Applied Physics for Engineering

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Preface

Science and technology have revolutionized our life. Day in and day out, we learn about some or other invention from different parts of the globe. We fantasize about some unreal things in science but are sure that they will become real soon, looking at the pace at which research and innovations are taking place. We encourage young minds to think out of the box and motivate them to take challenges. To make these young minds tread the less-travelled path we will have to equip them thoroughly with fundamentals and nuances of science, besides the current trends in the development of applications.

About the Book

Applied Physics for Engineering is written in a comprehensive manner, which will encourage students to learn the basics of engineering science. The topics are discussed thoroughly with well-illustrated figures and tables for conceptual understanding. The book presents the discussion of advanced topics at an introductory level.

Key Features

- Presents a detailed discussion on applications like electron microscopy, nanomaterials, and fabrication of ICs
- Includes features such as checkpoints and solved problems
- Includes chapter-end exercises and practice problems in each chapter

Content and Structure

The book contains eight chapters, which optimally fulfills the requirement of first year undergraduate students of all branches.

Chapter 1 deals with the basic concepts of uniform electric and magnetic field and the behaviour of the charged particle in it. Chapter 2 chapter covers the non-uniform electric field and related applications, while Chapter 3 is related to atomic or modern physics that talks about relevance of quantization in atomic world.

Chapters 4, 5, and 6 cover significant topics related to optics namely laser, interference, and polarization illustrating fundamental concepts of these phenomena exhibited by light along with their relevant applications.

Chapter 7 is about semiconductor physics and its contribution to the development of electronics, while Chapter 8 deals with some of the recent advances in the field and includes discussion on nanotechnology, integrated circuits, and electron microscopy.

Acknowledgement

I would wish to register special gratitude to Dr. Preeti Bajaj, Director, G. H. Raisoni College of Engineering, Nagpur for her encouragement in the pursuit of higher learning and for her support and constant encouragement. I am also thankful to my colleagues for their cooperation during the development of this book.

I am indebted to my family for their patience during the entire process of writing of this book.

I would also like to acknowledge with gratitude the unflinching support and guidance provided by the editorial team at Oxford University Press, India who made it possible to publish this book in time. Suggestions and feedback for the book are welcome and can be sent to butey.bhavana@raisoni.net or bpbuety@gmail.com.

Bhavana P. Butey

Features of

Learning Objectives Each chapter of the book has a section 'Learning Objectives', which briefs about all the topics discussed in the chapter.

Learning Objectives

After going through this chapter, you will be able to:

- Explain the basics of non-uniform electric field
- Explain the working of electron gun which is based on the concept of electron lens
- Explain the functioning of the fundamental parts of CRT
- Explain the working of CRO based on the concept of uniform and non-uniform electric fields
- Highlight the applications of CRO

Figures Numerous well-illustrated figures are added to the book for better understanding of concepts.

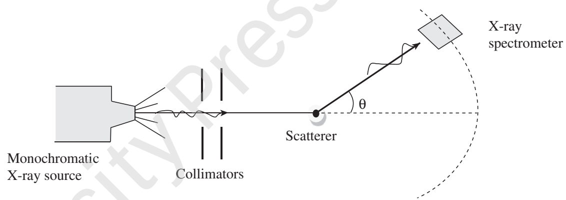


Fig. 3.2 Experimental arrangement of Compton effect

Checkpoint Each chapter is interspersed with sets of multiple choice questions to check the understanding of concepts.

Checkpoint 3

1. In the static magnetic field, work done by the field is always
   (a) maximum (c) zero
   (b) negative (d) positive
2. When an electron moving with velocity $\vec{v}$ enters a uniform transverse magnetic field the Lorentz force alters the
   (a) magnitude of $\vec{v}$ (c) magnitude and direction of $\vec{v}$
   (b) direction of $\vec{v}$ (d) magnitude and direction of $\vec{v}$ remains unaffected
3. When a charged particle enters a uniform magnetic field at right angle to the direction of field, it will move along
   (a) helical path (c) circular path
   (b) straight line (d) its motion is unaffected
4. A charged particle will travel along helical path in a magnetic field only when its velocity vector is
   (a) along the field direction (c) perpendicular to the field direction
   (b) opposite to the field direction (d) at an angle with the field direction

Solved Numericals Each chapter supports numerous well-illustrated numerical examples.

Solved Numericals

1. Find the ratio of population of the two states in He–Ne laser that produces light of wavelength 6328 Å at 27^{0}C.

Solution: Eq. 4.14 gives the ratio of population as

$$\frac{N_2}{N_1} = \exp\ \exp\left[-\frac{E_2 - E_1}{kT}\right]\exp\left[\frac{\Delta E}{kT}\right]$$

Value of kT at room temperature (that is 27^{0}C) is 0.026 eV. The difference in energy levels emitting light of wavelength λ is given by

$$\Delta E = h\nu = \frac{hc}{\lambda}$$

Substituting the values of $h = 6.63 \times 10^{-34}$ J-s, $c = 3 \times 10^8$ m/s and 1 eV = 1.6×10^{-19} J, we get

3. Calculate l_{coh} for Hg vapour lamp emitting in the green portion of spectrum at wavelength of 546.1 nm with emission bandwidth $\Delta\nu = 6 \times 10^{+8}$ Hz.

Solution: $\tau = \frac{1}{\Delta\nu} = 16.6 \times 10^{-10}$ s

The coherence length, l_{coh} is given by

$$l_{coh} = c\tau = 3 \times 10^8 \times 16.6 \times 10^{-10} \text{m}$$
$$= 0.49 \text{ m}$$

4. If the ray width of the 10.6 μm transition of a CO_2 laser is 60 MHz; calculate the coherence length of the laser. If the cavity length is 2m, show that not more than one mode will oscillate.

the Book

Summary Summary at the end of each chapter enables quick revision of important concepts discussed in the chapter.

Summary

- Laser light is different from ordinary source of light in view of its exceptional directionality, high monochromaticity, intensity, and remarkable coherence.
- Laser light can be obtained by exceeding the rate of stimulated emission over stimulated absorption and spontaneous emission.
- The rate of stimulated emission can be maximized by creating population inversion in the medium and enclosing the medium between two mirrors.
- Out of three pumping schemes, the four-level scheme is more efficient than the three-level pumping scheme.
- Choice of pumping mechanism depends on the state of the material (solid, liquid, or gas) in which laser is to be obtained.
- Ruby laser is a solid-state laser that uses the optical pumping mechanism and belongs to the three-level pumping scheme. It is a high wattage laser source that gives out highly directional and intense light.
- Helium–Neon laser is a gas laser, which uses electrical pumping mechanism and belongs to the four-level pumping scheme. It is a low-wattage laser source that gives out highly directional, monochromatic, and coherent light.
- Semiconductor laser is a low-wattage, highly compact laser source, which uses p-n junction diode, made of semiconducting material.
- Laser holograms contain three-dimensional information of the object. This is a reliable method of storing large

Self-study Questions

1. (a) What is laser? How does it differ from an ordinary source of light?
   (b) Discuss the properties of laser.
2. Explain spatial coherence and temporal coherence.
3. Explain the terms coherence length and coherence time. Derive the expression for the coherence length of a wave in terms of line width $\Delta\lambda$ corresponding to frequency band $\Delta\nu$.
4. (a) What is stimulated absorption?
   (b) Explain the three quantum processes. Which processes are maximized in laser operation and how?
5. Explain the terms: stimulated emission, population inversion, pumping and metastable states. What is their

   (b) What is the role of resonant cavity length in supporting different frequency modes?
9. (a) What is the role of metastable state in laser media? Is it a necessary for occurrence of lasing?
   (b) In laser, active media should have preferably broad absorption band. Explain why?
10. (a) Describe the construction of a solid state ruby laser.
    (b) Describe the working of ruby laser using energy-level diagram.
    (c) Why are the end faces of the ruby rod silvered?
11. (a) Explain the construction and working of He–Ne laser. Explain the role of end mirrors in laser. What frequencies are amplified by this mirror system?

Review Questions Each chapter supports a wealth of questions to help students during exam preparation.

Numerical Problems Numerical problems are also given in the book in relevant chapters.

Numerical Exercises

1. Suppose that an X-ray with a wavelength of 3.75 pm is fired at a stationary electron such that the scattering angle is 85°. What is the resulting energy of the electron?
   (Ans. 1.96×10^{-14} J or 1.74×10^{5} eV)
2. During Compton collision with an electron, a photon of violet light (λ = 400 nm) is backward scattered through an angle 180°. How much energy is transferred to the electron in this collision?
   (Ans. 1.9×10^{-21} J or 3.76×10^{-5}ev)
3. Incident X-rays have a wavelength of 0.31 nm and are scattered by the free electrons in graphite. The scattering angle in is θ = 135.0°. What is the magnitude of the momentum of (a) the incident photon and (b) the scattered photon?
   (Ans. h = 6.626×10^{-34}J.s and c = 2.998×10^{8} m/s.) [(a) 21.23×10^{-25} kg.m/s (b) 21.11×10^{-25} kg. m/s]

   Calculate (a) wavelength of scattered radiation at 60°. (b) energy of recoiled electron.
   [Ans. (a) 0.0372 A (b) 166.67 KeV])
5. A photon collides on the graphite target at right angles. Find the percentage change in the energy of the photon when (a) Gamma ray photon with wavelength 0.013 Å is used (b) visible photon of wavelength 6000 Å is used.
   [Ans. (a) 65.14 % (b) 4.05×10^{-4} %])
6. A bacterium (mass = 2×10^{-15} kg) in the blood is moving at 0.33 m/s. What is the de Broglie wavelength of this bacterium? (Ans. 10.04×10^{-19} m)
7. How fast does a proton have to move in order to have the same de Broglie wavelength as an electron moving at the speed of 4.5×10^{6} m/s?
   (Ans. 2.45×10^{3} m/s)

Contents

Kinematics of Electron in Electric and Magnetic Field

1

Learning Objectives

After going through this chapter, you will be able to:

- Gain knowledge about the various concepts of electric field, electric potential, and magnetic field
- Understand the motion of electron in uniform electric field
- Learn the motion of electron in uniform magnetic field
- Solve problems related to uniform electric and magnetic fields
- Solve application-based problems related to uniform electric and magnetic fields

1.0 Introduction

Today's technology is strongly driven by electronics. A majority of instruments, appliances, and gadgets is based on the customized electron movement in the circuits due to application of electric or magnetic field. Charged particles travel in a straight-line path in vacuum and behave just as any point mass particle, when it is in a free state. Under the influence of the electric and magnetic fields, the charged particles can be subjected to the desired forces and hence, predictable trajectories/behaviour can be obtained.

In this chapter, the particle properties of electron and the basic concepts of electric and magnetic fields are discussed. How various combinations and configurations of electric and magnetic field can be used to control the particle trajectories is discussed as cases using simple mathematics. The applications of various cases are also stated.

1.1 Electric Field - Basic Concepts

We are surrounded by many devices that work on the phenomenon of electromagnetism. Electromagnetism involves the interaction of electric and magnetic fields. Let us first learn the basics of electric field.

