdx$

Hence, co-energy in Eq. (2.20) is re-written in terms of the electromechanical force developed as:

$$dE'_f(i, x) = \psi di + Fdx \tag{2.21}$$

Alternately, $dE'_f(i, x)$ in partial derivative form is expressed as:

$$dE'_f(i, x) = \frac{\partial E'_f(i, x)}{\partial i}di + \frac{\partial E'_f(i, x)}{\partial x}dx \tag{2.22}$$

Therefore, comparing Eqs. (2.21) and (2.22) we express flux linkage and force in terms of the magnetic co-energy as: $\psi = \dfrac{\partial E'_f(i, x)}{\partial i}$ (2.23) and $F = \dfrac{\partial E'_f(i, x)}{\partial x}$ (2.24)

Expression for co-energy is derived as the area underneath the curve adjacent to i axis:

$$E'_f(i, x) = \int_0^i \psi(i, x)\, di \tag{2.25}$$

Considering the magnetic path to have linear characteristics as before we express Eq. (2.25) in terms of self-inductance with $L(x) = \dfrac{\psi(i, x)}{i}$, i.e., $\psi(i, x) = iL(x)$, such that:

$$E'_f(i, x) = \int_0^i iL(x)\, di = \frac{1}{2}i^2 L(x) \tag{2.26}$$

Thus, final expression for electromechanical force derived from co-energy Eq. (2.25) can be obtained as:

$$F = \frac{\partial E'_f(i, x)}{\partial x} = \frac{d}{dx}\left[\frac{1}{2}i^2 L(x)\right] \quad \text{or,} \quad F = \frac{1}{2}i^2 \frac{d}{dL}\left[L(x)\right]\frac{dL(x)}{dx}$$

or, $$F = \frac{1}{2} i^2 \frac{dL(x)}{dx} \tag{2.27}$$

Comparing Eqs. (2.16) and (2.27), we see that either way we obtain the same expression for the electromechanical force developed.

According to Eq. (2.11), *electromechanical force developed can be expressed as the negative partial derivative of energy stored in the magnetic field against linear displacement*. On the other hand, Eq. (2.24) tells us that *the same force can also be expressed as positive partial derivative of co-energy of the magnetic system against linear displacement*.

This force always tends to move the plunger in a direction such that the air gap is reduced and hence stored energy in the system is reduced, i.e., the system always tries to achieve a stable position of minimum field energy.

Let A (in m^2) and l (in m) are cross sectional area and length the magnetic flux path at the air gap in Figure 2.2. Then the above expression of co-energy Eq. (2.26) can be written in a different way by expressing the self-inductance as: $L(x) = \frac{\psi(i,x)}{i}$

So that, Eq. (2.26) becomes: $E'_f(i,x) = \frac{1}{2} i^2 \frac{\psi(i,x)}{i} = \frac{1}{2} i\psi(i,x) = \frac{1}{2} iN\phi(i,x)$

(Since $\psi = N\phi$ where N is number of turns made over the magnetic core and ϕ is the flux flowing through the magnetic field.)

Let us now introduce the term *co-energy density* which is nothing but is co-energy divided by the total volume (Al) through which the flux is passing:

$$E'_V = \frac{E'_f}{Al} = \frac{1}{2}\frac{iN\phi}{Al} = \frac{1}{2}\frac{Ni}{l}\frac{\phi}{A} = \frac{1}{2}HB = \frac{1}{2}\mu H^2 \tag{2.28}$$

A singly-excited magnetic system can also have a rotating actuator as shown in Figure 2.4. The linearly movable plunger is replaced by a rotating element free to rotate in the air gap between the two pole faces. In a manner similar to the way force is developed to actuate linear displacement, torque is developed in a rotating system. Expression for the torque developed can be found out in exactly similar manner to obtain:

In terms of magnetic energy stored:

$$T = -\frac{\partial E_f(\psi,\theta)}{\partial \theta} \tag{2.29}$$

In terms of magnetic co-energy: $$T = \frac{\partial E'_f(i,\theta)}{\partial \theta} \tag{2.30}$$

And finally: $$T = \frac{1}{2} i^2 \frac{dL(\theta)}{d\theta} \tag{2.31}$$

where θ is the angular displacement of the rotor from axis of the stator poles as marked in Figure 2.4.

The torque acts in such a way that the rotor is rotated in a direction so as to bring the rotor in alignment with the stator poles, thereby reducing the reluctance offered to the complete flux path. Since this torque is created by variation in reluctance of the magnetic circuit, it is often referred to as the *reluctance torque*. If the rotor is not of the salient pole structure as shown in Figure 2.4, but is of a smooth cylindrical structure, then reluctance seen by the stator flux does not vary with rotation of the rotor, and hence *no reluctance torque* will be developed.

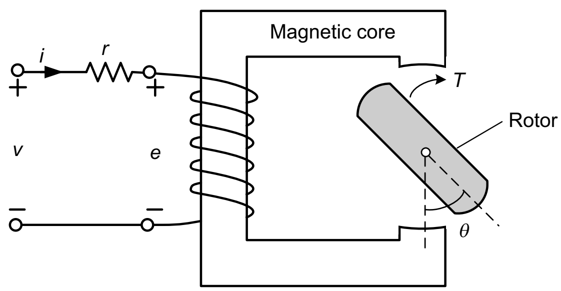


Fig. 2.4 Singly-excited rotating electromechanical energy conversion system

2.3.2 Doubly Excited Magnetic System

Model of a doubly excited magnetic system is shown in Figure 2.5. The stator is excited from a source that sends the exciting current i_1 through N_1 number of turns in the stator coil. The rotor also has an exciting coil with N_2 number of turns that is supplied from a second source that passes a current of i_2 through the rotor coil. As indicated in Figure 2.5, the rotor initial position is at an angle θ with respect to the stator pole axis. Due to interaction of the two magnetic fields, one from the stator and the second from the rotor, the rotor experience a torque that tends to rotate the rotor. Expression for the torque developed on rotor can be derived using concepts of magnetic energy and co-energy as before.

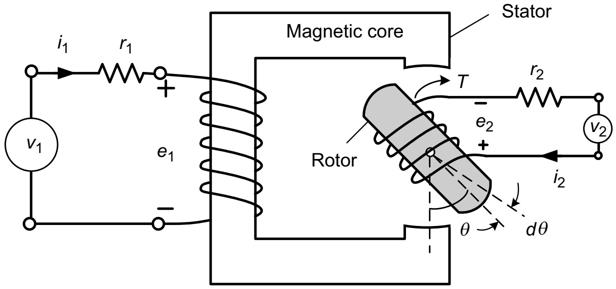


Fig. 2.5 Doubly excited rotating electromechanical energy conversion system

Let the rotor rotate by a small incremental angle $d\theta$ in a small time dt under influence of the torque.

Incremental amount of energy supplied to the magnetic field is due to combined action of two sources, one in the stator and the other in the rotor:

$$dE_e = e_1 i_1 dt + e_2 i_2 dt = i_1 d\psi_1 + i_2 d\psi_2 \tag{2.32}$$

where ψ_1 and ψ_2 are flux linkages in stator and rotor coils due to currents i_1 and i_2, respectively.

From energy balance concept in electromechanical systems, this input electrical energy can be expressed as summation of magnetic energy stored and mechanical energy developed:

$$dE_e = dE_f + dE_m \tag{2.33}$$

The differential amount of mechanical work done by the torque T when rotor moves by incremental angle $d\theta$ is

$$dE_m = T d\theta \tag{2.34}$$

Thus, expression for magnetic energy stored can be written for a doubly excited system as:

$$dE_f = dE_e - dE_m \quad \text{or,} \quad dE_f = i_1 d\psi_1 + i_2 d\psi_2 - T d\theta \tag{2.35}$$

Note that this energy stored in the magnetic field is a function of the two flux linkages ψ_1 and ψ_2, and is also dependent on angular position of rotor θ. Thus, in partial differential form, magnetic energy stored can be written as:

$$dE_f(\psi_1,\psi_2,\theta) = \frac{\partial E_f(\psi_1,\psi_2,\theta)}{\partial \psi_1} d\psi_1 + \frac{\partial E_f(\psi_1,\psi_2,\theta)}{\partial \psi_2} d\psi_2 + \frac{\partial E_f(\psi_1,\psi_2,\theta)}{\partial \theta} d\theta \tag{2.36}$$

Comparing Eqs. (2.35) and (2.36), the torque developed T can now be expressed in terms of magnetic energy stored as: $T = -\dfrac{\partial E_f(\psi_1,\psi_2,\theta)}{\partial \theta}$ (2.37)

Recall that magnetic energy stored and co-energy are related by the following expression [refer Eq. (2.17)]:

$$E_f + E_f' = i\psi = i_1\psi_1 + i_2\psi_2$$

Thus, magnetic co-energy has the expression: $E_f' = i_1\psi_1 + i_2\psi_2 - E_f$ (2.38)

In differential form: $dE_f' = \psi_1 di_1 + \psi_2 di_2 + i_1 d\psi_1 + i_2 d\psi_2 - dE_f$ (2.39)

Combining Eqs. (2.34) and (2.39), we have an alternate expression for differential co-energy:

$$dE_f' = \psi_1 di_1 + \psi_2 di_2 + T d\theta \tag{2.40}$$

Note that this co-energy denotes the area of ψ-i curve adjacent to the i axis. Thus, it is a function of the two currents i_1 and i_2, and is also dependent on angular position of rotor θ. Thus, in partial differential form, magnetic co-energy can be written as:

$$dE_f'(i_1,i_2,\theta) = \frac{\partial E_f'(i_1,i_2,\theta)}{\partial i_1}di_1 + \frac{\partial E_f'(i_1,i_2,\theta)}{\partial i_2}di_2 + \frac{\partial E_f'(i_1,i_2,\theta)}{\partial \theta}d\theta \tag{2.41}$$

Comparing Eqs. (2.40) and (2.41), the torque developed T can also be expressed in terms of magnetic co-energy as:

$$T = \frac{\partial E_f'(i_1,i_2,\theta)}{\partial \theta} \tag{2.42}$$

According to Eq. (2.36), *electromechanical torque developed in a doubly excited system can be expressed as the negative partial derivative of energy stored in the magnetic field against angular displacement.* On the other hand, Eq. (2.42) tells us that the *same torque can also be expressed as positive partial derivative of co-energy of the magnetic system against angular displacement*.

Neglecting magnetic saturation and hysteresis, the flux linkages ψ_1 and ψ_2 may be expressed in terms of self and mutual inductance of the two coils as: $\psi_1 = L_1 i_1 + M_{12} i_2$

$$\psi_2 = L_2 i_2 + M_{21} i_1 \tag{2.43}$$

where L_1 is self-inductance of stator winding, L_2 is self-inductance of rotor winding, and M_{12} (or M_{21}) is mutual inductance between stator and rotor windings.

$$L_1 = \frac{N_1^2}{S_1},\ L_2 = \frac{N_2^2}{S_2},\ M_{12} = M_{21} = M = \frac{N_1 N_2}{S_{12}} \tag{2.44}$$

where S_1, S_2, and S_{12} are the reluctances offered to stator flux, rotor flux, and resultant of stator and rotor flux respectively.

Assuming linear magnetization characteristics (linear ψ-i curve) for both stator and rotor as shown in Figure 2.6, the total magnetic energy stored in stator and rotor combined is given by summation of areas of the two triangles OABO of Figure 2.6(a) in stator and DEFD of Figure 2.6(b) in rotor:

$$E_f = E_{fs} + E_{fr} = \Delta\text{OABO} + \Delta\text{DEFD} = \frac{1}{2}i_1\psi_1 + \frac{1}{2}i_2\psi_2 \tag{2.45}$$

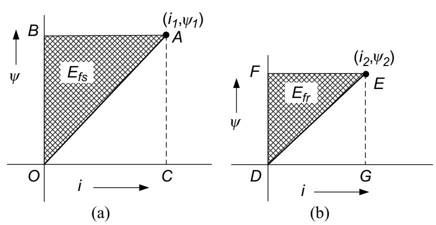


Fig. 2.6 Linear ψ-i characteristics indicating magnetic energy stored in (a) stator (b) rotor

Putting values of ψ_1 and ψ_2 from Eq. (2.43), we can write total magnetic energy stored as:

$$E_f = \frac{1}{2}i_1(L_1 i_1 + M_1 i_2) + \frac{1}{2}i_2(L_2 i_2 + M_2 i_1) \quad \text{or,} \quad E_f = \frac{1}{2}i_1^2 L_1 + \frac{1}{2}i_1 i_2 M_{12} + \frac{1}{2}i_2^2 L_2 + \frac{1}{2}i_1 i_2 M_{21}$$

Since, $M_{12} = M_{21} = M$, we have: $E_f = \frac{1}{2}i_1^2 L_1 + \frac{1}{2}i_2^2 L_2 + i_1 i_2 M$ (2.46)

When the stator and rotor windings are switched on and the rotor starts rotating, then the incremental change in energy stored in magnetic field can be calculated by differentiating Eq. (2.46) with respect to i_1, i_2 and also noting that L_1, L_2, and M will depend on position of the rotor θ. Thus:

$$dE_f(i_1,i_2,\theta) = i_1 L_1 di_1 + i_2 M di_1 + i_2 L_2 di_2 + i_1 M di_2 + \frac{1}{2}i_1^2 dL_1(\theta) + \frac{1}{2}i_2^2 dL_2(\theta) + i_1 i_2 dM(\theta) \tag{2.47}$$

From Eq. (2.32), the differential form of electrical energy supplied with respect to currents and rotor position can be written as: $dE_e(i_1,i_2,\theta)=i_1 d\psi_1+i_2 d\psi_2=i_1 d(L_1 i_1+M_{12} i_2)+i_2 d(L_2 i_2+M_{21} i_1)$

or, $dE_e(i_1,i_2,\theta)=i_1 L_1 di_1+i_1^2 dL_1(\theta)+i_1 M_{12} di_2+i_1 i_2 dM_{12}(\theta)+i_2 L_2 di_2+i_2^2 dL_2(\theta)+i_2 M_{21} di_1+i_1 i_2 dM_{21}(\theta)$

Putting $M_{12}=M_{21}=M$, we have:

$$dE_e(i_1,i_2,\theta)=i_1 L_1 di_1+i_1^2 dL_1(\theta)+i_1 M di_2+i_1 i_2 dM(\theta)+i_2 L_2 di_2+i_2^2 dL_2(\theta)+i_2 M di_1+i_1 i_2 dM(\theta)$$

or, $$dE_e(i_1,i_2,\theta)=i_1 L_1 di_1+i_1^2 dL_1(\theta)+i_1 M di_2+i_2 L_2 di_2+i_2^2 dL_2(\theta)+i_2 M di_1+2i_1 i_2 dM(\theta) \quad (2.48)$$

The differential amount of mechanical work done during incremental rotation of $d\theta$ is given from Eq. (2.33) by:

$$dE_m=Td\theta$$

From the law of conservation of energy we have:

$$dE_e=dE_m+dE_f \quad \text{or,} \quad dE_m=dE_e-dE_f$$

Subtracting Eqs. (2.47) from (2.48), we obtain:

$$dE_m=Td\theta=\frac{1}{2}i_1^2 dL_1(\theta)+\frac{1}{2}i_2^2 dL_2(\theta)+i_1 i_2 dM(\theta) \quad (2.49)$$

∴ Electromechanical torque developed can be expressed from Eq. (2.49) as:

$$T=\frac{1}{2}i_1^2\frac{dL_1(\theta)}{d\theta}+\frac{1}{2}i_2^2\frac{dL_2(\theta)}{d\theta}+i_1 i_2\frac{dM(\theta)}{d\theta} \quad (2.50)$$

In case the doubly excited system has translational motion instead of rotational motion, that is, one electromagnet is held stationary while the other electromagnet is free to move linearly on a plane due to interaction of their electromagnetic fields, then the electromechanical force (F) developed can be developed in a similar manner to obtain:

$$F=\frac{1}{2}i_1^2\frac{dL_1(x)}{dx}+\frac{1}{2}i_2^2\frac{dL_2(x)}{dx}+i_1 i_2\frac{dM(x)}{dx} \quad (2.51)$$

where, x is the amount of linear displacement of the movable element.