1.1.1 Electric Charge

There exist two kinds of electrical charges in nature, namely positive charges and negative charges. Like charges repel and unlike charges attract each other. Matter in its neutral state contains equal amounts of positive charges and negative charges. The charges, either positive or negative, are actually a collection of elementary charges carried by fundamental particles, namely the proton and electron. Therefore any positive or negative charge Q can be detected and can be written as

$$Q = \pm ne$$

where n is an integer and takes values 1, 2, 3…. (1.1)

The charge is a scalar quantity and its SI unit is coulomb. In Eq. (1.1), 'e' is the elementary charge that has value $e = 1.6 \times 10^{-19}$ C. Both electron and proton have the charge of magnitude 'e'. When a physical quantity such as charge can have only discrete values rather than any value, we say that the quantity is *quantized*. Electrical charge is therefore quantized. Thus for example, a particle can have a charge of $+11e$ or $-5\ e$ or no charge at all; but it cannot have a charge of $3.57e$.

1.1.2 Coulomb Force

Charles Augustin Coulomb in 1785 first measured the electrical attraction and repulsion quantitatively and deduced the law that governs them. Thus, according to Coulomb, the force of attraction or repulsion between unlike and like charges is inversely proportional to the square of the distance between them.

$$\vec{F} = \frac{Q_1 Q_2}{4\pi \varepsilon_0 \varepsilon_r r^2} \tag{1.2}$$

where Q_1 and Q_2 are point charges separated by a distance r, ε_0 is called the permittivity of free space and its value is $\varepsilon_0 = 8.85 \times 10^{-12}$ F/m, and ε_r is called the relative permittivity of the medium in which the charges are kept. The force between two static charged particles is often referred to as *Coulomb force* and it is a vector quantity. Coulomb force will be maximum when the charges are in vacuum and it gets reduced due to the presence of medium as $\varepsilon_r > 1$ for all medium except vacuum and air. The Coulomb force is specified in Newton and acts along the line joining the two point charges, Q_1 and Q_2.

1.1.3 Electric Field

Many common forces might be referred to as contact forces. We exert force on an object by coming into contact with it; for example, you push or pull a box. Similarly, a tennis racket exerts force on a ball when they make contact with each other. On the other hand, both gravitational force and electrical force act over the distance. A force exists between two objects even when they are not in contact with each other. Any charge sets up its electric field in the space surrounding it. Thus, the electrical interaction takes place through the electric field. Hence, electrostatic force can come into play even when charges are not in actual contact.

We can investigate the electric field surrounding a charge or group of charges by measuring the force on a small positive test charge. By a test charge we mean a charge so small that the force it exerts does not significantly alter the distribution of the other charges that cause the field which is being measured.

The force on a tiny positive charge q_0 placed at point P in the vicinity of a charged object is shown in Fig. 1.1.

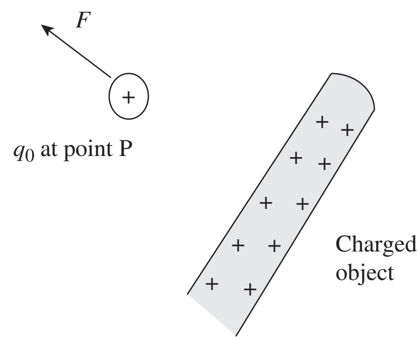


Fig. 1.1 A positive test charge q_0 placed at point P near a charged object

The electric field is a vector quantity and is defined in terms of the force on such a positive test charge. In particular, the electric field $\vec{E}$ at any point in space is defined as the force $\vec{F}$ exerted on a tiny positive test charge at that point divided by the magnitude of the test charge q_0.

$$\vec{E} = \frac{\vec{F}}{q_0} \tag{1.3}$$

Electric field has magnitude as given by Eq. (1.3), and its direction is that of the force $\vec{F}$ that acts on the positive charge (see Fig. 1.2). To define the electric field within some region we must similarly measure it at all points in the region. The SI unit of electric field is Newton per coulomb (N/C). An electric field has *unit intensity* if it exerts a force of one Newton upon one Coulomb. Although we use a positive test charge to define the electric field of a charged object, that field exists independently of the test charge.

Ideally, $\vec{E}$ is defined as the limit of F/q_0 as q_0 is taken smaller and smaller, approaching zero. The reason for defining $\vec{E} = \dfrac{\vec{F}}{q_0}$ as (with $q_0 \to 0$) is that $\vec{E}$ does not depend on the magnitude of the test charge q_0. This means that $\vec{E}$ describes only the effect of the charges creating the electric field at that point.

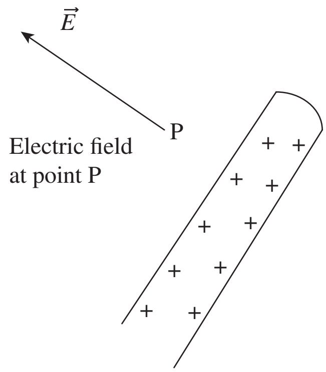


Fig. 1.2 Electric field E at point P produced by the charged object

To find the electric field at distance r from a single point charge Q, we assume a test charge q at that point. The coulomb force on test charge q is

$$\vec{F} = \frac{qQ}{4\pi\varepsilon_0\,\varepsilon_r\,r^2} \tag{1.4}$$

Therefore, electric field at that point would have magnitude

$$\vec{E} = \frac{qQ}{4\pi\varepsilon_0\,\varepsilon_r\,r^2} \times \frac{1}{q} \tag{1.5}$$

$$\vec{E} = \frac{Q}{4\pi\varepsilon_0\,\varepsilon_r\,r^2} \tag{1.6}$$

Thus, $\vec{E}$ is independent of test charge q; it depends only on the charge Q which produces the field.

1.1.4 Field Lines

Michael Faraday, who introduced the idea of electric fields in the 19th century thought of the space around the charged body as filled with lines of force. It is found to be convenient to represent and visualize electric field in terms of lines of force. The lines of force start from positive charge and end upon negative charge.

The relation between field lines and electric field is (1) the tangent to the line of force at any point gives the direction of the electric field $\vec{E}$ at that point. (2) The number of lines of force at any point passing through the cross-sectional area is proportional to the magnitude of electric field in that region as shown in Fig 1.6. The second relation means, field $\vec{E}$ is said to be strong if field lines are close and weak when the lines are far apart in that region. In a uniform electric field, the lines are straight, parallel, and uniformly spaced. In non-uniform fields, the field lines are curved and spaced non-uniformly. Figures 1.3, 1.4, and 1.5 show the electric field lines in different cases.

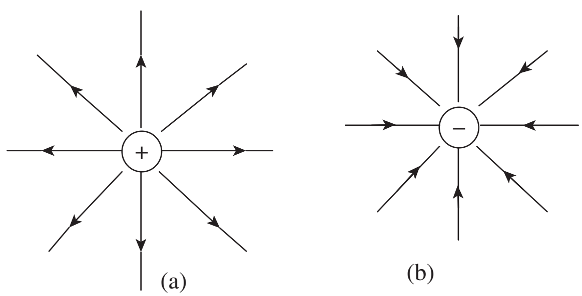


Fig. 1.3 Electric field lines (a) near a single positive point charge (b) near a single negative point charge

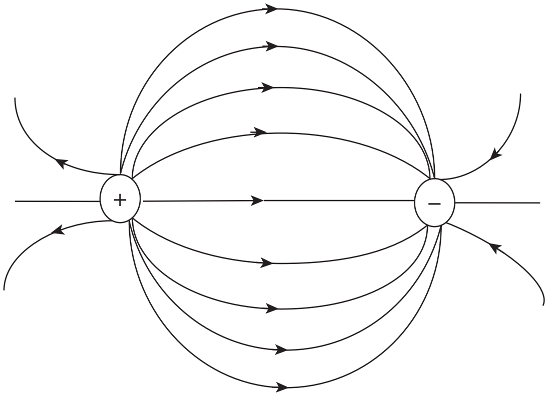


Fig. 1.4 Electric field lines near two equal charges of opposite sign

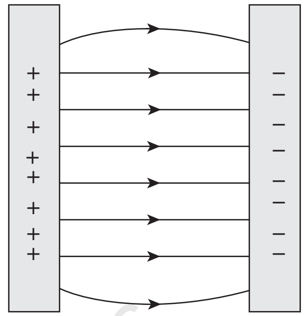


Fig. 1.5 Uniform electric field produced by a parallel plate capacitor. Field is non-uniform at the edges

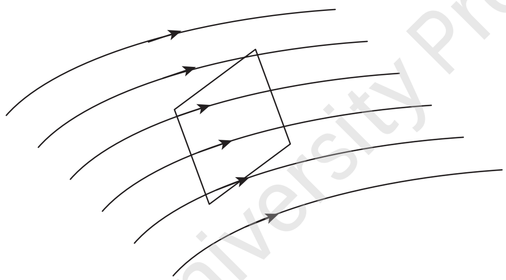


Fig. 1.6 Conventional representation of electric field intensity by the number of lines of force passing perpendicularly through a unit area

1.1.5 Equipotential Surfaces

An equipotential surface is a surface on which the potential has the same value at all points. No work is done on a charged particle by an electric field when the particle moves between two points i and f on the same equipotential surface as shown in Fig. 1.8. The equipotential surfaces produced by a point charge or a spherically symmetrical charge distribution are a family of concentric spheres as shown by dotted lines in Fig. 1.7. For a uniform field, the surfaces are a family of planes perpendicular to the electric field lines and thus to E, which is always tangent to these lines as shown in Fig. 1.8.

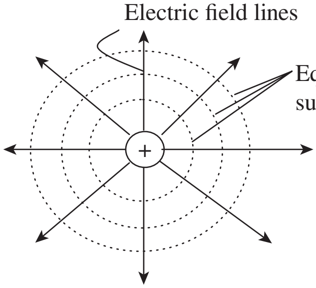


Fig. 1.7 Field of a point charge

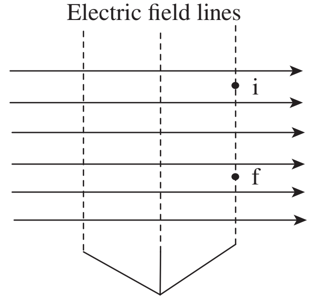


Fig. 1.8 Electric field lines and equipotential surfaces

1.2 Concepts of Electric Potential

In the previous sections we have identified one of the basic forces in the world, that is, the electric force. In this section we will associate energy viz. potential energy with the electrical force.

1.2.1 Electric Potential

Imagine a test charge q_0, for example, a proton in 'free fall' in an electric field. The motion of the test charge should be analysed in terms of energy transfer. So in particular we must be able to assign to the test charge an electric potential energy U, whose value for a given test charge depends only on the position of that charge in the electric field. In case of gravitation, if a test body falls or moves from an initial point i to a final point f, we define the difference in its gravitational potential energy as

$$\Delta U = U_f - U_i = -w_{if} \tag{1.7}$$

where w_{if} is the work done by the gravitational force on the body.

The same equation can be used as a definition of the difference in the electric potential energy of a test charge q_0 as it moves from an initial point i to a final point f, in an electric field. The difference in the electric potential energy of a test charge between these points is the negative of the work done by the electrostatic force via the electric field on the test charge during its motion. Thus, the field itself does the work on the charge. Now from definition of difference in electric potential energy of a test charge between two points, we move to the definition of electric potential energy of a test charge at a single point. We will first assign the value zero to potential energy at point i which is at infinity.

Now

$$U = -w_{\infty f} \tag{1.8}$$

Thus, the potential energy U of a test charge q at any point is equal to the negative of the work done $-w_{\infty f}$ on the test charge by the electric field. Electric potential energy is the energy of a charged object in an external electric field and is measured in joules. Now the potential energy of a point charge in an electric field depends on the magnitude of the charge as well as the field. However, potential energy per unit charge has a unique value at any point in an electric field.

Thus the potential energy per unit charge $\frac{U}{q}$ is independent of the magnitude of test charge q_0 and is characteristic of any electric field. Electric potential is a property of the field itself, whether or not a charged object has been placed in it. It is measured in J/C or volt. The quantity $\frac{U}{q}$ is called *electric potential* which is a scalar quantity.

$$V = \frac{U}{q} \tag{1.9}$$

$$\Delta V = V_f - V_i \tag{1.10}$$

$$= \frac{U_f}{q} - \frac{U_i}{q}$$

$$= \frac{\Delta U}{q} \tag{1.11}$$

$$\Delta V = V_f - V_i = \frac{-w_{if}}{q} \tag{1.12}$$

Thus, if $-w_{if}$ is positive, negative, or zero, then potential at f will be less, greater than, or same as potential at i.

1.2.2 Calculating the Potential Difference from Field

We can calculate the potential difference between any two points i and f in an electric field if we know the electric field vector all along any path connecting these points. To find the potential from $\vec{E}$ field, we find the work done on a positive test charge by the field as the charge moves from i to f.

Consider an arbitrary electric field represented by field lines as shown in Fig. 1.9 and a positive test charge q_0 that moves along the path from point i to point f. At any point on the path, an electrostatic force $q_0\vec{E}$ acts on the charge as it moves through the differential displacement ds.

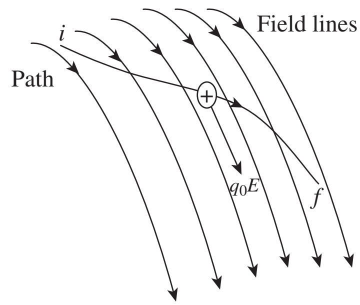


Fig. 1.9 A test charge q_0 moves from point i to point f along the path shown in a non-uniform electric field. During a displacement ds, an electrostatic force q_0E acts on the test charge. This force points in the direction of the field line at the location of the test charge

The differential work dw done on a particle by a force $\vec{F}$ during displacement ds is

$$dw = \vec{F}.\, ds \tag{1.13}$$

Using Eq. (1.3)

$$dw = q_0\, \vec{E}.\, ds \tag{1.14}$$

To find the total work w_{if} done by the field on the particle as it moves from point i to point f, we add via integration the differential work done on the charge for all the differential displacements ds along the path.

$$w_{if} = q_0 \int_i^f E.\, ds \tag{1.15}$$

If we substitute the total work w_{if} from Eq. (1.15) in Eq. (1.12), we get

$$V_f - V_i = -\int_i^f E.ds \tag{1.16}$$

Thus the potential difference $V_f - V_i$ between any points i and f in an electrical field is equal to the negative of the line integral (meaning the integral along the path) of $E.ds$ from point i to point f. If the electric field is known throughout a certain region, Eq. (1.16) allows us to calculate the difference in potential between any two points in the field. As the electrostatic force is conservative, all paths give the same result. If we choose the potential at point i to be zero and at point f as V, then Eq. (1.16) becomes

$$V = -\int_i^f E.\, ds \tag{1.17}$$

If the field is uniform, that is, $\vec{E}$ is constant over the path and can be moved outside the integral then we get

$$V = -Ed$$

$$\text{or } E = -\frac{V}{d} \tag{1.18}$$

Here d is the distance between the initial point 'i' and final point 'f'.

Potential due to point charge

Consider a point P at a distance r from a fixed point charge of magnitude q as shown in Fig. 1.10. In order to use Eq. (1.17) we imagine that a positive test charge moves from infinity to point P. As the path followed by the test charge does not matter, we consider a line that extends from infinity to point P along the radius from the point charge q.

We now evaluate the dot product ($\vec{E}$.ds) in Eq. (1.17) along the path taken by the test charge. The test particle is at some intermediate point, at distance r' from the point charge. The angle between $\vec{E}$ and ds is 180 degree. Using this angle and writing the magnitude of the displacement ds as dr' we can write the dot product of Eq. (1.17) as

$$V_f - V_i = -\int_i^f \vec{E}.\,ds \qquad (1.19)$$

$$V = -\int_i^f \vec{E}.\,ds \qquad (1.20)$$

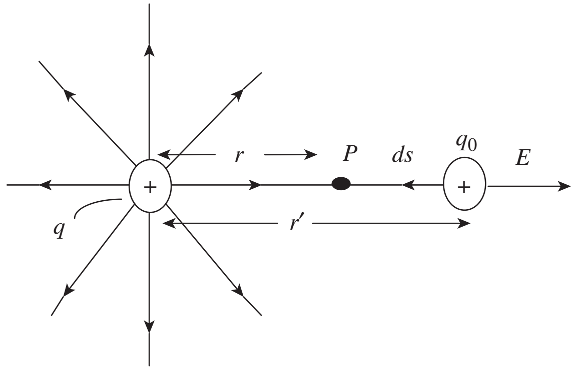


Fig. 1.10 The positive point charge q produces an electric field E and an electric potential V at point P

$$= -\int_\infty^r \frac{q}{4\pi\varepsilon_0\,\varepsilon_r r'^2}\,dr'$$

$$= \frac{-q}{4\pi\varepsilon_0\,\varepsilon_r}\left[\frac{-1}{r'}\right]_\infty^r = \frac{-q}{4\pi\varepsilon_0\,\varepsilon_r}\left[-\frac{1}{r}\right]$$

$$\therefore V = \frac{q}{4\pi\varepsilon_0\,\varepsilon_r r} \qquad (1.21)$$

Thus, the potential V at any point around a positive point charge is positive, relative to zero potential at infinity.

1.2.3 Calculating Field from the Potential

Let us see how to find the electric field when we know the potential. Figure 1.11 shows the cross sections of a family of closely spaced equipotential surfaces. The potential difference in between each pair of adjacent surfaces is 'dV'.

The field $\vec{E}$ at any point P is perpendicular to the equipotential surface through P. Suppose that a positive test charge q_0 moves through a displacement ds from one equipotential surface to the adjacent surface, then by using Eq. (1.12), we get

$$dw = -q_0\,dV$$

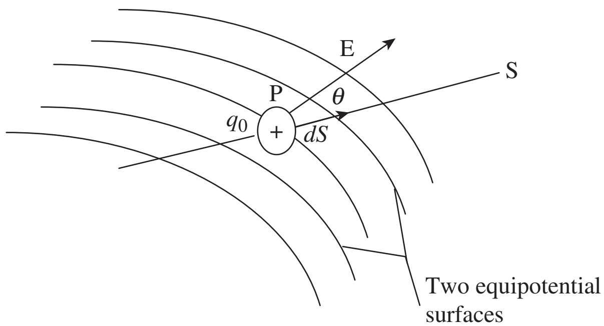


Fig. 1.11 A test charge q_0 moves distance ds from one equipotential surface to another. The displacement ds makes an angle θ with the direction of the electric field $\vec{E}$

Work done by the field may also be written using Eq. (1.14) as

$$dw = q_0\vec{E}.\,ds$$

Equating the two equations of dw, we get

$$-q_0 dV = q_0 \vec{E}.\,ds$$

$$-q_0 dV = q_0 E(\cos\theta) ds$$

$$E\cos\theta = \frac{-dV}{ds} \tag{1.22}$$

$$E_s = -\frac{dV}{ds} \tag{1.23}$$

We have added a subscript to $\vec{E}$ and switched to partial derivative to emphasize that Eq. (1.23) involves only the variation of V only along a specified axis (here it is S axis). The component of E in any direction is the negative of the rate of change of electric potential with distance in that direction. Thus,

$$E_x = -\frac{dV}{dx},\ E_y = -\frac{dV}{dy}, \text{ and } E_z = -\frac{dV}{dz} \tag{1.24}$$

Thus, if we know the value of V for all points in the region around a charge direction, that is, $V(x,y,z)$ we can find the component of $\vec{E}$ at any point by taking its partial derivatives.

1.3 Basic Concepts of Magnetic Field

So far we have examined the fundamental concepts related to electric field. Now in this section we will learn how a magnetic field can be set up in a region and the magnitude and direction of the force produced by it.

1.3.1 Current and Magnetic Field

Charge generates electric field and electric field exerts forces on charges. This can be written as

Charge ↔ electric field ↔ Charge

Similarly a magnet produces a magnetic field B at all points in the space around it. Magnetic fields are produced by permanent magnets or electromagnets. In electromagnets a coil is wound around an iron core and current is sent through the coil. Here the strength of the magnetic field depends on the magnitude of current. By using the analogy with electric field we can write,

Current ↔ magnetic field ↔ Current

Current generates magnetic field and magnetic field exerts force on current. But from where does the magnetic field come? Experiments show that the magnetic field is developed from the moving electric charges. A magnetic field is set up only when the charge is moving. In a permanent magnet, the moving charges are the electrons in the atoms that make up the magnet. In electromagnets the moving charges are electrons drifting through the coils of wire that surround these magnets. Thus, moving charges in an atom or current set up in a wire produces the magnetic field. If we place a moving charge or a wire carrying current in the magnetic field, the magnetic field will act on it. We can define magnetic field by finding the force acting on a test charge. When a test charge q moves with velocity $\vec{v}$ in the magnetic field of strength $\vec{B}$, then the force exerted by the magnetic field is given by

$$\vec{F}_L = q(\vec{v} \times \vec{B}) \tag{1.25}$$

This equation represents

(1) Magnetic force F_L always acts perpendicular to the velocity vector, that is, it can neither speed up nor slow down a moving charge particle, but can only deflect its direction of motion. Thus, magnetic force can change only the direction of the particle's velocity v but cannot change the kinetic energy of the particle.

(2) A magnetic field exerts no force on the charge that is at rest or moves parallel or anti-parallel to the field.

$$F_L = qvB\sin\theta = 0 \text{ when } \theta = 0° \text{ or } 180°.$$

(3) The maximum value of deflecting force occurs when the test charge is moving perpendicular to the magnetic field, that is, when $\theta = 90°$.

$$\vec{F}_L = qvB \tag{1.26}$$

(4) The magnitude of deflecting force is directly proportional to q and $\vec{v}$. The greater the charge of the particle and the faster it is moving, the greater the magnetic deflecting force. If the particle is stationary or electrically neutral then no magnetic force acts on it.

(5) The direction of magnetic deflecting force depends on the polarity of the charge, q. The direction of force is determined for the positive charge using the right-hand thumb rule for cross product (see Fig. 1.12). The curling of fingers indicates the velocity vector turned into the magnetic field vector and the thumb denotes the direction of the magnetic force. For the negative charge the direction of magnetic force will be in opposite direction as given by the thumb rule.