2.3.3 Electromagnetic Torque and Reluctance Torque

The total electromechanical torque produced by a doubly excited magnetic system is given by Eq. (2.50). In such a system, if only the stator is excited and no supply is given to the rotor coils, the rotor current $i_2 = 0$. Thus,

Eq. (2.50) reduces to: $T=\frac{1}{2}i_1^2\frac{dL_1(\theta)}{d\theta}$ (2.52)

Similarly, if only the rotor is excited and no supply is given to the stator, then stator current $i_1 = 0$. Torque

developed is then: $T=\frac{1}{2}i_2^2\frac{dL_2(\theta)}{d\theta}$ (2.53)

It is thus seen that in a doubly excited magnetic system, even if one of the coils is not energized, torque is still developed enabling the rotor to rotate in appropriate direction. This is because reluctance to the flux path changes with rotor position, i.e., self-inductance of stator and rotor are both dependent on the rotor position. These components of torque given by Eqs. (2.52–2.53) that are present in a doubly excited salient pole rotor structure as shown in Figure 2.5 are called *reluctance torques*. *In a salient pole structure, where the air gap between stator* *and rotor is different at different orientation of the rotor, reluctance torque is present due to tendency of magnetic flux to search for lowest reluctance path.* In either case, whether the stator or the rotor is excited, magnetic flux passing between stator and rotor always tend to follow a minimum reluctance path and thereby a reluctance torque is produced that rotates the rotor in clock-wise direction as shown in Figure 2.5.

If the salient pole rotor is now replaced by a smooth cylindrical rotor, then total torque on the rotor can be derived from Eq. (2.50) as following with both stator and rotor coils excited:

$$T = \frac{1}{2} i_2^2 \frac{dL_2(\theta)}{d\theta} + i_1 i_2 \frac{dM(\theta)}{d\theta} \tag{2.54}$$

Note that the reluctance torque term $T = \frac{1}{2} i_1^2 \frac{dL_1(\theta)}{d\theta}$ is absent in Eq. (2.54) because the reluctance S_1 seen by the stator flux does not change with movement of the rotor; surface of rotor being smooth cylindrical, making the air gap between stator poles and rotor surface constant. Thus, L_1 is constant and $\frac{dL_1(\theta)}{d\theta} = 0$.

In addition, if the stator poles surface facing the air gap is also cylindrical and stator coils are uniformly distributed around the cylindrical stator surface, then reluctance S_2 seen by the rotor flux also does not change with rotor position and hence $\frac{dL_2(\theta)}{d\theta} = 0$. In such a configuration with both stator and rotor surface facing each other are smooth cylindrical, then torque developed on the rotor reduces to

$$T = i_1 i_2 \frac{dM(\theta)}{d\theta} \tag{2.55}$$

This component of total torque $i_1 i_2 \frac{dM(\theta)}{d\theta}$ that depends on currents in both stator and rotor and also on the rate of change in mutual inductance M with respect to angular displacement θ is known as the *electromagnetic torque*. Electromagnetic torque can thus only be developed in a device when both stator and rotor carry current, and are mutually coupled.

Physically, production of electromagnetic torque can be explained by the fact that north and south poles developed by the stator electromagnet tends to attract the opposite poles created in rotor by the rotor electromagnet. Direction of rotation of the rotor due to electromagnetic torque thus depends on direction of current in both the coils.

Note that direction of reluctance torque does not depend on direction of current in either of the two coils, since in any case the rotor is always pulled to a direction where the air gap between stator and rotor can be made minimum.

2.4 GENERAL CONCEPTS OF ROTATING MACHINE

A device that converts mechanical energy into electrical energy by taking help of magnetic field is called generator. A device that converts electrical energy into mechanical energy using magnetic field is called *motor*.

2.4.1 Generator

According to the Faraday's law of electromagnetic induction, when a coil is rotated in a magnetic field, an EMF is induced in it. If a number of such coils are connected in series and they are collectively rotated in the magnetic field, then total EMF of summation of all coils is induced in the whole winding. Actually, this winding is placed on a laminated core and this total structure where EMF is generated is called the armature of the machine. The other portion of the machine where field flux is generated is called the field. When load is connected with armature terminal, then current starts to flow through the armature and an armature field is also created.

Figure 2.7 shows the basic configuration of a generator where a coil is made to rotate in a magnetic field by an external applied mechanical torque, T_m. Magnetic flux lines of forces are found to flow from N-pole to S-pole. According to Fleming right-hand rule, the direction of induced EMF (e) in the coil is shown in the figure. As an external load R_l is connected with the coil, so current (I) also starts to flow in same direction as that of induced EMF. According to Fleming left-hand rule, this current interacts with the magnetic field and generate an internal torque, T_e inside the machine which opposes the external supplied torque T_m. If R_l is not present and the circuit is open, then the applied mechanical torque is only equal to frictional torque T_f. Therefore, due to the presence of R_l, the applied mechanical torque is equal to:

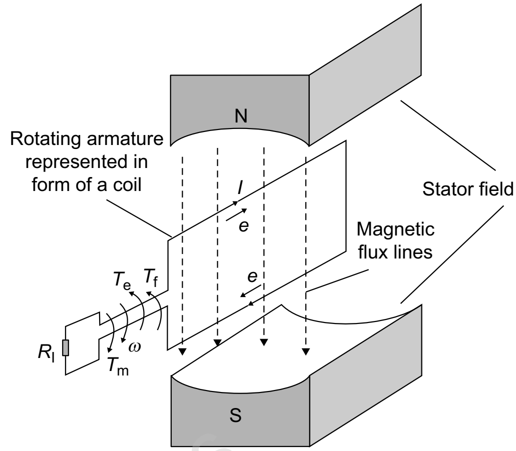


Fig. 2.7 Basic configuration of a generator

$$T_m = T_f + T_e \tag{2.56}$$

If, T_f is considered negligible, then: $T_m = T_e$

The current $I = \dfrac{e}{R_l + r}$, where r is resistance of the coil.

Then, $e = IR_l + Ir$

Multiplying both sides by current I, we have: $eI = I^2R_l + I^2r$

Here,

eI = Electrical power developed inside the machine

I^2R_l = Power consumed by the load

I^2r = Copper loss in the generator coil

Since power is defined as the rate of change of energy, and since under steady state the magnetic flux in an electrical machine is fairly constant, the rate of change of stored magnetic energy $\dfrac{dE_f}{dt} = 0$. Thus, the energy balance equation in a generator given by Eq. (2.2) can be modified to write the *power balance equation* under steady state as:

Electrical power developed inside the machine = Mechanical power supplied to the machine – Frictional losses

or, $$eI = (T_m - T_f)\omega = T_e\omega \tag{2.57}$$

where ω is speed of the rotor in radian per second.

Equation (2.57) denotes the equivalence between electrical power developed in the generator with the internal torque and speed of rotation.

2.4.2 Motor

Figure 2.8 shows a coil which is supplied from a DC source of V volt. The coil is placed in the magnetic field created by the N-pole and S-pole of the machine. The N-pole and S-pole constitute the stator field of the machine. When a number of such coils connected in series in the form a winding is placed on a magnetic core, then that is called rotor of the machine. For the time being, let the coil in Figure 2.8 is considered rotor of the machine. Magnetic core of the rotor is not shown in Figure 2.8 to enhance visual clarity.

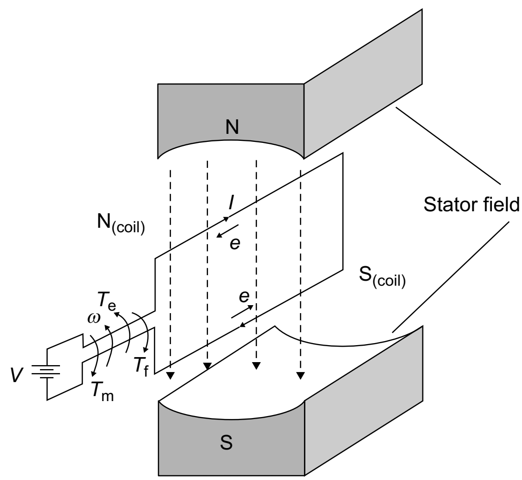


Fig. 2.8 Basic configuration of a motor

According to Fleming's left hand rule, the coil will experience a torque (T_e) in anticlockwise direction. Thus, the coil rotates in anti-clockwise direction. In order to maintain the rotation continuously, a special arrangement of brush and ring is made which is explained in more details in the chapter on DC machines. For the time being, it is assumed that rotation continues in anticlockwise direction. Now, according to Fleming's right hand rule, an EMF (e) is induced in the coil which opposes the supplied voltage (V) and therefore also opposes the supplied current.

Hence, the electrical equation associated with the coil is

$$V - e = Ir, \text{ where } r \text{ is resistance of the coil}$$

Multiplying both sides by current I, we have:

$$VI = eI + I^2 r$$

Here,

$I^2 r$ = Copper loss in the motor coil
eI = Electrical power available for conversion to mechanical power
VI = Electrical Power supplied

Thus, in the same way as for a generator, the *power balance equation* for a motor under steady state (with $\frac{dE_f}{dt} = 0$) can be written as:

Power supplied = Electrical power available for conversion to mechanical power + Copper loss

The torque (T_e) generated in anticlockwise direction needs to overcome the frictional torque, T_f and also has to carry the mechanical load torque, T_m on the motor. Therefore:

$$T_e = T_m + T_f \qquad \text{or,} \qquad \omega T_e = \omega T_m + \omega T_f \tag{2.58}$$

where ω is the rotational speed of the machine in radian per second.

In Eq. (2.58), ωT_e is the total mechanical power developed inside the machine which is available after conversion from electrical power.

Then,
$$eI = \omega T_e \tag{2.59}$$

Note the equivalence between Eq. (2.57) in a generator and Eq. (2.59) in a motor. This clearly indicates the interchangeability between a generator and a motor.

From the elementary definition of motor and generator, the following observations can be made:

(a) In case of generator, induced EMF e and current I are in same direction whereas in case of motor the induced EMF e and current I are in opposite directions.
(b) In case of generator, the internally developed torque, T_e is opposite to direction of rotation of the rotor or rotor speed ω, whereas in case of motor the internal developed torque, T_e and motor direction of rotation or rotor speed ω, is same.
(c) The direction of frictional torque T_f is always opposite to direction of rotation of the machine whether it is motor or generator.

2.5 PHYSICAL CONCEPT OF TORQUE PRODUCTION IN ELECTRICAL MACHINES

As explained earlier, in rotating machine, a North-South pole pair arises in both stator and rotor part of the machine. The North-South pole pair of the rotor always tries to align with that of the stator and hence a torque is produced in order to create this alignment. Two examples of how a continuous torque is created between stator and rotor are shown in Figures 2.9 and 2.10.

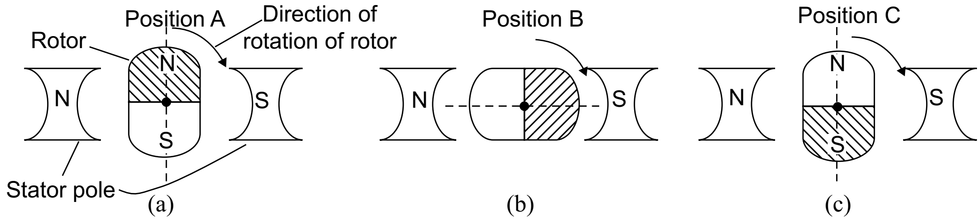


Fig. 2.9 Continuous torque production in a rotating machine by changing polarity of the rotor

Figure 2.9 shows that the arrangement in the rotor is so made that the polarity of a rotor is always at N-pole when it is at position A. When the rotor crosses the stator North-South pole axis, then the rotor pole strength is removed as shown in Figure 2.9(b). Due to inertia, the rotor crosses the stator axis and then the rotor polarity also gets interchanged as shown in Figure 2.9(c). So, there is a continuous torque in clockwise direction and rotation prevails in the system. This type of concept is used in DC machine.

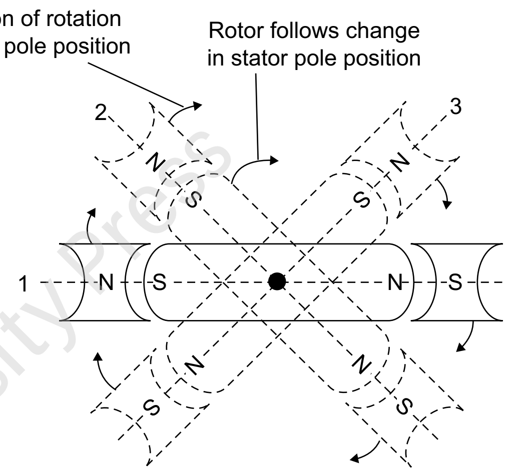


Fig. 2.10 Continuous torque production in a rotating machine by shifting stator pole position

In Figure 2.10 another way of rotor rotation is described. Here, the stator pole position is continuously shifted with time. Thus, it also exerts a continuous torque on the rotor and continuous rotation of the rotor is maintained. This mechanism is followed mainly in AC machines such as induction machine and synchronous machine.