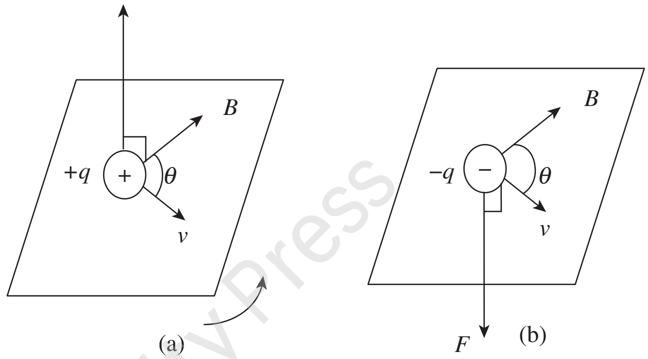


Fig. 1.12 Right hand rule to determine the direction of Lorentz force – (a) On a positive charge (b) On a negative charge

The SI unit of $\vec{B}$ is tesla or wb/m^2 (1 Tesla = 10^4 gauss).

If a charged particle moves through a region in which both electric and magnetic fields are present, then the resultant force is found by combining both the forces.

$$\vec{F} = q\vec{E} + q\,(\vec{v}\times\vec{B}) \tag{1.27}$$

This is sometimes called the *Lorentz equation*.

In next sections we will learn the behaviour of a charged particle in uniform electric and magnetic field which forms the basis of the countless electromagnetic devices.

Checkpoint 1

1. When an electron moves on an equipotential surface, its potential energy
   (a) increases
   (b) decreases
   (c) remains constant
   (d) becomes zero
2. In an uniform electric field, the field lines are
   (a) parallel
   (b) perpendicular
   (c) curved
   (d) none of these
3. If an electron moves through a potential difference of 100V, its kinetic energy is
   (a) 100 J
   (b) 16×10^{-19} J
   (c) 100 eV
   (d) none of these
4. An electric field has unit intensity if it
   (a) exerts a force of one Newton upon one Coulomb.
   (b) takes one joule to carry one Coulomb through it.
   (c) exerts a force of one Dyne upon one electron.
   (d) exerts a force of 2×10^{-7} Newton on 1 one metre long wire carrying one ampere.

1.4 Motion of Electron In Uniform Electric Field

1.4.1 Electron Motion in Longitudinal Uniform Electric Field

A uniform electric field can be generated with the help a pair of parallel plates A and B of area '*a*' separated by a small distance '*d*'. The length of the plates should be large as compared to '*d*'. If a dc voltage source is connected between the plates, then one plate acquires a negative charge and the other a positive charge of equal magnitude and an electric field is produced in the region between the plates. If the separation *d* of the plates is small compared to their size, the electric field in the region will be fairly uniform except near the edges. The direction of electric field is from positive plate A towards the negative plate B or along the X-axis as shown in Fig. 1.13.

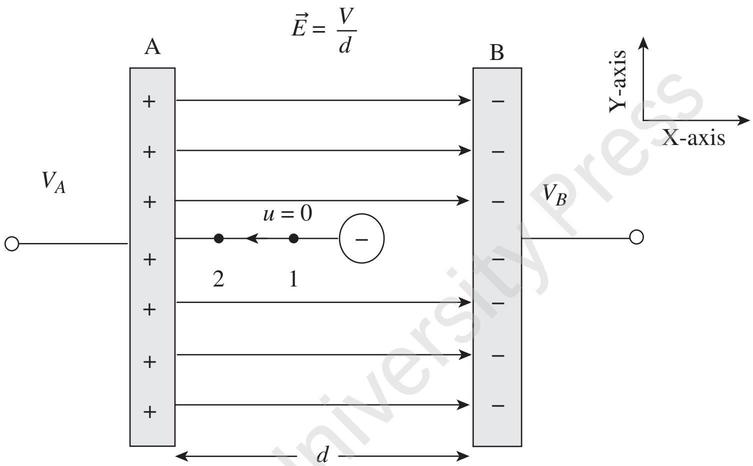


Fig. 1.13 Motion of electron in uniform longitudinal electric field

If the potential difference between the plates A and B is V volts, the electric field strength E is given by Eq. (1.18), that is, $E = \frac{V}{d}$. These two metal plates which form a parallel plate capacitor are kept in an evacuated space so that the particle paths are not distorted by the collisions with atmospheric gas molecules. Let us consider an electron of mass 'm' and charge '*e*', which is initially at rest in the uniform electric field. As soon as the field is established, the electron experiences a force due to the electric field, which is given by

$$\vec{F} = -e\vec{E} \tag{1.28}$$

The negative sign indicates that the force $\vec{F}$ accelerates the electron in a direction opposite to that of $\vec{E}$. According to the second law of Newton, the acceleration is given by

$$\vec{a} = \frac{\vec{F}}{m}$$

$$\vec{a} = \frac{-e\vec{E}}{m} \tag{1.29}$$

The negative sign in Eq. (1.29) is dropped in subsequent expressions as it indicates only that the direction of acceleration is opposite to that of the electric field. In the above equation $e, \vec{E}$, and m are constants and therefore the electron acceleration is constant, that is, the electron is moving with uniform acceleration. As the electron has constant acceleration, we can apply the equations of kinematics to know the motion of uniformly accelerated electron. The motion resembles that of a freely falling body in a gravitational field. By using the kinematic equation for finding the final velocity and initial condition that at $x = 0$, $u = 0$ (electron is initially at rest), we can write the final velocity as

$$v = u + at = 0 + at$$

Using Eq. (1.29) for acceleration, the final velocity becomes

$$v = \frac{eEt}{m} \tag{1.30}$$

Using kinematic equation for displacement, we can find the displacement along X-direction at any time 't' as given by

$$x = ut + \frac{1}{2}at^2 = 0 + \frac{1}{2}at^2$$

Substituting for acceleration we get

$$x = \frac{eEt^2}{2m} \tag{1.31}$$

To find the kinetic energy of an electron after it has travelled some distance x, using the kinematic equation, we get

$$v^2 = u^2 + 2as = 0 + 2ax$$

Substituting for acceleration we get

$$v^2 = \frac{2Eex}{m}$$

Therefore kinetic energy of an electron is given by

$$\text{Kinetic energy} = \frac{1}{2}mv^2 = \frac{1}{2}m \times \frac{2Eex}{m} = Eex \tag{1.32}$$

Thus, kinetic energy of an electron in longitudinal electric field is directly proportional to the distance travelled, because electric field E and charge of electron e are constants.

1.4.2 Energy Acquired by Electron in Electric Field

Referring to Fig.1.13 the electric field $\vec{E}$ causes electron motion towards the positively charged plate A. The force on the electron can be written by using Eq. (1.24) as

$$F = -eE = e\frac{dV}{dx} \tag{1.33}$$

Work is done on the electron by the electric field in moving it towards the plate A. The work done dw in moving the electron through an infinitesimal distance dx is given by

$$dw = F.dx$$

Total work done in moving the electron from point 1 to point 2 by using Eq. (1.33) is given by

$$w_{12} = \int_1^2 F.dx = \int_1^2 e\frac{dV}{dx}.dx$$

$$w_{12} = \int_1^2 edV = e(V)_1^2$$

Therefore,

$$w_{12} = e(V_2 - V_2) \tag{1.34}$$

In Eq. (1.34) V_1 and V_2 are the potentials at positions 1 and 2 respectively. The electron moves in the direction of increasing potential. According to work–energy theorem, the work done by the field in moving the electron is equal to the kinetic energy acquired by the electron.

According to Newton's Second Law $\vec{F} = ma$

$$= \frac{d}{dt}(mv) = m.\frac{dv}{dt}$$

As the work done is given by $dw = F.dx$

Substituting force in terms of velocity by using the above equation, we get

$$W_{12} = \int_1^2 F.dx = \int_1^2 m\frac{dv}{dt}dx = \int_1^2 m\frac{dv}{dx}\times\frac{dx}{dt}dx$$

$$= \int_1^2 mvdv = m\left(\frac{v^2}{2}\right)_1^2$$

$$\therefore W_{12} = \frac{1}{2}m(v_2^2 - v_1^2) \tag{1.35}$$

where v_1 and v_2 are electron velocities at positions 1 and 2, respectively.

Equating Eqs (1.34) and (1.35), we get

$$e(V_2 - V_1) = \frac{1}{2}m(v_2^2 - v_1^2) \tag{1.36}$$

Equation (1.36) indicates that the electron gains energy from the electric field. Assuming that the electron starts from rest and accelerates through a potential difference of V volts, we can rewrite Eq. (1.36) as

$$eV = \frac{1}{2}mv^2 \tag{1.37}$$

Equation (1.37) gives us an important relation between the kinetic energy and the potential energy of any charged particle accelerated through potential difference of V volts. As the amount of kinetic energy acquired is usually very small compared to a joule, the energy is expressed as an atomic unit, viz. electron volt. One *electron volt* is defined as the energy acquired by an electron when it is accelerated through a potential of one volt and is given by

$$1\text{ eV} = 1.602\times10^{-19}\text{ J} \tag{1.38}$$

For expressing the energy of atomic particles, electron volt unit is widely used. The higher values of energies are expressed as

$$1\text{ MeV}(1\text{ million eV}) = 10^6\text{eV and 1B eV}(1\text{ billion eV}) = 10^9\text{ eV}$$

The final or the terminal velocity v is given as

$$v = \sqrt{\frac{2eV}{m}} \tag{1.39}$$

For electron, substituting the values of its charge and mass, viz. 1.6×10^{-19} C and 9.1×10^{-31} kg we get

$$v = 5.93\sqrt{V} \times 10^5 \text{ m/s} \tag{1.40}$$

The application of longitudinal uniform electric field is to accelerate charged particles, that is, to impart kinetic energy to the charged particles.

1.4.3 Electron Motion in Transverse Uniform Electric Field

Let 'A' and 'B' be two plane parallel metal plates of length 'l' oriented horizontally and separated by distance 'd' as shown in Fig. 1.14. A potential difference of V volts is applied between the plates which produces a vertically acting uniform electric field (along negative Y-direction) directed from plate A to B. The electron enters the electric field in the positive X-direction with an initial velocity v_0 .The initial velocity of the electron is acquired when it is accelerated through a potential difference V_A as given by

$$v_0 = \sqrt{\frac{2eV_A}{m}} \tag{1.41}$$

As the electron enters the plates, it will experience a force due to electric field along Y-direction which is given by

$$\vec{F} = e\vec{E}$$

As the electric field acts only in the Y-direction, there will be no force along either X- or Z-direction. Therefore acceleration of the electron along the X- and Z-directions is zero and thus velocity is constant. Since the electrons move initially along the X-direction with an initial velocity, v_0 it will continue to travel