To find the torque expression for a machine, a cross-sectional view of a two-pole machine with stator and rotor is shown in Figure 2.11(a). It can be assumed that a sinusoidally distributed magnetic field with respect to space angle is present in the air gap of the machine for both the field of stator and rotor. The axis of stator field is represented as stator MMF, F_s and the rotor field is represented as rotor MMF, F_r as shown in Figure 2.11(a and b). *The axis actually represents position of peak value of F_s and F_r*. The north-south pole position of both stator and rotor are kept on the axis of F_s and F_r respectively. Figure 2.11(c) shows the sinusoidal distribution of field MMF with respect to space angle in case of stator MMF, F_s. The position of N and S in Figure 2.11(c) is also shown at position of maximum MMF value.

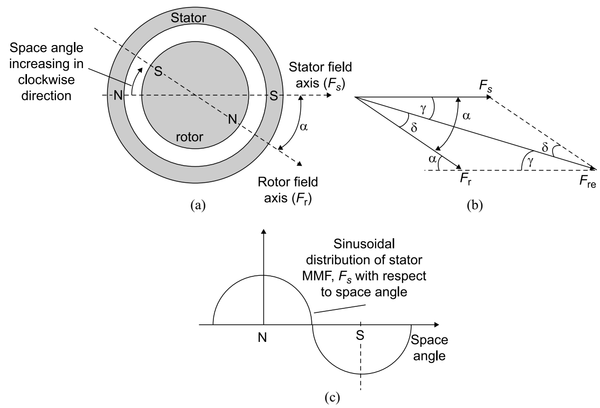


Fig. 2.11 (a) Stator and rotor of a machine and their respective field axis (b) Stator field, rotor field and resultant field (c) MMF distribution in space angle is considered to be sinusoidal

Later on, in detail discussions at various chapters, it will be found that for all the machines (DC machine, induction machine, synchronous machine, etc.) the angle between F_s and F_r will remain same when any machine is running under steady state. In brief, the following assumptions are made for determining the expression for torque production in a machine:

(a) Stator and rotor fields are sinusoidally distributed in the air gap between stator and rotor.
(b) The air gap is very less. So, all the flux lines are flowing radial and no tangential component is present.

(c) The field intensity H, radially along the air gap is constant in magnitude. Therefore, MMF across the air gap is: $F = H \cdot g$, where g is the radial air gap distance between stator and rotor.
(d) Because of all the above three assumptions, the MMF of both stator and rotor fields are sinusoidal and the flux density of both fields are also sinusoidal.

Let the angle between F_s and F_r be α. Then, the resultant MMF F_{re} can be expressed as:

$$F_{re}^2 = F_s^2 + F_r^2 + 2F_sF_r\cos\alpha \tag{2.60}$$

Reluctance of magnetic core part of the machine is considered to be negligible and only the reluctance of air gap length is considered. Therefore, *peak value* of resultant magnetic field intensity (H_{re}) at the air gap is

$$H_{re} = \frac{F_{re}}{g}$$

Average value of co-energy density in the air gap space according to Eq. (2.28) is:

$$E'_V = \frac{1}{2}\mu_0 \times [\text{average value of (field intensity)}^2]$$

For a sinusoidal distributed magnetic field, the field intensity in the air gap is also sinusoidally distributed around the space. Therefore, average value of (field intensity)^2 throughout the space angle $= \frac{1}{\pi}\int_0^{\pi}(H_{re}\sin\theta)^2 = \frac{1}{2}H_{re}^2$.

Then, average value of co-energy density over the air gap space $E'_V = \frac{1}{4}\mu_0 H_{re}^2$

Volume of the total air gap space $V = \pi DLg$, where D is mean diameter of the air gap; L is axial length of the stator or rotor.

Hence, putting value of $H_{re} = \frac{F_{re}}{g}$, total co-energy in the air gap can be found out as:

$$E'_f = \frac{\pi}{4}\mu_0 H_{re}^2 DLg = \frac{\pi}{4}\mu_0\left(\frac{F_{re}}{g}\right)^2 DLg = \frac{\pi}{4g}\mu_0 F_{re}^2 DL \tag{2.61}$$

Putting Eq. (2.60) in Eq. (2.61) we have:

$$E'_f = \frac{\pi}{4g}\mu_0 F_{re}^2 DL = \frac{\pi}{4g}\mu_0 DL\left[F_s^2 + F_r^2 + 2F_sF_r\cos\alpha\right] \tag{2.62}$$

Therefore, torque developed in the system following Eq. (2.42) is derived as *the positive partial derivative of co-energy of the magnetic system against angular displacement*:

$$T = \frac{dE'_f}{d\alpha} = -\frac{\pi}{2g}\mu_0 DLF_sF_r\sin\alpha \tag{2.63}$$

The above expression Eq. (2.63) is true for a two-pole machine.

In general, for P-pole machine, the torque expression can be modified as:

$$T = -\left(\frac{P}{2}\right)\frac{\pi}{2g}\mu_0 DLF_5F_r\sin\alpha \tag{2.64}$$

From the above expression, it is found that torque is proportional to the stator and rotor MMFs and the sine of angle between their axes. The negative sign in Eq. (2.64) indicates that the torque acts in a direction such that it tends to decrease the displacement angle α.

From Figure 2.11(b): $F_s\sin\alpha = F_{re}\sin\delta$ and $F_r\sin\alpha = F_{re}\sin\gamma$

The torque equation Eq. (2.64) can now be modified in alternate forms as:

$$T = -\left(\frac{P}{2}\right)\frac{\pi}{2g}\mu_0 DLF_r F_{re} \sin\delta \qquad (2.65)$$

$$T = -\left(\frac{P}{2}\right)\frac{\pi}{2g}\mu_0 DLF_s F_{re} \sin\gamma \qquad (2.66)$$

Thus, the torque expression can be again expressed as proportional to the stator MMF and the resultant MMF and sine of angle between them or it may be also be seen as proportional to the rotor MMF and the resultant MMF and sine of angle between them.

2.6 MECHANICAL DEGREE AND ELECTRICAL DEGREE

In Figure 2.12(a), a two pole AC generator is shown. The rotor of the machine is an electromagnet with fixed North and South poles. Previously, it was assumed that the MMF is sinusoidally distributed in space with respect to space angle (θ_m). Now, it is assumed that the flux (ϕ) distribution in space is also sinusoidal with respect to space angle. Further explanation on flux distribution is given in Chapter 11 on Construction, Principle, Operation and Performance of Synchronous Machine. When the rotor is at position I as shown in Figure 2.12, the flux is nearly zero at point near conductor A and maximum of value ϕ_m at position of stator (point X) just along the N-pole. So, EMF induced in conductor A at that instant is zero. Figure 2.12(b) shows the flux distribution curve for the rotor position I in bold line. As the rotor rotates from position I to position II, the distribution of flux in space also shifts from position I to position II (shown by dotted line) as shown in Figure 2.12(b). Thus, flux cutting with conductor A is found to increase sinusoidally. Then, EMF induced in conductor A will also increase sinusoidally. So, if the total time of one complete rotation of the rotor is T, then in time T, one complete sinusoidal flux wave will cut the conductor A. Then, one complete cycle of EMF wave is generated in conductor A within the same time T as given in Figure 2.12(d).

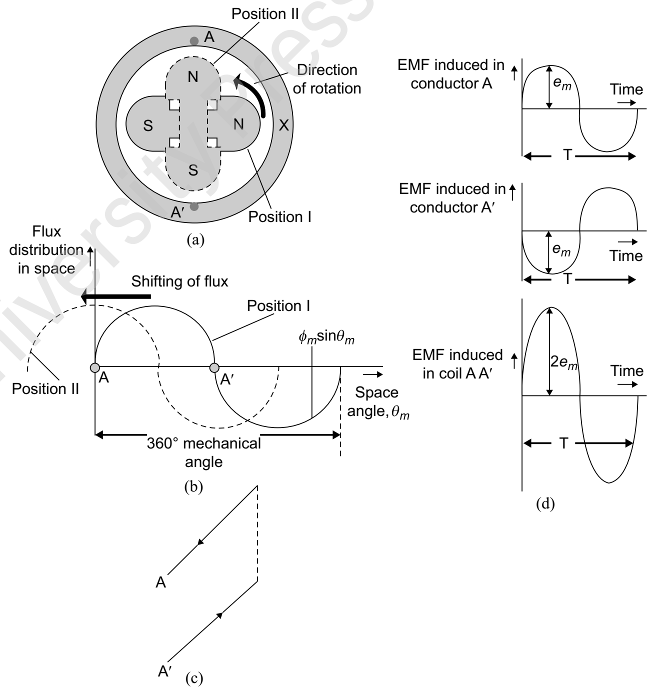


Fig. 2.12 Sinusoidal EMF generation in a coil of a two pole machine

Similarly, conductor A′ is under the influence of negative flux cutting as shown in Figure 2.12(b). The EMF induced in A′ is also sinusoidal but lagging by 180° to that of conductor A. Now, as shown in Figure 2.12(c), conductor A and A′ are connected in series from behind and therefore the induced EMF in the coil A-A′ is the summation of induced EMF of conductor A and A′ Figure 2.12(d).

Similarly, for a four pole machine (Figure 2.13(a)), it can be assumed that the flux distribution in space has two complete sine waves within the same 360° mechanical angle (Figure 2.13(b)). Then this flux distribution in space follows the function $\phi_m sin2\theta_m$. According to the present position of the rotor as shown in Figure 2.13(a), the distribution in space is shown by the bold line curve in Figure 2.13(b). Now, for one complete rotation of the rotor, two sinusoidal flux waves cut conductor A_1. If the total time taken for one complete rotation of the rotor is T, then in time T, two sinusoidal EMF waves are induced in conductor A_1 (Figure 2.13(c)). Figure 2.13(a) shows that position of conductor A_1 with respect to N-pole is same as position of conductor A_1' with respect to S-pole. Therefore, induced EMF in conductor A_1' is 180° out of phase with respect to conductor A_1. Then, the coil A_1-A_1', is induced with an EMF which is the summation of EMF induced in conductor A_1 and conductor A_1' (Figure 2.13(c)). Figure 2.13(a) also shows another pair of conductors A_2 and A_2'. The position of conductor A_2 is same as that of A_1 on the flux distribution graph (Figure 2.13(b)). Also, position of A_2' is similar to A_1'. Therefore, the coil A_2-A_2' also induces with the same EMF as that of coil A_1-A_1' (Figure 2.13(c)). Then coil A_1-A_1' and coil A_2-A_2', can be connected in parallel as shown in Figure 2.13(d).

In the above explanation, it is mentioned that perpendicular component of flux distribution in space for two pole machine is $\phi_m sin\theta_m$ and for four pole machine is $\phi_m sin2\theta_m$.Therefore, generalising for P number of poles, perpendicular component of flux distribution in space can be represented as $\phi_m \sin\frac{P}{2}\theta_m$. RMS value of this distribution is: $\phi_f = \frac{\phi_m}{\sqrt{2}}$

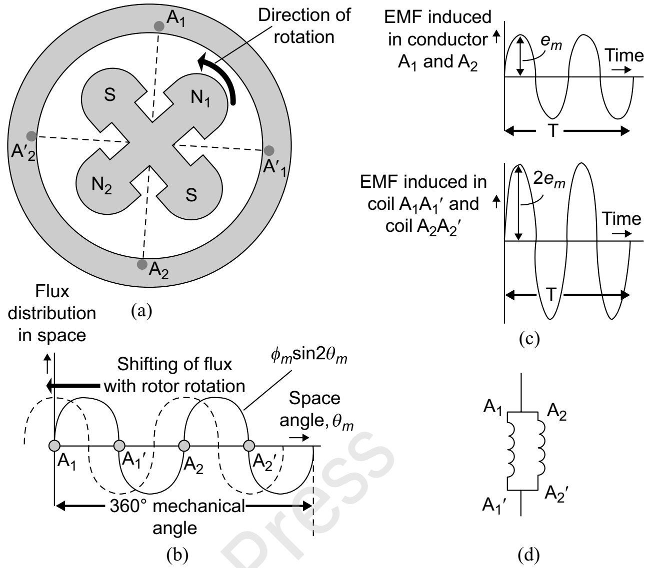


Fig. 2.13 Sinusoidal EMF generation in coils of a four pole machine

In addition, from Figure 2.12 and Figure 2.13, it can be seen that:

360° mechanical rotation generates $\frac{P}{2} \times 360°$angle of electrical wave.

$\Rightarrow$1° mechanical rotation generates $\frac{P}{2}$ degree angle of electrical wave

Therefore, when:

$P = 2$, 1° mechanical angle = 1° electrical angle
$P = 4$, 1° mechanical angle = 2° electrical angle
$P = 6$, 1° mechanical angle = 3° electrical angle

Therefore, in general:

1° electrical angle = $\frac{2}{P}$ degree mechanical angle

1 full cycle of electrical wave = $\frac{2}{P}$ cycle of mechanical rotation

If the frequency of generation of the electrical wave is f Hz, then:

f full cycles of electrical wave generated per second = $\frac{2 \times f}{P}$ cycle of mechanical rotation occurring per second

Thus to generate an electrical wave of frequency f Hz,

Speed of the rotor should be = $\frac{2 \times f}{P}$ rotation per second

or, speed of the rotor should be $\frac{120 \times f}{P}$ rotation per minutes (rpm).

This speed is called synchronous speed, N_s.

i.e., $$N_s = \frac{120 \times f}{P} \tag{2.67}$$

2.7 WINDING OF AC MACHINES

Normally, AC machines are of two types: Induction machine and synchronous machine. The winding nature of the armature (where EMF is generated) of the machine is, however, similar in both types of machine. For induction machine, the winding nature of the field (where magnetic flux is generated) is also same as that of the armature. For synchronous machine, the field arrangement is made by simple placement of concentric coils around a magnetic core. So, the main concern in any AC machine is how armature winding is to be arranged. Before going into the details of armature winding, certain important factors related to winding pattern is discussed in the following sections:

2.7.1 Distributed Winding—EMF Polygon and Distribution Factor

Figure 2.14(a) shows that a winding is formed in the outer stator by connecting three coils 1-1′, 2-2′, and 3-3′ in series. The rotor has a pair of poles with initial position as shown.Since each of the three coils are having same position with respect to the poles, induced EMF in each of them are same. These are represented by the phasors e_1, e_2, and e_3 for the coils 1-1′, 2-2′ and 3-3′, respectively, as shown in Figure 2.14(b). These three EMFs are shown to be all of same magnitude and are along the same phase. Then total EMF induced in the winding is summation of these three EMFs and is represented by e_{t1} which is equal to $3e_1$ (or $3e_2$ or $3e_3$). However, a practical machine does not have this type of concentrated (undistributed) winding arrangement. Rather, a distributed winding arrangement as shown in Figure 2.15(a) is present in all practical machines.

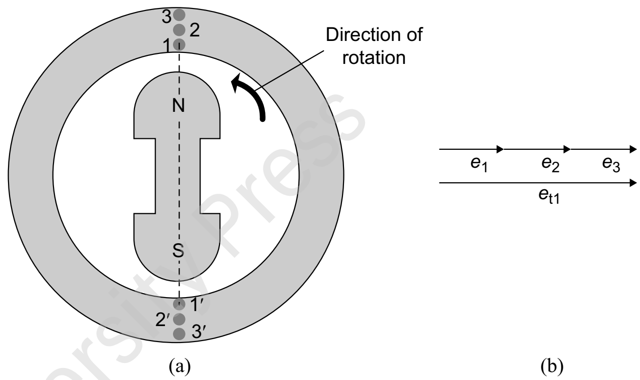


Fig. 2.14 (a) Undistributed winding arrangement (b) EMF induced in coil 1-1′, 2-2′ and 3-3′

Figure 2.15(a) shows a distributed winding where the three coils 1-1′, 2-2′ and 3-3′ do not occupy the same position in the stator, but are placed in slots separated from each other. These three coils are connected in series to form the total winding. The angle between two adjacent slots where the conductor 1, 2 and 3 are placed is α. The EMF induced in coils 1-1′, 2-2′ and 3-3′ are e_1, e_2 and e_3 respectively. For the rotor rotation shown in Figure 2.15(a), the EMF e_3 has a time phase lag of α angle with respect to e_2 (Figure 2.15(c)). Similarly, e_2 has a time phase lag of α angle with respect to e_1 (Figure 2.15(c)). However, the magnitude of e_1, e_2 and e_3 are same. This is because similar rate of flux cutting (Figure 2.15(b)) will occur in coils 2-2′ and 3-3′ when the space distribution of flux shifts by α angle and 2α angle, respectively, from the position of the coil 1-1′ as determined by direction of rotation of rotor. Therefore, the resultant EMF induced in the winding is the phasor sum of e_1, e_2 and e_3 and it is represented by e_{t2}.

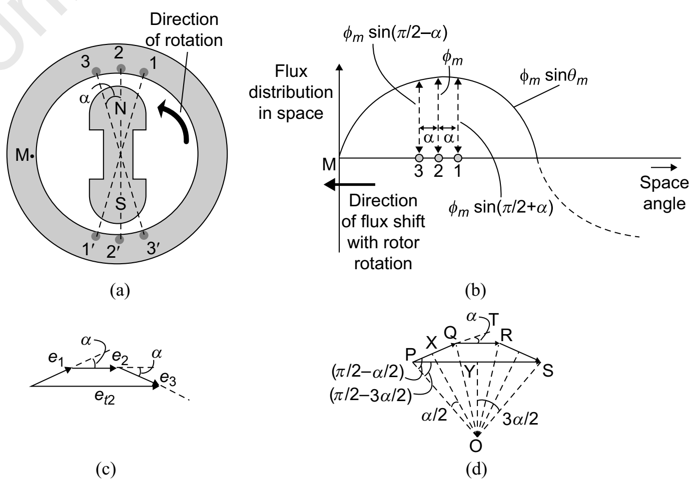


Fig. 2.15 (a) Distributed winding (b) position of conductor with respect to distribution of flux in space (c) phasor in coil 1-1', 2-2' and 3-3' (d) EMF polygon

Distribution factor (k_d) (or *breadth factor*) is defined as the ratio of induced EMF with distributed winding to that of concentrated (undistributed) winding.

In the present case, $k_d = \dfrac{e_{t2}}{e_{t1}}$.

To find out e_{t2}, the diagram of Figure 2.15(d) is taken into consideration.

In Figure 2.15(d), PS represent e_{t2}, PQ represent e_1, QR represent e_2 and RS represent e_3.

PQRS is part of a polygon of equal arm where O is the centre of the polygon.

Then,

$$OP = OQ = OR = OS$$

$$\angle PQR = \pi - \angle TQR = \pi - \alpha$$

$\therefore \angle PQO = \dfrac{1}{2}\angle PQR = \dfrac{\pi}{2} - \dfrac{\alpha}{2}$ [A line drawn from a vertex of an equal arm polygon to the centre of the polygon bisects the angle at the vertex of the polygon].

Then, $\angle PQO = \angle QPO = \dfrac{\pi}{2} - \dfrac{\alpha}{2}$ [Triangle ΔPQO is an isosceles triangle with equal arms OP and OQ. Therefore, $\angle PQO = \angle QPO$]

Then, $\angle POQ = \pi - \angle PQO - \angle QPO = \pi - \left(\dfrac{\pi}{2} - \dfrac{\alpha}{2}\right) - \left(\dfrac{\pi}{2} - \dfrac{\alpha}{2}\right) = \alpha$

OX is the line drawn from O perpendicular to the line PQ. As POQ is an isosceles triangle, then

$$\angle POX = \frac{1}{2}\angle POQ = \frac{\alpha}{2}$$

Now, $PX = OP\cos\angle XPO = OP\cos\left(\dfrac{\pi}{2} - \dfrac{\alpha}{2}\right) = OP\sin\dfrac{\alpha}{2}$

Thus, $PQ = 2PX = 2OP\sin\dfrac{\alpha}{2}$ Then, $e_1 = PQ = 2OP\sin\dfrac{\alpha}{2}$

Therefore, magnitude of $e_3 = e_2 = e_1 = 2OP\sin\dfrac{\alpha}{2}$

Then for concentrated (undistributed) winding, $e_{t1} = 3 \times 2OP\sin\dfrac{\alpha}{2}$

From property of polygon with equal arm and from property of isosceles triangle, $\angle YOP = \angle YOS = 3\dfrac{\alpha}{2}$

Now, $SY = OS\sin 3\dfrac{\alpha}{2}$ $\therefore PS = 2SY = 2OP\sin 3\dfrac{\alpha}{2}$ [As $OS = OP$]

Thus, total resultant EMF $e_{t2} = PS = 2OP\sin 3\dfrac{\alpha}{2}$

Therefore, distribution factor for the winding of Figure 2.15:

$$k_d = \frac{e_{t2}}{e_{t1}} = \frac{2OP\sin 3\frac{\alpha}{2}}{3 \times 2OP\sin\frac{\alpha}{2}} = \frac{\sin\frac{3\alpha}{2}}{3\sin\frac{\alpha}{2}}$$

Now, if there are n number of coils (instead of only three coils as shown in Figure 2.15) distributed in a winding, then distribution factor (k_d) for n number of coils will be: $k_d = \dfrac{\sin\frac{n\alpha}{2}}{n\sin\frac{\alpha}{2}}$ (2.68)

Expression for distribution factor Eq. (2.68) can now be utilized to obtain expression for induced EMF in a distributed winding.

Let, RMS value of EMF induced in a single coil = e

∴ RMS value of EMF induced in an concentrated (undistributed) winding with n number of coils = $e \times n$

Since distribution factor (k_d) is defined as the ratio of induced EMF with distributed winding to that of concentrated winding, therefore, RMS value of EMF induced in a distributed winding with n number of coils can be expressed as: $E_f = k_d \times e \times n$ (2.69)

The expressions for k_d Eq. (2.68) and E_f Eq. (2.69) obtained so far are valid for a two-pole machine with single turn in a coil. Also, the slots are single layered, i.e., a single coil is placed in a pair of slots. If the number of poles is more, and if the number of turns in a coil are more, and if the slots are having multiple layers of coils, then the following modifications need to be applied:

The quantity n which was defined in Eq. (2.68) as the number of coils, would have to be redefined as the *number of slots per pole per phase* to be used for calculation of k_d from Eq. (2.68). The quantity $n\alpha$ in that case is known as the *phase spread* and is expressed in electrical degrees or radians. Note that *phase spread* indicates the angle $\angle POS$ in Figure 2.15(d) where the number of slots per pole per phase is $n = 3$ and the angular displacement between slots = α.

RMS value of EMF induced in a distributed winding should use this new value of k_d and have a final modified expression: $E_f = k_d \times e \times T$ (2.70)

where T is number of turns per phase

2.7.2 Advantages of Distributed Winding

Graphical summation of EMF waveforms e_1, e_2 and e_3 are shown for concentrated winding in Figure 2.16 (a) and for distributed winding in Figure 2.16(b). In each case, for simplicity of representation e_1, e_2 and e_3 are taken as square wave-shape. For undistributed winding, the resultant EMF in the winding is represented by e_{t1} and for distributed winding the resultant EMF in the winding is represented by e_{t2}. Figure 2.16(a) shows that shape of e_{t1} is similar to shape of the induced EMFs in each coil; i.e., similar to the wave-shapes of e_1 or e_2 or e_3. Figure 2.16(b) shows that e_{t2} has a waveform that changes in steps. From the two figures, it is observed that the shape of e_{t2} in Figure 2.16(b) is coarsely like a sine wave as compared to the rectangular wave shape of e_{t1} in Figure 2.16(a). Therefore, by increasing the number of coils in a winding and by properly distributing them along the armature periphery, it is possible to improve the wave shape of induced EMF close to a sinusoidal waveform. Generation of sinusoidal EMF is always desirable for an AC generator. If the generated EMF is non-sinusoidal, then performance of any machine that is being supplied from the generator will get complicated and harmonics will be generated.

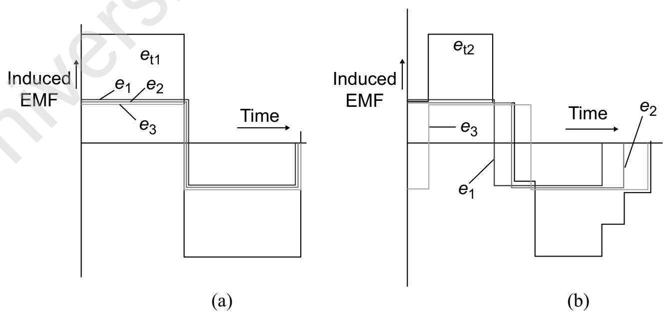


Fig. 2.16 EMF waveform for (a) undistributed winding (b) distributed winding

2.7.3 Coil Pitch and Pitch Factor

Another term related to AC machine armature winding is the *pitch* factor. A full pitch winding is a winding in which the two coil sides of a coil are placed in such a way that position of one coil side with respect to one pole is same as that of the other coil side with respect to the adjacent opposite pole. Thus, according to Figure 2.17(a), if the coil is made up of two coil sides R and R′, then it is a full pitch coil since while coil side R in under N pole, the other coil side R′ of the same coil is under S pole. *Angular distance between these two full-pitched coil sides is* *π electrical angle* as shown in Figure 2.17(c) [for a two-pole machine, the distance happens to be π mechanical angle as well]. In coil R-R′, say the EMF induced in conductor R is e_R and in conductor R′ is e_R'. They are same in magnitude and phase as their positions with respect to the respective poles are same. The phasor diagram is shown in Figure 2.17(e) and the resultant EMF across the coil terminals is thus $e = 2e_R$.

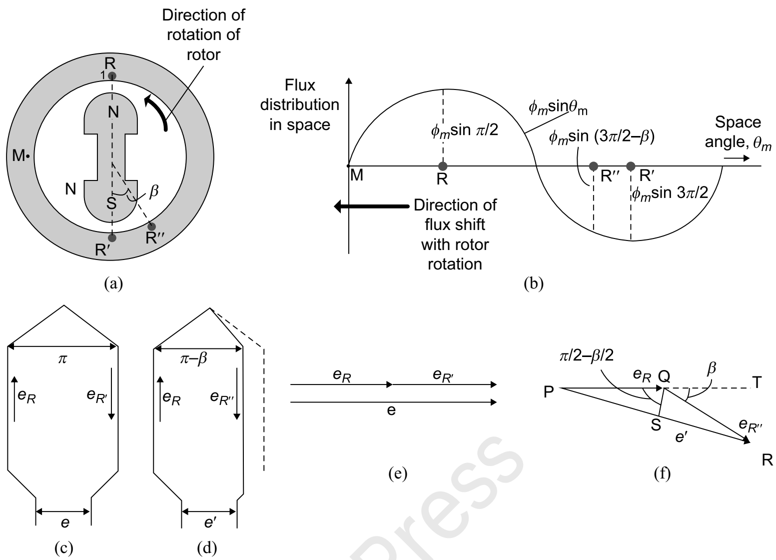


Fig. 2.17 Full pitch and short pitch winding

Now, if the coil is formed with coil sides R and R″ (Figure 2.17(a)), then it is called a short pitch (chorded) winding. The angular distance between the coil sides is less than π electrical angle (Figure 2.17(d)). Let EMF induced in conductor R″ is e_R''. With the rotor rotating in the direction shown in Figure 2.17(a), the EMF e_R'' (Figure 2.17(f)) has a time phase lag of β angle with respect to e_R (Figure 2.17(e)). However, the magnitude of e_R and e_R'' are same. This is because similar rate of flux linkage (Figure 2.17(b)) will occur in conductor R″ when the space distribution of flux shifts by β angle from the position of the conductor R′. Therefore, the resultant EMF induced in the coil R-R″ is the phasor sum of e_R and e_R'' and it is represented by e'.

Pitch factor (k_p) or *coil span factor* is defined as the ratio of induced EMF with short pitch winding to that of full pitch winding. In the present case, $k_p = \dfrac{e'}{e}$.

To find out e', the diagram of Figure 2.17(f) is taken into consideration. ΔPQR is an isosceles triangle with PQ representing e_R and QR representing e_R''. Then, $PQ = QR$.

Also, ∠TQR = ∠QPR + ∠QRP [Property of triangle: external angle of a triangle is sum of two interior opposite angles]

$\angle QPR = \angle QRP = \dfrac{\beta}{2}$ [As QPR is an isosceles triangle with $PQ = QR$, Then $\angle QPR = \angle QRP$]

Now, drop a perpendicular bisector QS on the base PR of ΔPQR.

$$PS = PQ\cos\angle QPR = PQ\cos\frac{\beta}{2} \qquad \therefore\ PR = 2PS = 2PQ\cos\frac{\beta}{2}$$

Thus, induced EMF in the short pitch coil R-R″ is $e' = PR = 2PQ\cos\dfrac{\beta}{2}$

Induced EMF in the full pitch coil R-R′ is: $e = 2e_R = 2PQ$

Then, pitch factor (k_p) with respect to the coil can be expressed as: $$k_p = \frac{e'}{e} = \frac{2PQ\cos\frac{\beta}{2}}{2PQ} = \cos\frac{\beta}{2} \qquad (2.71)$$

Pitch factor with respect to one coil is applicable to the whole winding assuming the winding to be uniformly would all around the armature.

Expression for pitch factor Eq. (2.71) can now be utilized to obtain expression for induced EMF in a short pitch coil once the EMF in a full pitch coil is known.