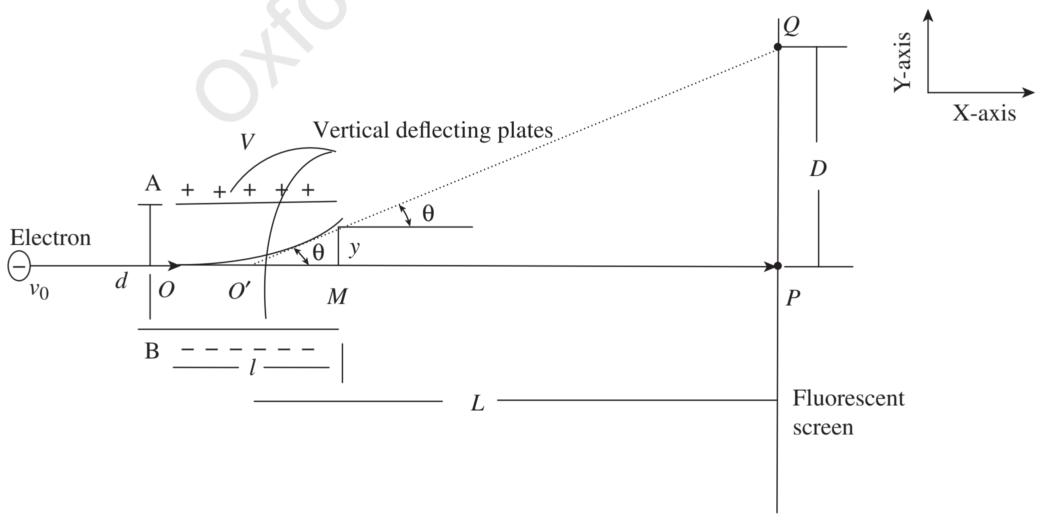


Fig. 1.14 Electron motion in transverse uniform electric field

along the X-axis with the same velocity. As electric field is acting along Y-direction, the force experienced by electron along Y-direction is

$$\vec{F}_y = -m\vec{a}_y \tag{1.42}$$

$$e\vec{E} = -m\vec{a}_y$$

$$\vec{a}_y = \frac{-e\vec{E}}{m} = \text{constant} \tag{1.43}$$

Using initial conditions of zero velocity along Y-direction, velocity of the electrons at any time 't' by using kinematic equation for velocity is

$$v_y = 0 + a_y t \tag{1.44}$$

Substituting Eq. (1.43) in Eq. (1.44) we get

$$v_y = \frac{-eEt}{m} \tag{1.45}$$

Using the initial condition of zero displacement $y_0 = 0$ and $u_y = 0$ the displacement of the electron in the Y-direction is

$$y = 0 + \frac{1}{2} a_y t^2 \tag{1.46}$$

$$y = \frac{-eEt^2}{2m} \tag{1.47}$$

Along X-direction $a_x = 0$ so the distance x travelled by the electron in the time interval t is given by

$$x = v_0 t + 0$$

$$x = v_0 t \tag{1.48}$$

From Eq. (1.48), we can write

$$t = \frac{x}{v_0} \tag{1.49}$$

Using Eqs (1.47) and (1.49) we get

$$y = -\frac{eEx^2}{2mv_0^2} \tag{1.50}$$

This represents vertical deflection over the horizontal distance travelled by the electron in the transverse electric field. Ignoring the negative sign as it indicates direction, the magnitude of vertical displacement can be written as

$$y = kx^2 \tag{1.51}$$

where $k = \dfrac{eE}{2mv_0^2}$ is a constant. Equation (1.51) is that of a parabola; hence, we can say that an electron travels along a parabolic path in the transverse uniform electric field. Thus, the deflection produced at any point within the plates can be found out by using Eq. (1.50).

The vertical deflection produced due to the electric field cannot be measured as we cannot see electrons. If we keep a fluorescent screen in the path of the electron when it ejects out of the electric field, then we can measure the deflection produced by the applied electric field. The whole assembly is kept in an evacuated glass tube called cathode ray tube abbreviated as CRT of which the front face is the screen.

After leaving the deflection plates, that is, electric field, the electron travels along a straight line as shown in Fig. 1.14. The electrons hit the fluorescent screen enabling the measurement of the deflection produced due to transverse electric field. The straight line along which electrons move after exiting the deflecting plates is a tangent to the curve at the end of the plates. If this tangent is extended backwards, then it intersects the X-axis at point O' as shown in Fig. 1.14. This point O' is called the *virtual source of electrons* as they appear to come from this point. This apparent origin of electrons is at the centre of the deflection plates and the screen is kept at a distance 'L' from it. Let us obtain an expression for the deflection produced on the screen.

The slope of the tangent to the parabolic path is obtained by differentiating Eq. (1.50) and dropping the negative sign,

$$\frac{dy}{dx} = \frac{eEx}{mv_0^2} \tag{1.52}$$

The slope of the parabola at the end of the plates is obtained by using $x = l$ (where l is the length over which electric field acts) in the above equation,

$$\therefore \frac{dy}{dl} = \frac{eE}{mv_0^2} l \tag{1.53}$$

$$\frac{dy}{dl} = \tan\theta = \frac{eE}{mv_0^2} l \tag{1.54}$$

From the geometry of the Fig. 1.14, the deflection 'D' produced on the screen is given by

$$D = L \tan\theta \tag{1.55}$$

Substituting the value of tan θ by using Eq. (1.54), we get

$$D = \frac{LeEl}{m{v_0}^2} \tag{1.56}$$

Here v_0 is the initial velocity acquired by an electron when accelerated through a potential V_A. So substituting Eq. (1.41), we get

$$D = \frac{LeEl}{m}\left(\frac{m}{2ev_A}\right) = \frac{LEl}{2V_A} \tag{1.57}$$

As electric field $\vec{E}$ is given by $\qquad \vec{E} = \frac{V}{d}$

Equation (1.57) can also be written as $\qquad D = \frac{LlV}{2dV_A} \qquad (1.58)$

Angular displacement of the electron path in the electric field is given by

$$\theta = \tan^{-1}\left(\frac{D}{L}\right) \tag{1.59}$$

Substituting value of D from Eq. (1.58), we get

$$\theta = \tan^{-1}\left(\frac{lV}{2\,dV_A}\right) \tag{1.60}$$

Time spent or transit time of the electron in the electric field is given by

$$t = \frac{l}{v_0} \tag{1.61}$$

The deflection sensitivity, 'S' is defined as the deflection produced per unit deflecting voltage applied to deflecting plates. It is given by

$$S = \frac{D}{V} = \frac{lL}{2\,dV_A} \tag{1.62}$$

From the expression of sensitivity it is clear that we can increase it by increasing the values of L and l. But we cannot increase L (distance of screen from the centre of the deflecting plates) beyond a certain limit as it will make the cathode ray tube longer which will make it difficult to handle. Similarly we cannot increase the length of the plates l , because a large deflection of electrons is not possible as electrons may hit the plates instead of coming out. The distance between the plates d cannot be reduced for the same reason. Hence, optimum values of L, l and d (these are called tube constants) are selected by the manufacturer for the desired sensitivity of the tube. From Eq. (1.62) sensitivity is inversely proportional to the accelerating potential.

$$S \propto \frac{1}{V_A} \tag{1.63}$$

According to Eq. (1.63) sensitivity will decrease if the accelerating potential is more. This is because the electrons will be moving very fast and will not stay in the field for sufficient time for field to show its effect. Thus, the sensitivity is affected by the accelerating potential.

The reciprocal of the deflection sensitivity is called *deflection factor* 'G', which is given by

$$G = \frac{1}{s} = \frac{2dV_A}{lL} \tag{1.64}$$

The application of transverse uniform electric field is to deflect the charged particle along the desired trajectory. Thus, we can manipulate the path of the charged particle by using the transverse electric field.

1.4.4 Electron Projected at an Angle into Uniform Electric Field

Let 'A' and 'B' be two plane parallel metal plates oriented horizontally and separated by distance 'd' as shown in

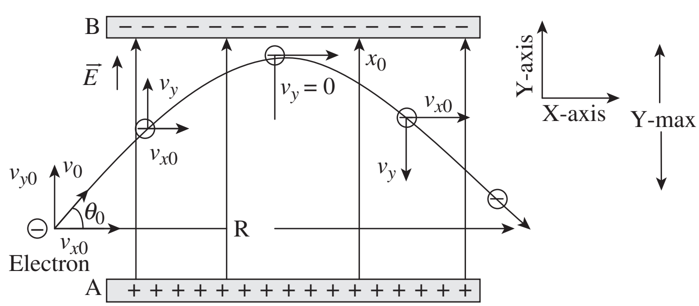


Fig. 1.15 Projectile motion of an electron in a uniform electric field.

Fig. 1.15. A potential difference of V volts is applied between the plates which produces a vertically acting uniform electric field (along Y-direction) directed from plate A to B. Suppose an electron is projected into a uniform electric field at an angle θ_0 with the X-axis and with an initial velocity v_0. The electric field acts in the positive Y-direction and the electron gets accelerated in the negative Y-direction. It means that the electron is projected such that it travels against the force due to electric field. The acceleration of electron is constant and the motion of electron will be very much similar to that of a projectile in a gravitational field. The equations of kinematics in two dimensions can be extended to the present case.

The velocity component in X-direction v_0 remains constant as no field is acting along X-direction, while v_y decreases initially and again increases when the electron reverses its path. The initial velocity components along X- and Y-directions are,

$$v_{x0} = v_0 \cos\theta_0$$

$$v_{y0} = v_0 \sin\theta_0 \tag{1.65}$$

The components of final velocity at time t along X- and Y-directions are

$$v_x = v_{x0} = v_0 \cos\theta_0 = \text{constant} \tag{1.66}$$

$$v_y = v_{y0} + at = v_0 \sin\theta_0 + at \tag{1.67}$$

From Eqs (1.66) and (1.67) we can obtain X- and Y-coordinates for the electron at any time by using kinematic equation as follows.

X-coordinate:

$$S = ut + \frac{1}{2}at^2$$

$$\therefore x = v_{x0}t = v_0 \cos\theta_0 t \tag{1.68}$$

Y-coordinate:

$$y = v_{y0}t + \frac{1}{2}at^2 = (v_0 \sin\theta_0)t + \frac{1}{2}at^2 \tag{1.69}$$

From Eq. (1.68)

$$t = \frac{x}{v_0 \cos\theta_0}$$

Substituting t in Eq. (1.69), we get

$$y = v_0 \sin\theta_0\left(\frac{x}{v_0\cos\theta_0}\right) + \frac{1}{2}a\left(\frac{x}{v_0\cos\theta_0}\right)^2$$

$$y = (\tan\theta_0)x + \frac{1}{2}a\left(\frac{x^2}{v_0^2\cos^2\theta_0}\right)$$

Rearranging the terms we can rewrite the equation as

$$y = (\tan\theta_0)x + \left(\frac{a}{2v_0^2\cos^2\theta_0}\right)x^2$$

$$y = ax + bx^2 \tag{1.70}$$

The quantities in the bracket are constants, $a = \tan\theta_0$ and $b = \dfrac{a}{2v_0^2 \cos^2\theta_0}$.

Equation (1.70) is the equation of a parabola. Therefore the trajectory of an electron projected into a uniform electric field is a parabola. Various parameters of motion of a charged particle can be obtained similar to the projectile in a gravitational field.