From Eq. (2.70), we know that RMS value of EMF induced in a full pitch distributed coil with T number of turns is given by: $E_{\mathrm{f}} = k_{\mathrm{d}} \times e \times T$

Since pitch factor (k_{p}) is defined as the ratio of induced EMF with short pitch winding to that of full pitch winding, therefore, RMS value of EMF induced in a short pitch distributed winding can be expressed as:

$$E_{\mathrm{f}} = k_{\mathrm{p}} k_{\mathrm{d}} \times e \times T \tag{2.72}$$

The product of pitch factor (k_{p}) and distribution factor (k_{d}) is referred to as the *winding factor* (k_{w}):

$$k_{\mathrm{w}} = k_{\mathrm{p}} k_{\mathrm{d}} \tag{2.73}$$

2.7.4 Advantage of Using Short Pitch Winding (short chorded winding)

It has been assumed so far that the flux distribution in space is always sinusoidal. However, in practice, the distribution of flux in space is not purely sinusoidal, and therefore EMF generated in AC generator will also have harmonics. Due to symmetry of the EMF wave around the vertical axis, only odd harmonics like 3rd, 5th, 7th and so on will be present. Higher the harmonic order, lesser is its effects due to reduced magnitude of the higher harmonic voltages. So, the more dominant harmonics that need to be taken care of while attempting to produce a smooth sinusoidal waveform are the 3rd and 5th harmonic components. Figure 2.18 shows samples of fundamental, 3rd harmonic and 5th harmonic EMF that may be generated in an AC generator due to non-sinusoidal distribution of air-gap flux.

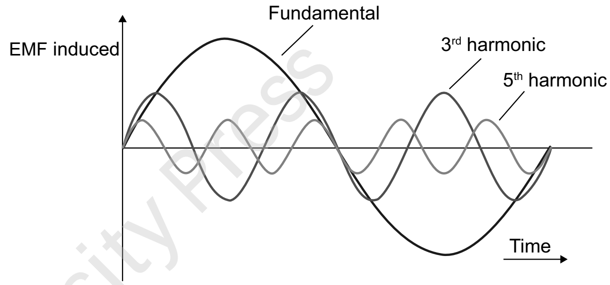


Fig. 2.18 EMF generation in an AC generator where harmonics are present

When the EMF waveform is composed of fundamental and harmonics, RMS values of which are E_1, E_3, E_5, etc., effective value of the resultant voltage E is:

$$E = \sqrt{E_1^{\,2} + E_3^{\,2} + E_5^{\,2} + \ldots} \tag{2.74}$$

In a three-phase machine, the phase angle difference between fundamental signals of consecutive phases is always 120°. Therefore, the third harmonic voltages present in the three-phases have a phase difference of 3×120° = 360° or 0° between them. Thus, for a three-phase balanced system, not only that the third harmonic voltages in three-phases are equal in magnitude, they are also in the same phase. In a star connected machine, the line voltage is measured as the potential difference between any two-phase terminals. Since the third harmonic voltage is same in all three-phases, when the line voltage of a star connected machine is calculated, the third harmonic voltages get cancelled out.

Effects of third harmonic voltages in individual phase coils can also be eliminated by proper choice of coil span, i.e., by proper choice of short pitching angle. Let e_{R1} and $e_{\mathrm{R''1}}$ be the fundamental induced EMF in conductors R and R″, respectively, of the short pitch coil R-R″ shown in Figure 2.17. Let the short pitch angle $\beta = \dfrac{\pi}{3}$, i.e., coil span between R and R″ is $\dfrac{2\pi}{3}$. Thus, the time phase difference between the EMF e_{R1} and $e_{\mathrm{R''1}}$ is $\dfrac{\pi}{3}$ as shown in Figure 2.19(a). Thus, the fundamental induced EMF in coil R-R″ is e_1 which is summation of e_{R1} and $e_{\mathrm{R''1}}$. Now, let e_{R3} and $e_{\mathrm{R''3}}$ be the

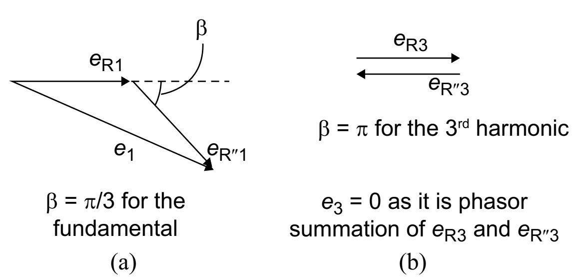


Fig. 2.19 Fundamental induced EMF and 3rd harmonic induced EMF in short pitch coil

3rd harmonic induced EMF in conductors R and R″, respectively. Then, the time phase difference between the EMF e_{R3} and $e_{R''3}$ is $\frac{3\pi}{3}$ which is equal to π as shown in Figure 2.19(b). Fig 2.19(b) shows that the induced EMFs due to 3rd harmonic in the two coil sides of coil R-R″ is equal and opposite to each other such that they are summing up to zero and is represented by e_3. Thus, using short pitching by $\frac{\pi}{3}$ angle, the third harmonic voltage can be eliminated in a coil and therefore in the whole winding. In general, if it is desired to eliminate the n^{th} harmonic component from the phase voltage, the short pitch angle should be $\frac{1}{n}$ times the full pitch.

While manufacturing a machine, the flux distribution in space gives estimation about which harmonics of induced EMF is liable to be present in the generated EMF waveform of the AC generator. The short pitching of the winding is done accordingly to cancel out the most prominent harmonics that may be present in the generated voltage waveform.

2.7.5 Winding Factors for Harmonic Waveforms

When the flux distribution and hence the induced EMF is assumed sinusoidal, then expression for induced EMF in a distributed winding is written following Eq. (2.72) as:

$$E_f = k_p k_d \times e \times T$$

The product of pitch factor (k_p) and distribution factor (k_d) is referred to as the *winding factor* (k_w):

$$k_w = k_p k_d$$

In case the flux wave deviates from being sinusoidal, the induced EMF also becomes non-sinusoidal. Thus, the EMF signal will have different harmonics in addition to the fundamental frequency component signal. The distribution factor and pitch factor both will have different values corresponding to different harmonics.

Referring to Section 2.7.1, Eq. (2.68), the distribution factor for the fundamental component of voltage is given by: $k_{d1} = \dfrac{\sin\frac{n\alpha}{2}}{n\sin\frac{\alpha}{2}}$

Then distribution factor for the m^{th} harmonic will be given by: $k_{dm} = \dfrac{\sin\frac{nm\alpha}{2}}{n\sin\frac{m\alpha}{2}}$ (2.75)

Note that for the m^{th} harmonic, the phase difference between voltages of adjacent coils is $m\alpha$.

In the same way, pitch factor will also have different values for different harmonic voltages. If the chording angle is β electrical degrees for the fundamental component of voltage, then following Section 2.7.3 and Equation (2.71), the pitch factor is given by $k_p = \cos\frac{\beta}{2}$. For the m^{th} space harmonic flux, the short chording angle becomes $m\beta$ electrical degrees. Therefore, pitch factor for the m^{th} harmonic is

$$k_{pm} = \cos\frac{m\beta}{2} \tag{2.76}$$

As a result, winding factor for the m^{th} harmonic becomes:

$$k_{wm} = k_{pm} k_{dm} \tag{2.77}$$

where k_{dm} and k_{pm} are given by Eqs. (2.75) and (2.76).

Example 2.1 The induced EMF in each turn of a single-phase AC generator with uniformly distributed T single turn coils is 2 V (rms). Calculate EMF of the whole winding.

Solution: For single-phase AC generator with uniformly distributed winding, the phase spread is generally 180° (electrical).

Thus, in Eq. (2.68), $n\alpha = \pi$

Where,

n = number of slots per pole per phase

α = angle between each slots

When α is very small, $\sin\frac{\alpha}{2} \approx \frac{\alpha}{2}$

$$\therefore \text{Distribution factor } k_d = \frac{\sin\frac{n\alpha}{2}}{n\sin\frac{\alpha}{2}} = \frac{\sin\frac{n\alpha}{2}}{n\frac{\alpha}{2}} = \frac{\sin\frac{\pi}{2}}{\frac{\pi}{2}} = 0.637$$

Considering full pitch coil, the pitch factor $k_p = 1$

Given: EMF induced in single turn = e = 2V

∴ EMF induced in the whole winding with T turns is:

$$E_f = k_p k_d \times e \times T = 1 \times 0.637 \times 2 \times T \approx 1.273T$$

Example 2.2 Calculate the distribution factor for a three-phase, four-pole AC machine having single layer winding uniformly distributed among 36 slots.

Solution: Number of slots per pole $\frac{36}{4} = 9$

∴ Angle between each slot $\alpha = \frac{180°}{9} = 20°$

Number of slots per pole per phase $n = \frac{36}{4 \times 3} = 3$

$$\therefore \text{Distribution factor } k_d = \frac{\sin\frac{n\alpha}{2}}{n\sin\frac{\alpha}{2}} = \frac{\sin\frac{3\times 20°}{2}}{3\sin\frac{20°}{2}} = 0.9598$$

Example 2.3 The armature of a 20-pole AC generator has 180 stator slots with 6 conductors per slot. All the coils of a phase are in series. When the machine winding is connected as single-phase, the generated voltage is V_1. When the same machine winding is connected as three-phase star connection, the generated phase voltage is V_2. Assuming full pitch, single-layer winding, find the ratio V_1/V_2.

Solution: For single-phase winding, no. of coils in the phase,

$$T_1 = \frac{\text{Slots} \times (\text{Conductors/slot})}{2 \times \text{Number of phases}} = \frac{180 \times 6}{2 \times 1} = 540$$

Number of slots per pole per phase, $n_1 = \frac{180}{20} = 9$

As it is a single-phase machine, $\alpha = \frac{180°}{9} = 20°$

∴ Angle between each slot $n_1\alpha = \frac{180°}{1} = 180°$

$$\text{Distribution factor } k_{d1} = \frac{\sin\frac{n_1\alpha}{2}}{n_1\sin\frac{\alpha}{2}} = \frac{\sin\frac{180°}{2}}{9\sin\frac{20°}{2}} = 0.6398$$

For three-phase winding, no. of coils in each phase,

$$T_2 = \frac{180 \times 6}{2 \times 3} = 180$$

Number of slots per pole per phase, $n_2 = \frac{180}{(3 \times 20)} = 3$

As it is a three-phase machine, $n_2\alpha = \frac{180°}{3} = 60°$

Since the same machine is used, ∴ angle between each slot remains as $\alpha = 20°$

$$\text{Distribution factor } k_{d2} = \frac{\sin\frac{n_2\alpha}{2}}{n_2\sin\frac{\alpha}{2}} = \frac{\sin\frac{60°}{2}}{3\sin\frac{20°}{2}} = 0.9597$$

The phase EMF for first case, $E_{f1} = k_p k_{d1} \times e \times T_1$

The phase EMF for second case, $E_{f2} = k_p k_{d2} \times e \times T_2$

Assuming full pitch coils for both cases, $k_p = 1$

In both the cases, EMF per coil (e) would remain same.

∴ Ratio of generated voltages:

$$\frac{V_1}{V_2} = \frac{E_{f1}}{E_{f2}} = \frac{k_p k_{d1} \times e \times T_1}{k_p k_{d2} \times e \times T_2} = \frac{0.6398 \times 540}{0.9597 \times 180} = 2$$

Example 2.4 A two-pole, three-phase star connected AC generator with uniformly distributed winding of total 60 turns. In each of these turns e_t volts (rms) is induced. Calculate the phase and line voltage for the following two cases:

(a) phase spread is 120° with full pitch winding
(b) phase spread is 60° with four-fifth of full pitch winding

Solution: The induced voltage per phase,

$$E_f = k_p k_d \times e_t \times T$$

where,

e_t = Induced EMF per turn

T = Turns per phase = 60/3 = 20

Distribution factor, $k_d = \dfrac{\sin\frac{n\alpha}{2}}{n\sin\frac{\alpha}{2}} \approx \dfrac{\sin\frac{n\alpha}{2}}{n\frac{\alpha}{2}}$ (when α is small $\sin\alpha \approx \alpha$ in radians)

(a) for phase spread of 120°,

$$n\alpha = 120° = 2\pi/3$$

$$k_d = \frac{\sin\frac{2\pi}{6}}{\frac{2\pi}{6}} = 0.827$$

With full-pitch winding, $k_p = 1$

$$\therefore \quad E_f = 1 \times 0.827 \times e_t \times 20 = 16.54e_t$$

∴ Induced voltage between lines:

$$E_f(Line) = \sqrt{3}E_f = 28.65e_t$$

(b) for phase spread of 60°,

$$n\alpha = 60° = \pi/3$$

$$k_d = \frac{\sin\frac{\pi}{6}}{\frac{\pi}{6}} = 0.955$$

As the winding is short pitched by four-fifth of full pitch winding, then short-pitch angle, $\beta = \dfrac{180°}{5} = 36°$

∴ Pitch factor, $k_p = \cos\dfrac{\beta}{2} = \cos\dfrac{36°}{2} = 0.95$

$$\therefore \quad E_f = 0.95 \times 0.955 \times e_t \times 20 = 18.15e_t$$

∴ Induced voltage between lines:

$$E_f(Line) = \sqrt{3}E_f = 31.43e_t$$

Example 2.5 An eight-pole, three-phase machine has 36 number of stator slots and has a 120° phase spread winding. Find the distribution factor of the machine.

Solution: For 120° phase spread, $n\alpha = 120°$

n = number of slots per pole per phase $= \dfrac{36}{8 \times 3} = 1.5$

α = electrical angle between each slots

Then, $n\alpha = 1.5\alpha = 120°$

$\therefore \alpha = 80°$

Distribution factor, $k_d = \dfrac{\sin\frac{n\alpha}{2}}{n\sin\frac{\alpha}{2}} = \dfrac{\sin\frac{120°}{2}}{1.5\sin\frac{80°}{2}} = 0.898$

Example 2.6 An AC generator has a uniformly distributed full pitch winding.

(a) Find distribution factor when four-fifth of the slots are wound.
(b) Find ratio of single-phase output with four-fifth slots wound and single-phase output when all slots are wound.
(c) Find ratio of single-phase output with four-fifth slots wound and three-phase output when all slots are wound.
(d) Find ratio of single-phase output with all slots wound and three-phase output when all slots are wound.

Solution

(a) When four-fifth of all slots are wound:

Phase spread, $n\alpha = \dfrac{4}{5} \times \dfrac{180°}{m}$, where m is number of phases

n = number of slots per pole per phase
α = angle between two slots

For single-phase and four-fifth of slots being wound,

$$n\alpha = \frac{4}{5} \times \frac{180°}{1} = \frac{4\pi}{5}$$

When α is very less, $n\sin\dfrac{\alpha}{2} \approx \dfrac{n\alpha}{2}$

$$\therefore \text{Distribution factor, } k_d = \frac{\sin\frac{n\alpha}{2}}{\frac{n\alpha}{2}} = \frac{\sin\frac{4\pi}{10}}{\frac{4\pi}{10}} = 0.756$$

(b) When all slots are wound:

Phase spread, $n\alpha = \frac{180°}{m}$, where m is number of phases

For single-phase with all slots being wound,

$$n\alpha = \frac{180°}{1} = \pi$$

When α is very less, $n\sin\frac{\alpha}{2} \approx \frac{n\alpha}{2}$

$$\therefore \text{Distribution factor, } k_d = \frac{\sin\frac{n\alpha}{2}}{\frac{n\alpha}{2}} = \frac{\sin\frac{\pi}{2}}{\frac{\pi}{2}} = 0.6366$$

$\therefore$Ratio of single-phase output with four-fifth slots wound to that when all slots are wound = ratio of (distribution factor × no. of turns)

$$= \frac{0.756 \times \frac{4T}{5}}{0.6366 \times T} = 0.95$$

where T is total number of turns when all slots are wound.

(c) For three-phase winding with all slots being wound,

$$n\alpha = \frac{180°}{3} = \frac{\pi}{3}$$

$$\therefore \text{Distribution factor, } k_d = \frac{\sin\frac{n\alpha}{2}}{\frac{n\alpha}{2}} = \frac{\sin\frac{\pi}{6}}{\frac{\pi}{6}} = 0.955$$

With same voltage induced per turn, three-phase output that is available across terminals will be $\sqrt{3}$ times the EMF induced per phase. Remember that the total T number of turns will be distributed among three-phases.

Thus, ratio of single-phase output with four-fifth slots wound to three-phase output when all slots are wound

$$= \frac{0.756 \times \frac{4T}{5}}{\sqrt{3} \times 0.955 \times \frac{T}{3}} = 1.096$$

(d) Ratio of single-phase output to three-phase output when all slots are wound $= \frac{0.6366 \times T}{\sqrt{3} \times 0.955 \times \frac{T}{3}} = 1.155$.

Example 2.7 A four-pole AC generator has a three-phase winding distributed among 120 slots. The coils are short pitched in such a way that if one coil side lies in slot number 1, the other side of the same coil lies in slot number 27. Calculate the winding factor for (a) fundamental and (b) third harmonic frequency voltages.

Solution: n = Number of slots per pole per phase $= \frac{120}{4 \times 3} = 10$

For full pitch coils, the phase spread in electrical degrees = 180°, i.e., π radian

$\therefore \alpha$ = Electrical angle between two slots $= \frac{180°}{n} = \frac{180°}{10} = 18°$

For full pitch coils, the coil span is $= \frac{\text{Slots}}{\text{Pole}} = \frac{120}{4} = 30$ slots

For the given short pitch coil, the coil span is (27 – 1) = 26 slots.

$\therefore$ Coil span in electrical degrees

$$= \frac{180°}{\text{Slots}/\text{Pole}} \times 26 = \frac{180°}{120/4} \times 26 = 156°$$

$\therefore$ Chording angle, i.e., short pitch angle

$\beta = (180° - 156°) = 24°$

(a) For fundamental frequency voltage component,

$$\text{Distribution factor, } k_{d1} = \frac{\sin\frac{n\alpha}{2}}{n\sin\frac{\alpha}{2}} = \frac{\sin\frac{10 \times 18°}{2}}{10 \times \sin\frac{18°}{2}} = 0.639$$

$$\text{Pitch factor, } k_{p1} = \cos\frac{\beta}{2} = \cos\frac{24°}{2} = 0.978$$

$$\therefore \text{Winding factor, } k_{w1} = k_{p1}k_{d1} = 0.978 \times 0.639 = 0.625$$

(b) For third harmonic frequency voltage component, Distribution factor,

$$k_{d3} = \frac{\sin\frac{3n\alpha}{2}}{n\sin\frac{3\alpha}{2}} = \frac{\sin\frac{3 \times 10 \times 18°}{2}}{10 \times \sin\frac{3 \times 18°}{2}} = 0.22$$

$$\text{Pitch factor, } k_{p3} = \cos\frac{3 \times \beta}{2} = \cos\frac{3 \times 24°}{2} = 0.81$$

$$\therefore \text{Winding factor, } k_{w3} = k_{p3}k_{d3} = 0.81 \times 0.22 = 0.178$$

2.7.6 Winding and Coil Groups

The winding of an AC machine is normally three-phase type as there are many inherent advantages for three-phase winding. This three-phase winding is symmetrically connected so that a balanced EMF is generated in the winding. A balanced three-phase EMF means that magnitude of EMF in each phase will be equal in magnitude and having a time phase difference of 120°.

A three-phase, two-pole winding of the armature of an AC machine is shown in Figure 2.20(a). According to the figure, there are six numbers of slots for each pole with two numbers of conductors in each slot. A coil group A_1-A_2, is shown with terminal A_1 and A_2. The coil group has two series connected coils whose upper coil sides are lying in slots 1 & 2 and lower coil sides are lying in slots 7 & 8. Similarly, the next coil group, B_1-B_2, is formed with respect to phase B. Their position with respect to the previous coil group is 120° apart. In that case, they will be formed by the upper coil sides in slots 5 & 6 and lower coil-sides in slots 11 & 12. The third coil group, C_1-C_2, is linked with phase C and will be 120° out of phase with the coil group B_1-B_2. Then, upper coil sides in slots 9 & 10 and lower coil sides in slots 3 & 4 will get involved.

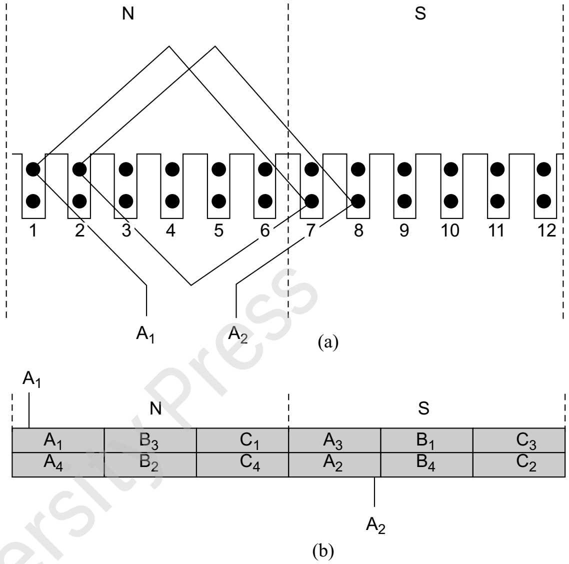


Fig. 2.20 (a) three-phase, two-pole, double layer winding of the stator (b) phase spread of the winding

Figure 2.20(b) shows the phase spread on the armature surface. A_1 and A_2 are the terminals of the coil group A_1-A_2 situated on top zone, A_1 and on bottom zone, A_2. Similarly, B_1 and B_2 zones are related to B_1-B_2 coil group and C_1 and C_2 zones are linked with coil group C_1-C_2. Figure 2.20(b) shows that another set of coil groups A_3-A_4, B_3-B_4, C_3-C_4, is possible which are covering the zones A_3 & A_4, B_3 & B_4, and C_3 & C_4, respectively. However, from Figure 2.20(b) it is observed, coil group A_3-A_4 is lying 180° out of phase with respect to coil group A_1-A_2. Therefore, EMF induced in coil group A_3-A_4 is also 180° out of phase with respect to EMF induced in coil group A_1-A_2. However, the magnitudes of EMF induced in both the coil groups are same as their positions with respect to the poles (N-pole for coil group A_1-A_2 and S-pole for coil A_3-A_4) are same. Therefore, they can be connected in parallel or in series by choosing proper terminals as shown in Figure 2.21(a) and 2.21(b). Voltage $V_{A1\text{-}A2}$ and $V_{A4\text{-}A3}$ are exactly identical and therefore the coil group A_1-A_2 and A_3-A_4 can be connected in series or parallel. Similarly, B_3-B_4 coil group is lying 180° out of phase with respect to B_1-B_2 coil group and C_3-C_4 coil group is lying 180° out of phase with respect to C_1-C_2 coil group. Therefore, they will also behave in the same manner as that of the explanation provided for A_3- A_4 coil group. When the coil groups are connected in parallel as shown in Figure 2.21(a), then they form two parallel paths for the flow of current in a single-phase.

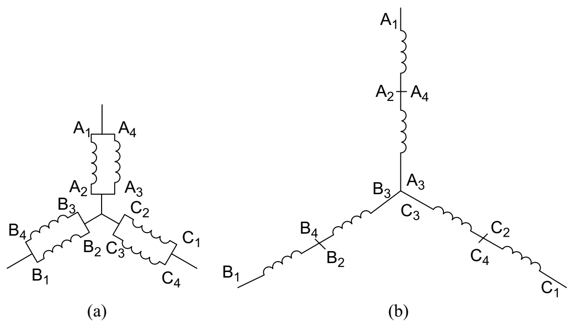


Fig. 2.21 (a) Coil groups connected in parallel (b) coil groups connected in series

Coil Pitch The coil pitch is the space angle (electrical) between two coil sides of the same coil and expressed in number of coil slots. According to Figure 2.20(a), the coil side placed on the top of slot 1 joins with the bottom coil sides placed at slot 7. Therefore, the coil pitch is 7 – 1 = 6.

Pole Pitch The pole pitch is equal to number of slots per pole. According to Figure 2.20(a), pole pitch = 12/2 = 6.

2.7.7 Different Types of Winding Arrangement

For a short pitch winding of the same machine, the lower coil sides are shifted by one slot position. Then, the zones of various phase spread is as shown in Figure 2.22 for a two-pole machine.

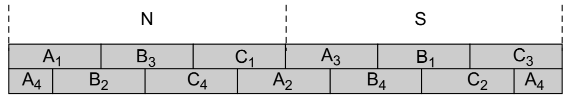


Fig. 2.22 Phase spread of short pitch double layer winding for three-phase, two-pole machines

The phase spread distribution of a four-pole machine is shown in Figure 2.23(a). The connection diagram of various coil groups is shown in 2.23(b). Thus, there are four numbers of parallel paths for flow of current in a phase.

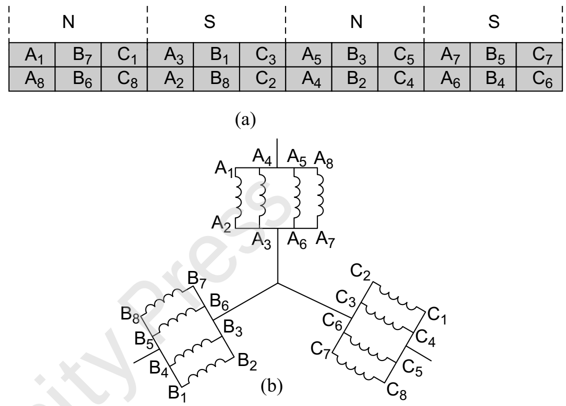


Fig. 2.23 (a) Phase spread of the full pitch winding for three-phase, four-pole machine (b) coil groups connection

In all the above mentioned three-phase winding, the phase spread is 60° and it is called narrow spread winding. Here, number of coil groups per phase per pair of poles is 2. Sometimes, wide spread winding is also used where the phase spread is 120°. Here, number of coil groups per phase per pair of poles is 1. A four-pole machine wide spread arrangement is shown in Figure 2.24. There are two coil groups per phase and the phase difference between induced EMF in them is 360°. Therefore, voltage phasor of all coil groups in a phase are identical in phase and magnitude for a widespread winding.

Let there be n number of coils in a coil group. These n numbers of coils are placed in n adjacent slots. Let α be the electrical angle between two adjacent slots. Then, the phase spread $n\alpha = \dfrac{180^\circ}{m}$ where m is the number of phases. For a two-phase machine, $n\alpha = 90^\circ$. For three-phase machine, $n\alpha = 60^\circ$ (for narrow spread winding).

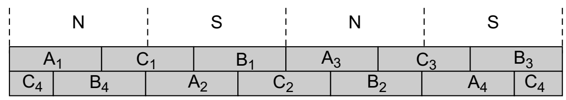


Fig. 2.24 Wide spread full pitch winding for three-phase, four-pole machine

2.8 MMF OF DISTRIBUTED AC WINDING

The armature of a machine has distributed winding. When current flows through this winding, a magnetic field is created by this current. The MMF space distribution of this field in the air gap between stator and rotor is discussed in the present section.

2.8.1 MMF Distribution in Space of a Single Coil

Figure 2.25 shows a two-pole AC machine with a cylindrical rotor. It has been assumed that a single coil A-A′ is present which has N number of turns and each conductor carries i current. The direction of current is shown in Figure 2.25. According to right hand grip rule, the direction of magnetic field flux is shown. The direction of flux also determines the North (N) and South (S) pole on the stator and also on the rotor. The MMF created by the coil is Ni. If the reluctance of the metallic core is considered negligible, then $\dfrac{Ni}{2}$ amount of MMF is consumed to create

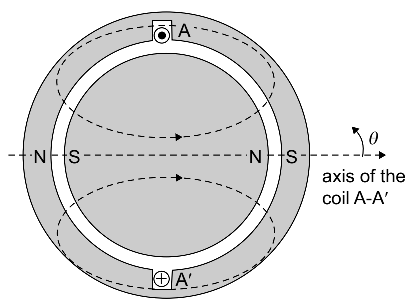


Fig. 2.25 A two-pole AC machine where direction of armature field is shown for a single coil

flux in the air gap between stator to rotor and the rest $\frac{Ni}{2}$ amount of MMF creates flux in the air gap between rotor and stator.

MMF or flux radially going from rotor to stator is considered positive and that from stator to rotor is considered negative.

The concept is made clearer when the stator and the rotor are laid down flat as shown in Figure 2.26.

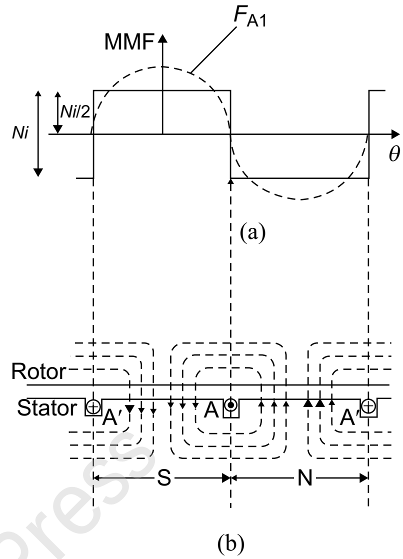


Fig. 2.26 (a) MMF distribution in the air gap for a single coil and the fundamental component of that MMF wave (b) The stator and the rotor of the machine is laid down flat and direction of armature field is shown for the coil A-A′

Figure 2.26 shows that MMF created by the coil A-A′ is distributed in space as rectangular wave with $+\frac{Ni}{2}$ creating flux from rotor to stator and $-\frac{Ni}{2}$ creating flux from stator to rotor. Fourier series analysis is applied on this MMF wave and the fundamental sine wave is drawn in the Figure 2.26.The expression of the fundamental wave is:

$$F_{A1} = \frac{4}{\pi}\left(\frac{Ni}{2}\right)\cos\theta \tag{2.78}$$

where θ is the angle with respect to axis of the coil which is same as the positive peak of the fundamental MMF wave.