The maximum distance that the charged particle reaches in Y-direction, Y_{max}

Using kinematic equation, $v^2 = u^2 + 2as$ along Y-direction, we get

$$v_y^2 = v_{y0}^2 + 2ay \tag{1.71}$$

At maximum height the Y-component of velocity reduces to zero.

$$0 = v_{y0}^2 + 2\,a\,Y_{max}$$

$$0 = (v_0 \sin\theta_0)^2 - 2aY_{max}$$

The negative sign is due to the deceleration

$$2aY_{max} = v_0^2 \sin^2\theta_0$$

$$Y_{max} = \frac{v_0^2 \sin^2\theta_0}{2a} \tag{1.72}$$

Time taken by the charged particle to reach the maximum distance in the Y-direction

Using kinematic equation, $v_y = v_{y0} + at$ along Y-direction and as at maximum height the Y-component of velocity reduces to zero, we get

$$0 = v_0 \sin\theta_0 + at$$

$$\therefore t = \frac{v_0 \sin\theta_0}{a} \tag{1.73}$$

Time of flight *T* It is the time taken by the charged particle to return to its original position along the X-direction which is exactly twice the time required to reach the maximum height.

$$T = 2t$$

Substituting the value of t from Eq. (1.73), we get

$$T = \frac{2v_0 \sin\theta_0}{a} \tag{1.74}$$

Range (R) It is the horizontal distance travelled (along X-direction) by the charged particle when it returns to its initial launch height. As the distance travelled along X-direction is due to X-component of the velocity, the distance covered along X-direction is the product of the X-component of the velocity and the time of flight.

$$\therefore \mathrm{R} = v_x T \tag{1.75}$$

Substituting Eq. (1.74) for T (time of flight), we get

$$R = v_0 \cos\theta_0 \left(\frac{2v_0 \sin\theta_0}{a}\right) \tag{1.76}$$

$$R = \frac{v_0^2}{a}(2\sin\theta_0 \cos\theta_0)$$

$$R = \frac{v_0^2 \sin 2\theta_0}{a} \tag{1.77}$$

Checkpoint 2

1. Electric field lines are always …… to equipotential surfaces.
   (a) perpendicular
   (b) parallel
   (c) opposite
   (d) none of the above
2. The application of longitudinal electric field is to
   (a) deflect the particle
   (b) decelerate the particle
   (c) accelerate the particle
   (d) none of the above
3. The deflection sensitivity of CRT in electrostatic case is
   (a) inversely proportional to V_A
   (b) does not depend on V_A
   (c) inversely proportional to $\sqrt{V_A}$
   (d) directly proportional to V_A
4. When an electron moves towards the negatively charged plates, it will
   (a) accelerate
   (b) decelerate
   (c) stops abruptly
   (d) none of the above
5. The terminal velocity of an electron is:
   (a) directly proportional to accelerating potential
   (b) indirectly proportional to accelerating potential
   (c) directly proportional to square root of accelerating potential
   (d) indirectly proportional to square root of accelerating potential
6. In an electric field the range covered will be maximum for the particles projected at same angle and with same velocity for particles with
   (a) higher specific charge
   (b) lower specific charge
   (c) zero specific charge
   (d) does not depend on specific charge
7. The electrostatic deflection sensitivity is independent of
   (a) tube constants
   (b) electric field
   (c) accelerating potential
   (d) none of the above
8. The deflection of an electron beam on a CRT screen is 10 mm. If the pre-accelerating anode voltage is halved and the potential between the deflection plates is doubled, the deflection of the electron beam will be
   (a) 80 mm
   (b) 20 mm
   (c) 40 mm
   (d) 10 mm
9. A charged particle projected at an angle with electric field will follow
   (a) circular path
   (b) helical path
   (c) parabolic path
   (d) straight line path
10. For an electron travelling with initial velocity 2×10^3 m/s, the time taken by the electron to travel the electric field region of 200 mm width is
   (a) 10^{-4} s
   (b) 10^2 s
   (c) 10^4 s
   (d) 200 s

1.5 Motion of Electron in Uniform Magnetic Field

A static magnetic field does not exert any force on a charged particle at rest. When the charged particle moving with a velocity v enters a magnetic field, it experiences a magnetic force as given by Eq. (1.25)

$$\vec{F}_L = q(\vec{v} \times \vec{B})$$

The infinitesimal work done by the magnetic field in moving the charged particle through a small distance dx is dw which is calculated as follows.

$$dw = \vec{F}_L \bullet dx = q(\vec{v} \times \vec{B}) \bullet dx$$

As velocity is distance per unit time, distance dx can be written as $dx = vdt$. Substituting this value of dx in the above equation we get

$$dw = q(\vec{v} \times \vec{B}) \bullet v\, dt = 0 \tag{1.78}$$

Thus, from Eq. (1.78) it is proved that a steady magnetic field does no work on the moving charge particles.

From the above result and work–energy theorem, the kinetic energy of the electron does not change due to the action of magnetic field. As force F_L and velocity are perpendicular to each other, we can write

$$F_L \bullet v = ma \bullet v = m\left(\frac{dv}{dt}\right) \bullet v = 0 \tag{1.79}$$

Therefore

$$\frac{d}{dt}\left(\frac{mv^2}{2}\right) = 0$$

$$\text{or } \frac{1}{2}mv^2 = \text{constant} \tag{1.80}$$

or

$$v = \text{constant}$$

Hence, we can say that applied magnetic field cannot change the speed of the electron. It can only change the direction of the velocity vector. Therefore, the force exerted on the moving charged particles by the magnetic field is called *deflecting force*.

1.5.1 Motion in Longitudinal Uniform Magnetic Field

If a charged particle (positive or negative) moves along the magnetic lines of induction, the magnetic force, also called the *Lorentz force*, acting on it will be zero as the velocity vector $\vec{v}$ is parallel to the magnetic field vector $\vec{B}$ as shown in Fig. 1.17.

$$\vec{F}_L = q(\vec{v} \times \vec{B}) = qvB \sin\theta = qvB \sin 0° = 0 \tag{1.81}$$

Similarly, if the charged particle moves (positive or negative) opposite to the field lines, then the magnetic force on it will be again zero as

$$\overrightarrow{F_L} = q(\vec{v} \times \vec{B}) = qvB \sin\theta = qvB \sin 180° = 0 \tag{1.82}$$

So, if the charged particle (positive or negative) enters along or opposite to the direction magnetic field, it continues to move along the initial direction of motion without any change in its speed and direction.

1.5.2 Motion in Transverse Uniform Magnetic Field

Let us consider an electron of charge 'e' entering in a uniform magnetic field $\vec{B}$ with its initial velocity $\vec{v}$ normal to the field. Magnetic field is acting into the plane of the page which is shown by crosses in Fig. 1.18. Due to the transverse magnetic field, maximum force acts on electrons.

$$\vec{F}_L = q(\vec{v} \times \vec{B}) = qvB\sin\theta = evB\sin 90° = evB \qquad (1.83)$$

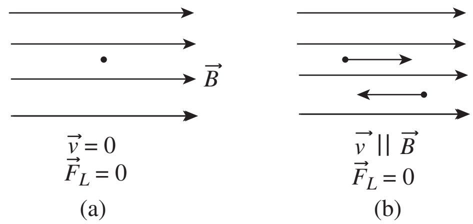


Fig. 1.17 Electron in a uniform magnetic field. (a) When a charged particle (positive or negative) is at rest (b) When a charged particle (positive or negative) is moving parallel or antiparallel

A magnetic force F_L continually deflects the electrons and because $\vec{v}$ and $\vec{B}$ are always perpendicular to each other, this deflection causes the electrons to follow a circular path of radius 'R'. Let us determine the parameters that characterize the circular motion of an electron.

The magnetic force exerted on a charged particle in a transverse magnetic field is always directed towards the centre. Hence, it is similar to the centripetal force which is given by

$$F_c = \frac{mv^2}{R} \qquad (1.84)$$

Equating the two forces we get

$$F_L = F_c$$

$$evB = \frac{mv^2}{R}$$

Solving for R

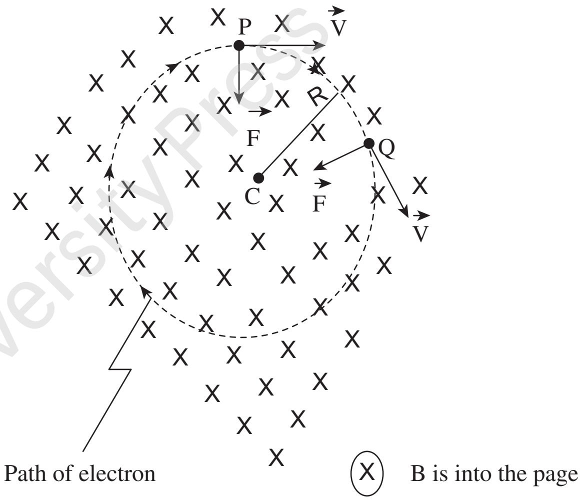


Fig. 1.18 Force exerted by a uniform magnetic field on a moving charged particle (in this case, an electron) produces a circular path

$$R = \frac{mv}{eB} \qquad (1.85)$$

The above equation gives the radius of the circular path. The radius of the circular path depends on the particle's momentum. The period 'T', that is, the time required to complete one revolution is equal to the circumference divided by the speed is obtained as follows.

$$T = \frac{2\pi R}{v} \qquad (1.86)$$

Substituting for R from Eq. (1.85), we get

$$T = \frac{2\pi m}{eB} \qquad (1.87)$$

The frequency f is given by

$$f = \frac{1}{T} = \frac{eB}{2\pi m} \qquad (1.88)$$

The angular frequency ω of the motion is $\omega = 2\pi f = 2\pi \times \dfrac{eB}{2\pi m}$

Therefore,
$$\omega = \frac{eB}{m} \tag{1.89}$$

The quantities T, f, and ω do not depend on the speed of the particle (for $v < c$). Hence, faster particles will move in large circles and slower particles in small circles, but all the particles with the same q/m take the identical time T to complete one revolution. The sense of rotation will be clockwise for electrons and it is counterclockwise for a positive charge. The application of transverse uniform magnetic field is to change the path of the charged particle along the curved path, which is employed in devices such as cyclotron and Bainbridge mass spectrograph.

Projected case Motion of an electron moving with uniform velocity v projected at an angle θ (acute or obtuse) into a uniform magnetic field $\vec{B}$.

Let us assume that the magnetic field is acting along the Z-direction as shown in Fig. 1.19.

A charged particle moves in a uniform magnetic field, with velocity $\vec{v}$ making an angle θ with the field direction. Let us resolve velocity $\vec{v}$ into rectangular components.

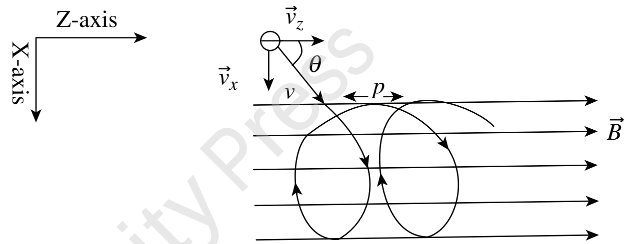


Fig. 1.19 An electron entering in a magnetic field at an angle θ spirals about the field lines

$$\vec{v}_x = \vec{v}\sin\theta \tag{1.90}$$

$$\vec{v}_z = \vec{v}\cos\theta \tag{1.91}$$

The magnetic force acting due to the component of velocity of the charged particle that is parallel to the magnetic field $\vec{v}_z$, is zero. Hence, the parallel component of velocity $\vec{v}_z$, is unaffected. Therefore, the charged particle moves at a constant speed along the direction of the magnetic field. The magnetic force due to the component of the velocity which is perpendicular to the magnetic field $\vec{v}_x$, is maximum. This force will cause the particle to execute uniform circular motion in a plane perpendicular to the magnetic field.

$$\vec{F}_L = q(\vec{v}_x \times \vec{B}) = qv_x B\sin\theta = ev_x B\sin 90^\circ = ev_x B$$

Thus, the parallel component of velocity $\vec{v}_z$ gives rise to the linear motion, whereas the perpendicular component of velocity $\vec{v}_x$ is responsible for the circular motion of the electron. The resultant motion of the electron will be superposition of the linear and circular motion which is a helical motion. Hence, due to the component of velocity parallel to the magnetic field, circular motion will drift at a constant speed along the magnetic field producing an overall helical motion.

The radius of the circular path can be found by equating the magnetic force on the charge with the centripetal force.

$$ev_x B = \frac{mv_x^2}{R} \tag{1.92}$$

$$R = \frac{mv_x}{eB}$$

$$R = \frac{mv\sin\theta}{eB} \tag{1.93}$$

The periodic time T for one revolution is $$T = \frac{2\pi R}{v_x}$$

Substituting for R from Eq. (1.93) we get $$T = \frac{2\pi mv\sin\theta}{eBv\sin\theta}$$

$$T = \frac{2\pi m}{eB} \tag{1.94}$$

The axis of the helix is along the field direction and the pitch of the helix is the linear distance covered by the electron in one revolution along the field direction.

$$p = v_z \times T = v\cos\,\theta \times T \tag{1.95}$$

$$p = \frac{2\pi mv\cos\theta}{eB} \tag{1.96}$$

The magnetic field acting at an angle with respect to the path of the charged particle is best used for focusing the charged particle. This principle finds application in electron microscopy.

1.6 Velocity Filter/Selector

A combination of electric and magnetic fields applied at right angles is called *crossed field configuration*. The crossed field configuration forms what is known as velocity selector and is a common occurrence in mass spectrometers and particle accelerators. The underlying physical principle is explained below.

In the crossed field configuration, a charged particle moving with velocity $\vec{v}$ enters in a direction perpendicular to both the fields. The fields are applied in such a way that for a given charged particle, the fields will exert the force in the same plane but in the opposite direction. A typical configuration which is shown in Fig 1.20 can be used for both positively and negatively charged particles. This configuration can be used to find the unknown velocity of the charged particle.

When the charged particle of unknown velocity enters into the configuration, both the fields will show their existence by exerting the force on it. Electric field $\vec{E}$ will exert a force of magnitude qE on the charged particle, while the magnetic field exerts a force qvB on the same particle. The electric force is in opposite direction to the magnetic force as shown in the figure. When these two forces are made equal in magnitude, they will cancel out each other and the charged particle will move undeflected in a straight line. Its velocity will be given by

$$F_E = F_L$$

Therefore, $$qE = qvB$$

Therefore, $$v = \frac{E}{B} \tag{1.97}$$

By adjusting the magnitude of any one of the fields, the forces can be made equal. Thus, this crossed field configuration can be used for determining the unknown velocity of a charged particle by knowing the magnitudes of the electric and magnetic fields.

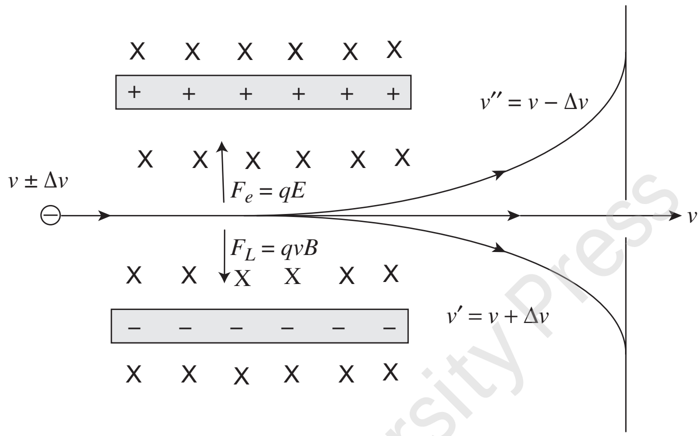


Fig. 1.20 Crossed field configuration as a velocity filter/selector

The crossed field configuration can also be used to filter out the charged particles with desired velocity from the stream of particles having spread in their velocity. The working of the crossed field configuration as a velocity filter or selector is explained below.

Only those crossed configurations can be used as a filter/selector, where the electric and magnetic field directions are such that the forces exerted by them on the charged particle are in the same plane, but are acting in opposite directions.

The desired velocity of the charged particle to be selected is obtained by adjusting the magnitudes of electric and magnetic field, such that $v = \frac{E}{B}$ · The stream of negatively charged particles having a small velocity spread is allowed to enter the configuration as shown in Fig. 1.20. The electric force is independent of the velocity of charged particles, but the magnetic force is dependent on the magnitude of velocity; hence, it will be more if velocity is more and vice versa. Hence, charged particles having more velocity than the selected velocity, that is, for $v' > v$, the force due to magnetic field is greater than the force due to electric field. Hence, charged particles with velocity higher than the selected velocity will be deflected along the direction of magnetic force which is in downward direction. For charged particles having velocity less than the selected velocity, that is, $v'' < v$, the magnetic force will reduce. Hence, electric force will be more and the charged particles will be deflected along the direction of the electric force which acts in the upward direction as shown in Fig. 1.20. Thus, the crossed configuration can be used to filter out or select the charged particles with desired velocity. Thus, a mono-velocity beam can be obtained by using a simple combination of static electric and magnetic field which finds application in many devices.

Checkpoint 3

1. In the static magnetic field, work done by the field is always
   (a) maximum
   (b) negative
   (c) zero
   (d) positive
2. When an electron moving with velocity $\vec{v}$ enters a uniform transverse magnetic field the Lorentz force alters the
   (a) magnitude of $\vec{v}$
   (b) direction of $\vec{v}$
   (c) magnitude and direction of $\vec{v}$
   (d) magnitude and direction of $\vec{v}$ remains unaffected
3. When a charged particle enters a uniform magnetic field at right angle to the direction of field, it will move along
   (a) helical path
   (b) straight line
   (c) circular path
   (d) its motion is unaffected
4. A charged particle will travel along helical path in a magnetic field only when its velocity vector is
   (a) along the field direction
   (b) opposite to the field direction
   (c) perpendicular to the field direction
   (d) at an angle with the field direction
5. The pitch of the helix described by a proton in uniform magnetic field will
   (a) decrease with increase in magnetic field strength
   (b) decrease with decrease in magnetic field strength
   (c) does not depend on magnetic field strength
   (d) none of these
6. For measuring the pitch in helical motion we use the velocity of an electron
   (a) along the field direction
   (b) opposite to the field direction
   (c) perpendicular to the field direction
   (d) none of the above
7. If an electron and a proton enter the same transverse uniform magnetic field, then
   (a) electron will make more revolutions
   (b) proton will make more revolutions
   (c) both electron and proton will make same number of revolutions
   (d) none of the above
8. The time required for the slower particle than the faster particle for completing one revolution in a transverse uniform magnetic field is
   (a) large
   (b) small
   (c) same
   (d) zero
9. In a transverse uniform magnetic field the charged particle will always describe the path in a plane
   (a) of the magnetic field
   (b) perpendicular to the magnetic field
   (c) which contains magnetic field direction and velocity vector
   (d) which contains magnetic field direction and force vector
10. A helium ion and a proton are travelling in the transverse uniform magnetic field along the circular path with the same tangential speed. The periodic time of the helium ion will be
    (a) half as that of proton
    (b) equal to that of proton
    (c) double as that of proton
    (d) one-third as that of proton
11. In velocity selector electric field acts along Z-direction and magnetic field along X-direction. A negatively charged particle moving faster than the selected velocity will deflect
    (a) along positive Z-direction
    (b) along X-direction
    (c) along Y-direction
    (d) along negative Z-direction

12. In a velocity filter, if E = 100 V/m and B = 0.01 T, the speed of the particle that passes undeflected through the field is
    (a) 10^4 m/s
    (b) 10^{-4} m/s
    (c) 1 m/s
    (d) 10^3 m/s
13. To select a charged particle moving with monovelocity $\vec{v}$ in velocity selector, we have to adjust
    (a) magnitude of E = magnitude of B
    (b) $E > B$
    (c) $B > E$
    (d) none of the above

Solved Numericals

1. A proton accelerates from rest in a uniform electric field of 1000 N/C. After some time it attains the speed of 5×10^5 m/s.

(a) Find the acceleration of the proton.

(b) How long does it take the proton to reach the above velocity?

(c) How far has it moved in this time?

(d) What is its kinetic energy at this time?

Given data: Electric field strength E = 1000 N/C, Initial velocity of the proton u = 0

Proton velocity after time t is $v = 5 \times 10^5$ m/s

Proton charge, $e = 1.602 \times 10^{-9}$ C

Proton mass $m = 1.67 \times 0^{-27}$ kg

Solution:

(a) $$a = \frac{eE}{m} = \frac{(1.602 \times 10^{-19} c)\,(1000\text{N/C})}{1.67 \times 10^{-27}\,\text{kg}}$$

$$= \frac{1.602 \times 10^{-16}}{1.67 \times 10^{-27}\,\text{kg}}$$

$$\therefore a = 9.6 \times 10^{10}\,\text{m/s}^2$$

(b) $$t = \frac{v}{a} = \frac{5 \times 10^5\,\text{m/s}}{9.6 \times 10^{10}\,\text{m/s}^2}$$

$$t = 5.2 \times 10^{-6}\,\text{S}$$

(c) $$s = \frac{1}{2}at^2 = \frac{1}{2}(9.6 \times 10^{10})\,(5.2 \times 10^{-6})^2$$

$$s = 2.59\ \text{m}$$

(d) $$\text{K.E.} = \frac{1}{2}mv^2 = \frac{1}{2}(1.67 \times 10^{-27})\,(5 \times 10^5)^2$$

$$= 41.75 \times 10^{-17}\,\text{kg}.\frac{m^2}{\text{s}^2} = 41.75 \times 10^{-17}\,\text{j}$$

$$\text{K.E.} = 2.61\,\text{keV}$$

2. A proton has an initial velocity of 2×10^5 m/s sin the X-direction. It enters a uniform electric field of 2.5×10^4 N/C in a direction perpendicular to the field lines.

(a) Find the time it takes for the proton to travel 0.03 m in the X-direction.

(b) Find the vertical displacement of the proton after it has travelled 0.03 m in the X-direction.

(c) Determine the components of proton velocity after it has travelled 0.03 m in the X-direction.

Given data: Initial velocity $v_0 = 2 \times 10^5$ m/s, Electric Field $E = 2.5 \times 10^4$ N/C, Proton charge, $e = 1.602 \times 10^{-19}$c, Proton mass $m = 1.67 \times 10^{-27}$ kg

Solution:

(a) Proton is not accelerated along x-direction. Time taken by proton to cover distance l is

$$t = \frac{1}{v_0} = \frac{0.03\ \text{m}}{2 \times 10^5\ \text{m/s}} = 1.5 \times 10^{-7}\,\text{s}$$

(b) $$y = \frac{eEt^2}{2\ \text{m}}$$

$$= \frac{(1.602 \times 10^{-19}\text{C})\,(2.5 \times 10^4 N/c)\,(1.5 \times 10^{-7}\,\text{s})^2}{2(1.67 \times 10^{-27}\,\text{kg})}$$

$$= 2.697 \times 10^{-2}\ \text{m} = 2.7\ \text{cm}$$

(c) $$v_y = \frac{eEt}{m}$$

$$= \frac{(1.602 \times 10^{-19}\text{C})\left(2.5 \times \frac{10^4 N}{c}\right)(1.5 \times 10^{-7}\,\text{s})}{(1.67 \times 10^{-27}\,\text{kg})}$$

$$v_y = 3.59 \times 10^5\ \text{m/s}$$

$$v_x = v_0 = 2 \times 10^5\ \text{m/s}$$

3. Hundred-volt electrons are introduced at A into a uniform electric field of 10^5 V/m, as shown in Fig. SN.1 The electrons emerge at the point B in time 5.39 ns.