The peak of this sinusoidal wave, $F_{AP} = \frac{4}{\pi}\left(\frac{Ni}{2}\right)$ (2.79)

From here onward, the fundamental MMF wave will only be considered and higher harmonics will be neglected as there magnitude is also less. Moreover, in the next section it will be shown that in distributed winding the higher order harmonics get cancelled.

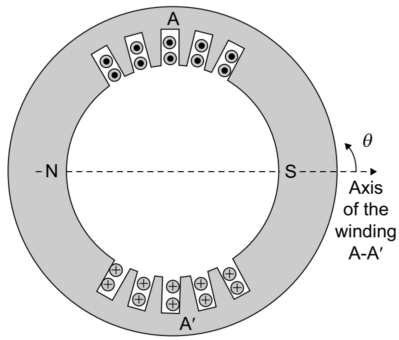


Fig. 2.27 Distributed winding placed on the armature of AC machine

2.8.2 MMF Distribution in Space of a Distributed Winding

Figure 2.27 shows a distributed two-layer winding. Let N be the number of turns in a single layer and i is the current flowing in a single turn. Then for two layer winding for each slot the MMF produced is $2Ni$. The MMF generated in the air gap produced by the winding is shown in Figure 2.28. It is found that the aggregate MMF produced by the winding is stepped with a step jump of $2Ni$. It is also noted from the Figure 2.28 that the MMF wave tends to become a sinusoidal wave. The upper half of the MMF wave locates for a position on the stator where S-pole of the stator exists. The lower half of the MMF wave corresponds to the N-pole position of the stator.

From this MMF wave, the fundamental sinusoidal wave and its peak value has to be determined. Let the number of turns connected in series in a single coil group = N_{cg}

Therefore, current i which is mentioned to be flowing through a single turn actually flows through all the N_{cg} turns in a single coil group.

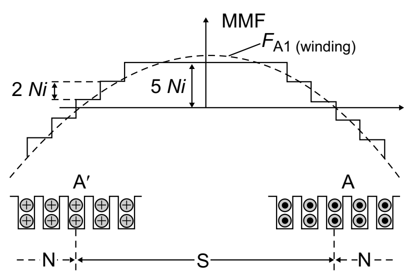


Fig. 2.28 MMF wave of the distributed winding

Then total Ampere-turn per coil group = $N_{cg}i$

Let the number of coil groups in the winding of a phase = A

Assume that the coil groups are connected in parallel as shown in Figure 2.21(a) and Figure 2.23(b).

Then total Ampere-turn per phase = $A(N_{cg}i)$

From Figure 2.21(a) and Figure 2.23(b), it can be understood that if i is the current flowing through a single coil group then for A number of coil groups connected in parallel in a single-phase, Ai amount of current flows through a single-phase.

Let I_a is the current in a single-phase. Then $I_a = Ai$

Then total Ampere-turn per phase = *Ncg Ia*

If there are P number of poles in the machine, then ampere-turn per phase per pole = $\frac{N_{cg}I_a}{P}$ (winding of the machine is considered to be concentrated)

Therefore, the peak value of the fundamental MMF wave for the concentrated winding

$$= \frac{4}{\pi}\left(\frac{N_{cg}I_a}{P}\right)$$

But, the actual winding is a distributed winding. Therefore, the peak will be diminished by a factor k_d which is the distribution factor of the machine. Hence, for distributed winding, the peak of the fundamental MMF wave is $F_{AP(\text{winding})} = k_d \frac{4}{\pi}\left(\frac{N_{cg}I_a}{P}\right)$ (2.80)

Therefore, the fundamental MMF waveform is expressed by the equation:

$$F_{A1(\text{winding})} = k_d \frac{4}{\pi}\left(\frac{N_{cg}I_a}{P}\right)\cos\theta \tag{2.81}$$

The angle θ is taken with reference to the axis where the N-pole and S-pole of the stator is shown.

It is thus advantageous to use distributed winding from the point of view of obtaining MMF waveform more close to a sinusoid.

2.9 MATERIALS USED IN ELECTRICAL MACHINES

All electrical machines, be it static devices like transformer, or be it rotating such as motors and generators, will comprise of the following basic components:

- Magnetic materials to be used as core
- Conducting materials in windings and coils
- Insulating materials for electrical isolation between different live parts

In addition, rotating machines will have bearings, fans etc. while transformer has various accessories that will be discussed in details in corresponding chapters.

2.9.1 Magnetic Materials Used in Electrical Machines

We have learnt earlier in Chapter 1 that the core of an electrical machine must be made of a good magnetic material so that the mutual flux can easily pass through it and chances of leakage are less. In addition to reducing flux leakage, a proper choice of core material also ensures lower power loss during magnetization, enough mechanical strength to support the windings and also optimized cost.

On the basis of relative permeability, we have seen in Section 1.9 that materials can be grouped into the following three categories for describing their magnetic properties:

- Ferromagnetic materials: $\mu_r >> 1$ (Iron, steel, nickel, cobalt)
- Paramagnetic materials: $\mu_r \approx 1$ (Aluminum, Magnesium)
- Diamagnetic material: $\mu_r < 1$ (Copper, Silver, Wood)

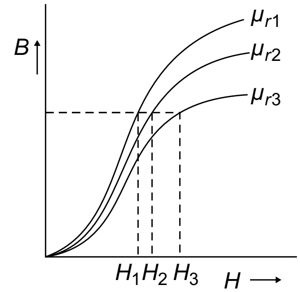


Fig. 2.29 Magnetization curves for different ferromagnetic materials

Because of their high permeability, i.e., high affinity towards magnetic flux, ferromagnetic materials are most popularly used as magnetic materials to build the core of an electrical machine.

Remember the relation $B = \mu_r H$, where H (AT/m) is the magnetizing force required to produce a magnetic flux density of B (Wb/m^2) in a magnetic material of relative permeability of μ_r.

The B–H curves for magnetic materials with different μ_r values are shown in Figure 2.29. The μ_r value determines the slope of the curve. Material with higher μ_r value will have a steeper B-H curve.

For the same value of flux density, a magnetic material with higher μ_r value will require less magnetizing force and hence less magnetizing current.

In Figure 2.29,

$$\mu_{r1} > \mu_{r2} > \mu_{r3} \qquad\qquad H_1 < H_2 < H_3$$

Since, $H = \dfrac{NI_m}{L}$

where N is the number of turns in the exciting coil, I_m is the magnetizing current and L is the length of flux path. Thus $I_{m1} < I_{m2} < I_{m3}$.

Thus, a magnetic material with higher relative permeability will draw less magnetizing current.

Thus, the power factor and hence losses and regulation are better in machines employing magnetic materials with higher permeability. Also, μ_r value if higher, the area of Hysteresis loop is less as shown in Figure 2.30 and thus the Hysteresis losses are also less.

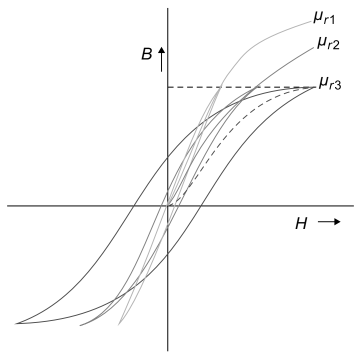


Fig. 2.30 Dependence on Hysteresis loop area of relative permeability

Thus, it is preferred to have a magnetic material with high value of relative permeability as the core material in an electrical machine.

In earlier days of electrical industry, the sheet material used for magnetic circuit of electrical machines and core of the transformer was iron with low content of carbon and other impurities. This had one major disadvantage of that of ageing. Ageing is the term used to denote the deterioration of magnetic performance over time while in service caused by increase in coercive force and Hysteresis loss which in turn caused cumulative overheating and subsequent breakdown. During the last century, however, it was discovered that a great improvement in magnetic properties of iron (steel) could be achieved by alloying Silicon with the steel in which the content of Si lies from about 0.3% to 4.5% by weight. The advantages of adding Si to steel are:

1. It virtually eliminates the ageing problem
2. It increases the permeability of steel, thereby requiring less magnetizing current
3. Increased permeability reduces the Hysteresis loss
4. It increases resistivity of steel, thereby reducing the Eddy current loss as well
5. It increases mechanical strength
6. It increases malleability of steel sheets

Unfortunately, addition of *Si* to steel has two major drawbacks also. As the percentage of Silicon increases, it is found that there is some loss of permeability at higher flux densities and loss of ductility. Therefore, above about 5% Si content, the resulting alloy is very brittle and cannot be punched or sheared.

An analytical expression for Eddy current loss was provided in Chapter 1:

$$P_e = K_e f^2 t^2 B_m{}^2 \text{ W/m}^3$$

where K_e is constant depending on the material properties including its resistivity, f is the supply frequency, t is the thickness of the material body, and B_m is maximum value of flux density in the material. It is thus apparent that in machines where there is chance of varying flux linking with the magnetic material, Eddy current loss can be greatly reduced by reducing the thickness 't' of the Si-steel core material. This is achieved by stacking thin sheets (laminations) of Si-steel together to make the entire thickness of core. These laminations are insulated from each other by layers of varnish so that flow of Eddy current across them is prevented. Figure 2.31 shows sample photographs of laminations used in transformers and stator of a rotating AC machine.

Fig. 2.31 Si-steel laminations (a) transformer (b) rotating AC machine

Blocks of Si-steel are passed through a series of rollers of progressively reduced spacing to form thin sheets of laminations from a thick block of steel as schematically shown in Fig 2.32.

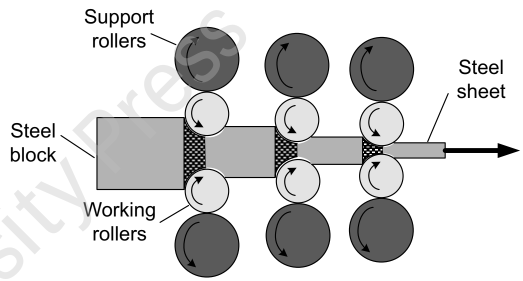


Fig. 2.32 Steel rolling mill producing laminations

2.9.2 Types of Si-steel Laminations Available

1. HRS—Hot rolled steel
2. CRGOS—Cold rolled grain oriented steel
3. CRNOS—Cold rolled non-oriented steel

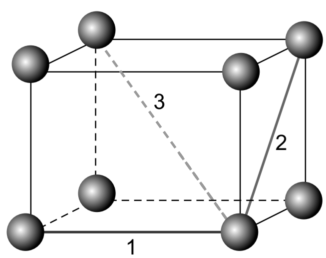
HRS

HRS is made when a block of Si-steel is rolled under high pressure to form thin laminations while the steel block is under red-hot condition. In HRS, however, the grains (Si-steel crystals) are randomly oriented. In this material there is no preferential direction of magnetization. Its magnetic properties are similar when magnetization is done in any direction. HRS is used as core material in rotating parts of a machine where the direction of flux always changes. HRS is cheaper, but its surface finish is not good as due to heating, expansion and contraction may happen and gas bubbles may be formed inside the surface.

CRGOS

Si-steel crystal lattice structure is shown in Figure 2.33:

1. Crystal edge
2. Face diagonal
3. Crystal diagonal (cube diagonal)



1. Crystal edge
2. Face diagonal
3. Crystal diagonal (cube diagonal)

Fig. 2.33 Si-steel crystal

While manufacturing CRGOS, a block of Si-steel is heat treated (annealed) with specific cycles of time and temperature, and then rolled in the form of thin laminations under cold condition in such a way that all the grains (crystal edges) are aligned along the direction of rolling. Magnetization is easiest in the direction of grain orientation (i.e., crystal edges). Along this direction, the material offers highest permeability and lowest iron losses as denoted in Figure 2.34.

If the flux lines are parallel to the grain orientation, then best magnetic properties are obtained. CRGOS is hence very effectively used as core material for large transformers.

Surface finish of CRGO is very smooth since it is rolled under cold condition, but the manufacturing cost is quite high.

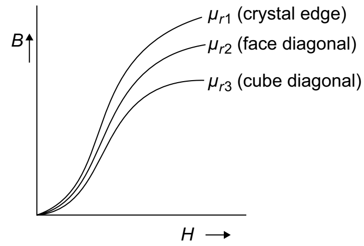


Fig. 2.34 *B–H* characteristics of CRGO steel when magnetization is done along different diagonals

CRNOS

A block of Si-steel rolled in the form of thin laminations under cold condition is called CRNOS. There is, however, random orientation of the grains in CRNOS resulting in no preferred direction of magnetization. CRNOS is used with rotating machines where the flux path is always changing. The surface finish of CRNOS is better than HRS and CRNOS is cheaper than CRGOS.

For use in transformers, CRGOS appears to be the best because flux path is always constant along the core. This reduces the losses and offers highest permeability. Cost of CRGOS is, however, more than HRS. In fact:

$$\frac{\text{Cost/kg of CRGOS}}{\text{Cost/kg of HRS}} = 1.25 \sim 1.35$$

It may seem that using CRGOS will be more costly than HRS when used as core material for a given transformer. However, note that CRGOS having higher permeability than HRS, higher values of flux densities are permissible in CRGOS since its saturation level is higher than that of HRS as shown in Figure 2.35.

Since higher flux densities are allowable in CRGOS, for same value of flux, cross sectional area requirement of core is less for CRGOS than HRS.

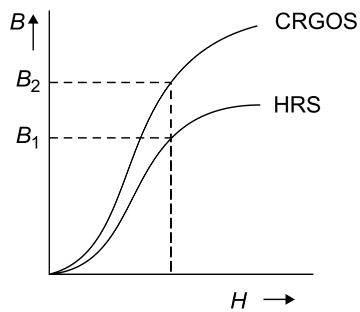


Fig. 2.35 Saturation value of CRGOS is higher than that of HRS

B2 = flux density with CRGOS

B_1 = flux density with HRS

For same flux ϕ in the core, the cross sectional area required for core when CRGOS and HRS are used can be compared as:

$$A_2 = \frac{\phi}{B_2} \text{ with CRGOS} \qquad A_1 = \frac{\phi}{B_1} \text{ with HRS}$$

Since $B_2 > B_1$, $A_2 < A_1$

Thus, though per kg cost of CRGOS is more than that of HRS, the total weight of core material required for CRGOS being less, the overall cost using CRGOS or HRS are often comparable.

2.9.3 Conductor Materials Used for Winding

Windings of DC and AC machines consist of the current-carrying conductors wound around the sections of the core or slots in the core, and these must be properly insulated, supported and cooled to withstand operational and test conditions. Copper and aluminum are the primary materials used as conductors in electrical machine windings. These conductors may be of circular or rectangular cross section, depending on the current and voltage ratings of the machine and are insulated using enamel, paper or cotton.

2.9.4 Insulating Materials in Electrical Machines

Electrical machines use varieties of insulating materials in different shapes and sizes for electrical isolation between two live parts at different potential or between one live part and ground potential. Insulating materials can be either of organic, inorganic or synthetic origin. Insulating materials are processed (both chemically and mechanically) for use in electrical machines of varying capacity.

In order to get more information on conductor materials used for winding, insulating materials in electrical machines, and uses of insulating materials in electrical machines, please go to online resources.

2.10 MATLAB EXAMPLES

MATLAB Example 2.1 A two-pole, three-phase star connected AC generator with uniformly distributed winding of total 60 turns. In each of these turns 5 volts (rms) is induced. Calculate the phase and line voltage for the following two cases:

(a) phase spread is 120° with full pitch winding
(b) phase spread is 60° with four-fifth of full pitch winding

```
N = 60;                                   %Number of turns  = 60
T=N/3;                                    % Number of turns per phase
Et=5;                                     % Voltage per turn = 5 V
%(a)    For phase spread of 120° and full pitch coils
Ph_Sprda=2*pi/3;                          %phase spread =120°
Kda=sin(Ph_Sprda/2)/(Ph_Sprda/2);         %Distribution factor
Kpa=1;                                    % Pitch factor=1 for full pitch coil
V_Pha=Kpa*Kda*Et*T;                       % Phase voltage induced
V_La=sqrt(3)*V_Pha;                       % Line voltage induced
%(b)    For phase spread of 60° and coil pitch as 4/5th of full pitch
Ph_Sprdb=pi/3;                            %phase spread =60°
Kdb=sin(Ph_Sprdb/2)/(Ph_Sprdb/2);         %Distribution factor
Beta=pi/5;                                %Short pitch angle (1/5th of pole pitch)
Kpb=cos(Beta/2);                          % Pitch factor for short pitch coil
V_Phb=Kpb*Kdb*Et*T;                       % Phase voltage induced
V_Lb=sqrt(3)*V_Phb;                       % Line voltage induced

disp(['(a)Phase spread is 120° with full pitch winding ']);
disp(['Distribution factor Kd = ', num2str(Kda)]);
disp(['Pitch factor  Kp = ', num2str(Kpa)]);
disp(['Per Phase induced EMF = ', num2str(V_Pha) ' V']);
disp(['Line voltage induced = ', num2str(V_La) ' V']);
disp(['(b)Phase spread is 60° with 4/5th of full pitch winding ']);
disp(['Distribution factor Kd = ', num2str(Kdb)]);
disp(['Pitch factor  Kp = ', num2str(Kpb) ]);
disp(['Per Phase induced EMF = ', num2str(V_Phb) ' V']);
disp(['Line voltage induced = ', num2str(V_Lb) ' V']);
```

Upon execution, the output is

```
>> ME2_1
(a)Phase spread is 120° with full pitch winding
Distribution factor Kd = 0.82699
Pitch factor  Kp = 1
Per Phase induced EMF = 82.6993 V
Line voltage induced = 143.2394 V
(b)Phase spread is 60° with 4/5th of full pitch winding
Distribution factor Kd = 0.95493
```

```
Pitch factor Kp = 0.95106
Per Phase induced EMF = 90.8192 V
Line voltage induced = 157.3035 V
```

MATLAB Example 2.2 A four-pole AC generator has a three-phase winding distributed among 120 slots. The coils are short pitched in such a way that if one coil side lies in slot number 1, the other side of the same coil lies in slot number 27. Calculate the winding factor for (a) fundamental and (b) third harmonic frequency voltages.

```
S = 120;             %Number of slots  = 120
P=4;                 %Number of poles  = 4
Ph=3;                %Number of phases  = 3
Short_Span=26;       % Coil span = 26 slots

n=120/(4*3);         % Slots per pole per phase
Ph_Sprd=pi;          % For full pitch coil
alpha=Ph_Sprd/n;     % Eletrical angle between slots per pole per phase

Slots_Per_Pole=120/4;
Coil_Pitch=(Ph_Sprd/Slots_Per_Pole)*Short_Span;

Beta=(Ph_Sprd-Coil_Pitch);      %Short pitch angle

%   (a)For fundamental frequency voltage component

Kda=sin(n*alpha/2)/(n*sin(alpha/2));   %Distribution factor
Kpa=cos(Beta/2);     % Pitch factor for short pitch coil

Kwa=Kpa*Kda;         % Winding factor

%   (d)For third harmonic frequency voltage component

Kdb=sin(3*n*alpha/2)/(n*sin(3*alpha/2));   %Distribution factor
Kpb=cos(3*Beta/2);     % Pitch factor for short pitch coil

Kwb=Kpb*Kdb;         % Winding factor

disp(['(a)For fundamental frequency voltage component']);
disp(['Distribution factor Kd = ', num2str(Kda)]);
disp(['Pitch factor  Kp = ', num2str(Kpa)]);
disp(['Winding factor Kw = ', num2str(Kwa)]);

disp(['(b)For third harmonic frequency voltage component']);
disp(['Distribution factor Kd = ', num2str(Kdb)]);
disp(['Pitch factor  Kp = ', num2str(Kpb) ]);
disp(['Winding factor Kw = ', num2str(Kwb)])
```

Upon execution, the output is

```
>> ME2_2
(a)For fundamental frequency voltage component
Distribution factor Kd = 0.63925
Pitch factor Kp = 0.97815
Winding factor Kw = 0.62528
```

```
(b)For third harmonic frequency voltage component
Distribution factor Kd = 0.22027
Pitch factor Kp = 0.80902
Winding factor Kw = 0.1782
```

SUMMARY

- Electromechanical systems deal with interaction of electrical, magnetic, and mechanical systems.
- Relationship between input energy, output energy, energy losses, and energy stored is governed by the law of conservation of energy in all types of machines.
- Electromechanical force (or torque) developed can be expressed as the negative partial derivative of energy stored in the magnetic field against linear (or angular) displacement.
- Electromechanical force (or torque) can also be expressed as positive partial derivative of co-energy of the magnetic system against linear (or angular) displacement
- Total torque in a doubly excited salient pole structure comprises reluctance torque and electromagnetic torque. Reluctance torque can act when at least one of the two structures (stator and rotor) is excited, whereas electromagnetic torque acts when both are excited together.
- In case of generator induced EMF *e* and current *I* are in same direction whereas in case of motor the induced EMF *e* and current *I* are in opposite directions.
- In case of generator, the internally developed torque, T_e is opposite to direction of rotation of the rotor or rotor speed ω, whereas in case of motor the internal developed torque, T_e and motor direction of rotation or rotor speed ω, is same.
- In case of generator induced EMF *e* and current *I* are in same direction whereas in case of motor the induced EMF *e* and current *I* are in opposite directions.
- 1° electrical angle = $\left(\frac{2}{P}\right)^{\circ}$ mechanical angle
- All practical rotating (circular) machine armatures have distributed winding where the coils do not occupy the same position, but are placed in slots separated from each other. These three coils are connected in series to form the total winding.
- Distribution factor (k_d) (or breadth factor) is defined as the ratio of induced EMF with distributed winding to that of concentrated (undistributed) winding. $k_d < 1$ for a distributed winding.
- By increasing the number of coils in a winding and by properly distributing them along the armature periphery, it is possible to improve the wave shape of induced EMF close to a sinusoidal waveform.
- Angular distance between the two coil sides of a full pitch coil is π electrical angle, i.e., its coil sides are exactly one pole pitch apart. In a short-pitch coil or a short-chorded coil, the angular distance between the coil sides is less than π electrical angle.
- *Pitch factor* (k_p) is defined as the ratio of induced EMF with short pitch winding to that of full pitch winding. $k_p < 1$ for a short-pitch coil.
- By proper choice of short-pitch angle, it is possible to cancel out the most prominent harmonics that may be present in the generated voltage waveform.
- RMS value of EMF induced in a short pitch distributed winding can be expressed as:
- $E_f = k_p k_d \times e \times T$, e is the EMF per turn, and T is the total number of turns
- The product of pitch factor (k_p) and distribution factor (k_d) is referred to as the *winding factor* (k_w): $k_w = k_p k_d$
- The shape of MMF developed by a single coil is rectangular; containing fundamental sinusoidal component and higher harmonics.
- A distributed winding gives rise to MMF wave closer to sinusoid.
- Magnetic materials used in electrical machines should preferably have high permeability, low losses, high mechanical strength. These properties can be improved in steel by adding silicon to it by less than 5% in weight.
- Copper and aluminum are most commonly used conductor material used in electrical machines. Both have their relative merits and demerits.
- Choice of insulating material to be used in electrical machines depend on several of its properties including resistivity, breakdown strength, dielectric constant, temperature withstanding capacity, chemical and mechanical stability, etc.

CHAPTER-END EXERCISES

Multiple-choice Questions

1. For a P-pole machine, the relation between electrical and mechanical degrees is given by
   (a) $\theta_{\text{elec}} = \frac{2}{P}\theta_{\text{mech}}$
   (b) $\theta_{\text{elec}} = \frac{4}{P}\theta_{\text{mech}}$
   (c) $\theta_{\text{mech}} = \frac{P}{2}\theta_{\text{elec}}$
   (d) $\theta_{\text{elec}} = \frac{P}{2}\theta_{\text{mech}}$
2. For eliminating n^{th} harmonic from the EMF generated in the phase of a three-phase alternator, the chording angle should be
   (a) $n \times$ full-pitch
   (b) $\frac{1}{n} \times$ full-pitch
   (c) $\frac{2}{n} \times$ full-pitch
   (d) $\frac{3}{n} \times$ full-pitch
3. The developed electromagnetic force and/or torque in electromechanical energy conversion system act in a direction tends to
   (a) increase the stored energy at constant flux
   (b) decrease the stored energy at constant flux
   (c) decrease the co-energy at constant MMF
   (d) decrease the stored energy at constant MMF
4. To eliminate fifth harmonic voltage from the phase voltage of a generator, the winding of the machine should be short pitched by an electrical angle of
   (a) 60°
   (b) 36°
   (c) 45°
   (d) 18°
5. A three-phase AC machine stator winding has a coil span of 156 electrical degrees. Its coil span factor is:
   (a) 0.914
   (b) 0.94
   (c) 0.978
   (d) 0.208
6. The main advantage of distributing the winding in slots is to
   (a) Reduce harmonics in the generated voltage
   (b) Make the winding mechanically stronger
   (c) Reduce the amount of copper required
   (d) Reduce size of the machine
7. Electromagnetic torque is present in a rotating machine when
   (a) The rotor has salient pole structure
   (b) When the air gap is uniform
   (c) When either the rotor or the stator coil is excited
   (d) When both the stator and rotor coils are excited
8. For linear electromagnetic circuit, which one of the following statements is true?
   (a) Stored energy is zero
   (b) Stored energy is less than co-energy
   (c) Stored energy is equal to co-energy
   (d) Stored energy is more than co-energy
9. Addition of 0.3 to 4.5% of silicon to iron _________ the electrical resistivity of iron
   (a) Increases
   (b) Decreases
   (c) Does not change
   (d) Reverses
10. Mica is a
    (a) Magnetic material
    (b) Conducting material
    (c) Insulating material
    (d) Semiconductor material

Critical Thinking Questions

1. 'For successful mechanical movement in a singly excited system, it is necessary that air gap between stator and the moving element is non-uniform'. Justify with reason whether the statement is true or false.
2. 'For achieving movement of the rotor of a cylindrical rotor motor, it is necessary that both stator and rotor be excited'. Justify with reason whether the statement is true or false.
3. In a doubly excited system, the salient pole rotor moves in a direction such that the stored energy in magnetic field is reduced, but the co-energy is increased: Justify with reason whether the statement is true or false.
4. 'Direction of rotation of the rotor in a singly excited machine with salient pole structure of rotor can be reversed by reversing the current direction in stator electromagnet'. Justify with reason whether the statement is true or false.
5. Derive the power balance equation in a basic generator from the original energy balance equation

of electromechanical energy conversion principles considering the law of conservation of energy.

6. 'Distribution factor and pitch factor reduces the EMF induced in every turn of a generator'. Justify with reason whether the statement is true or false.
7. If air does not have any hysteresis loss associated during its magnetization, then why simple air-core coils are not used in transformers?
8. What will happen if the percentage of silicon is increased beyond 5% in electrical grade steel?

Descriptive Questions

1. Based on the principle of conservation of energy, write an energy balance equation for a motor. Also, write briefly about the various energy terms involved.
2. Show that for a singly excited linear electromechanical energy conversion device, electromechanical force developed can be expressed as the negative partial derivative of energy stored in the magnetic field against linear displacement.
3. What is reluctance torque? Explain the difference between electromagnetic torque and reluctance torque. Explain whether a DC motor can develop any reluctance torque.
4. Show that the general expression for torque developed in a *P*-pole AC machine is proportional to the stator MMF, the resultant MMF and sine of angle between them.
5. What are the advantages of distributing a winding in rotating electrical machine? Show that $K_d = \dfrac{\sin\frac{q\gamma}{2}}{q\sin\frac{\gamma}{2}}$. where k_d = distribution factor, q = slots per pole per phase, γ = slot pitch in electrical radian.
6. What is EMF polygon? Find expression for the pitch factor of a short chorded winding.
7. Derive the relation between electrical angle and mechanical angle in relation to rotating machines.
8. What are the advantages of adding silicon to steel for use as core material in electrical machines?
9. What are the thermal classifications of insulating materials as per Indian standards?

Numerical Problems

1. Find the distribution factor of a three-phase, six-pole AC machine with 54 slots. **[Ans: 0.9598]**
2. Calculate the pitch factor of a coil having its two sides placed apart by $5/8^{th}$ of full pitch distance. **[Ans: 0.83]**
3. An AC machine has 4 poles with 64 slots. The coils are wound with $11/16^{th}$ faction of full pitch. Calculate pitch factor. **[Ans: 0.88]**
4. Determine the winding factor of an eight-pole, three-phase alternator having 72 slots, with 6 turns per coil when the winding is short pitched by 3 slots. **[Ans: 0.58]**
5. Determine the winding factors for a six-pole AC machine winding placed in 36 slots with coils of span $5/6^{th}$ of a pole pitch, when the winding is
   (a) Three-phase with phase spread of 60°
   (b) Three-phase with phase spread of 120°
   (c) Two-phase
   (d) Single-phase
   [Ans: (a) 0.929 (b) 0.837 (c) 0.876 (d) 0.622]
6. Find the fundamental, 3^{rd}, and 5^{th} harmonic distribution factors for a 16-pole, 144 slot alternator.
   [Ans: k_{d1} = 0.667, k_{d3} = 0.33, k_{d5} = 0.667]

Answers to Multiple-choice Questions

1. a, 2. b, 3. b, 4. b, 5. c, 6. a, 7. d, 8. c, 9. a, 10. c