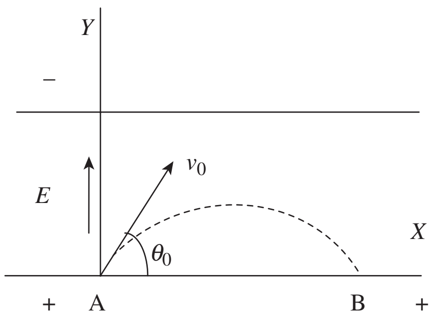


Fig. SN.1 Parabolic path of an electron in uniform electric field

(a) What angle does the electron make with the horizontal?

(b) What is the distance AB?

Given data: Electric field $E = 10^5$ V/m,

Time of flight $T = 5.39$ ns

Solution: The path of the electrons will be a parabola, as shown by the dashed curve in Fig. SN.1. The initial electron velocity is found using the equation

$$v_0 = 5.93 \times 10^5 \sqrt{V_A} \frac{\text{m}}{\text{s}} = 5.93 \times 10^5 \sqrt{100}$$

$$= 5.93 \times 10^6 \text{ m/s}$$

The acceleration due to electric field is

$$a = \frac{eE}{m} = \frac{1.6 \times 10^{-19} \times 10^5}{9.1 \times 10^{-31}} = 1.76 \times 10^{16} \text{ m/s}^2$$

(a) Here we must find θ_o first. The time given in the problem is the time of flight. The formula for time of flight is

$$T = \frac{2v_0 \sin \theta_0}{a}$$

$$\theta_0 = \sin^{-1}\left(\frac{T \times a}{2 \times v_0}\right) = \sin^{-1}\left(\frac{5.39 \times 10^{-9} \times 1.76 \times 10^{16}}{2 \times 5.93 \times 10^6}\right)$$

$$= \sin^{-1}(0.799)$$

$$\theta_0 = 53.12^0$$

(b) The distance AB is the range which is given by

$$R = v_0 \cos\theta_0 \times T$$

$$R = (5.93 \times 10^6 \cos 53.12)(5.39 \times 10^{-9})$$

$$= 19.18 \times 10^{-3} \text{ m} = 19.18 \text{ mm}$$

4. An electron with a velocity of 4.6×10^7m/s enters a uniform magnetic field of induction 0.085 *T* perpendicular to the field lines. Determine (a) the Lorentz force acting on the electron and (b) radius of the circle in which it moves.

Given data: Charge of electron $e = 1.6 \times 10^{-19}$ C, Mass of electron $\mathbf{m} = 9.1 \times 10^{-31}$ kg,

$v = 4.6 \times 10^7$ m/s, $B = 0.085$ T

Solution: (a)

$$F_L = evB = (1.602 \times 10^{-19} \text{C})\ (4.6 \times 10^7 \text{m/s})\ (0.085\text{T})$$

$$= 6.26 \times 10^{-13} \text{ N}$$

(b) $$R = \frac{mv}{eB} = \frac{(9.8 \times 10^{-31} \text{kg})\ (4.6 \times 10^7 \text{m/s})}{(1.60 \times 10^{-19} \text{C})\ (0.085\text{T})}$$

$$= 3.1 \times 10^{-3} = 3.1\text{mm}$$

5. A uniform magnetic field of 0.03 T exists along the x-axis. A proton is shot into the field with a speed of 8×10^6 m/s at an angle 30° with the x-axis. Find (a) the radius and (b) pitch of the helical path. (Mass of proton m = 1.67×10^{-27} kg)

Given data: Mass of proton = 1.67×10^{-27} kg, Charge of proton = 1.6×10^{-19} *C*, $v = 8 \times 10^6$ m/s, $B = 0.03$ T and $\theta = 30°$

Solution:

(a) $$R = \frac{mv \sin\theta}{qB} = \frac{(1.67 \times 10^{-27} \text{kg})\ (8 \times 10^6 m/s)\ (0.5)}{(1.602 \times 10^{-19} C)\ (0.3T)}$$

$$= 13.9 \times 10^{-2} \text{ m}$$

$$= 0.14 \text{ m}$$

(b) Pitch of the helical path

$$P = \frac{2\pi mv \cos\theta}{qB}$$

$$= \frac{2(3.143)(1.67 \times 10^{-27} \text{kg})(8 \times 10^6 \text{m/s})(0.866)}{(1.602 \times 10^{-19} C)\ (0.37)}$$

$$= 1.23 \text{ m}$$

6. An electron beam passes through a magnetic field of 4×10^{-3} Wb.m^2 and electric field of 5.4×10^3 V/m both acting simultaneously at the same point. Assuming the path of electrons remains unchanged, (a) calculate the electron speed. (b) If the electric field is switched off, what will be the radius of the circular path?

Given data:

$B = 4 \times 10^{-3}$ Wb.m^2 ; $m = 9.1 \times 10^{-31}$ kg

$E = 5.4 \times 10^3$ V/m ; $e = 1.602 \times 10^{-19}$ C

Solution:

(a) $$v = \frac{E}{B} = \frac{5.4 \times 10^3 \text{ V/m}}{4 \times 10^{-3} \text{ wb/m}^2} = 1.35 \times 10^6 \text{ m/s}$$

(b) $$R = \frac{mv}{eB} = \frac{(9.1 \times 10^{-31} \text{kg})\ (1.35 \times 10^6 \text{ m/s})}{(1.602 \times 10^{-19} \text{C})\ (4 \times 10^{-3} \frac{\text{Wb}}{\text{m}^2})}$$

$$\therefore R = 1.92 \times 10^{-3} \text{m} = 1.92 \text{ mm}$$

Numerical Exercises

1. An electron travelling with initial velocity of 4×10^6 m/s enters a region of uniform electric field of strength 4.5×10^3 N/C along the field lines.

(a) Find the deceleration of the electron.

(b) Determine the time it takes for the electron to come to rest after it enters the field.

(c) How far does the electron move in the electric field before coming to rest?

(Ans: 7.92×10^{14} m/s^2, 5.05×10^{-9} s, 0.0106 m)

2. A proton accelerates from rest in a uniform electric field of 600 N/C. After some time its speed is 4.5×10^6 m/s.

(a) Find the acceleration of the proton.

(b) How long does it take the proton to reach the above velocity?

(c) How far has it moved in this time?

(d) What is its kinetic energy at this time?

(Ans: 5.74×10^{10} m/s^2, 7.83×10^{-5} s, 175.45 m, 16.9×10^{-15} J)

3. An electron placed in an electric field experiences an upward acceleration of 3.6×10^3 m/s^2. Determine the acceleration of a proton in the same electric field.

(Ans: 1.95 m/s^2)

4. A uniform electric field exists in a region between two oppositely charged plates. An electron is released from rest at the surface of the negatively charged plate and strikes the surface of the opposite plate, 2 cm away, in time 1.5×10^{-8} s. What is the speed of the electron as it strikes the second plate? What is the magnitude of the electric field intensity?

(Ans: 1.3×10^6 m/s, 240 V/m)

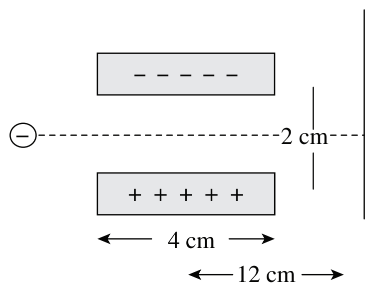
5. An electron is projected along the axis midway between the plates of a cathode ray tube with an initial velocity 3.8×10^6 m/s. The uniform electric field between the plates has an intensity of 2500 N/C and acts upwards.

(a) How far below the axis has the electron moved when it reaches the end of the plates?

(b) At what angle with the axis is it moving as it leaves the plates?

(c) How far below the axis will it strike the florescent screen?

(Ans: 24 mm, 50.63° 144 mm)



6. An electron beam is accelerated through a potential difference of 3000 V. It is then made to pass through a uniform electric field established perpendicular to its path by a set of rectangular parallel plates of length 5 cm. The distance between the plates is 2.5 cm and the potential difference is 60 V. Determine the angle of deflection of the electron beam and its transit time through the electric field.

(Ans: 1.14°, 1.53×10^{-9} s)

7. Find the electrostatic deflection factor of the cathode ray tube in volts per metre, if the distance between the plates is 0.01 m, length of plates is 0.045 m, and distance from the screen to the centre of plates is 0.33 m, for accelerating voltages of 5000 V and 10000 V.

(Ans: 13468 V/m, 6734 V/m)

8. An electron is projected with an initial velocity of 10^7 m/s and at an angle 30° to the horizontal into a uniform electric field of 5000 N/C.

(a) Find the maximum height to which the electron rises vertically.

(b) After what horizontal distance does the electron return to its original elevation?

(Ans: 1.4 cm, 9.8 cm)

9. A proton is projected at an angle of 47° to the horizontal at an initial speed of 6×10^5 m/s in a region of uniform electric field of 200 N/C.

(a) Find the time taken by the proton to return to its initial height.

(b) Find the horizontal displacement of the proton when it reaches the maximum height.

(Ans: 4.7×10^{-5} s, 9.4 m)

10. A singly charged carbon ion moving with the speed of 3×10^5 m/s enters a magnetic field of 7000 G at right angles to the field. Mass of the ion is 19.9×10^{-27} kg.

(a) What is the force that acts on the carbon ion?

(b) What is the centripetal acceleration of the ion?

(c) What is the radius of the circle in which the ion moves?

(Ans: 3.36×10^{-14} N, 1.68×10^{12} m/s^2, 0.053 m)

11. An electron of KE 5×10^3 eV enters a uniform magnetic field of induction 0.32 Wb/m^2 perpendicular to its direction of motion. Calculate its velocity and radius of the path.

(Ans: 4.2×10^7 m/s, 0.746 mm)

12. An α-particle flies into a homogeneous magnetic field perpendicular to its velocity. The angular momentum of the α-particle in the magnetic field is 1.33×10^{-22} kg.m^2/s. The induction of the magnetic field is 2×10^{-2} Tesla. Find the kinetic energy of the α-particle in eV. (Mass of the α-particle = 6.68×10^{-27} kg., charge = 3.2×10^{-19} C).

(Ans: 398 eV)

13. An electron projected at a velocity of 5×10^7m/s enters a uniform magnetic field at an angle of 30° with it. Calculate

(a) Magnetic flux density required for making the electron to follow a helical path of diameter 0.5 m.

(b) The time taken by the electron for completing one revolution.

(c) Pitch of the helix.

(Ans: 5.68×10^{-4}, 63 ns, 2.72 m)

14. A positive ion beam moving along the x-axis enters a region of uniform electric field of 5 kV/m along *y*-axis and magnetic field of 2 kG along z-axis. Calculate the speed of the ions which pass without deviation. What will happen to ions which are moving (a) faster, (b) slower, than these ions?

(Ans: 2.5×10^4 m/s)

15. An electron is accelerated through a potential difference of 1 kV and directed into a region between two parallel plates separated by 0.02 m with a 100V potential difference between them. If the electron moves perpendicular to the electric field between the plates, what magnetic field is necessary perpendicular to both the electron path and the electric field so that the electron travels in a straight line?

(Ans: 2.67×10^{-4})

Summary

- Electric field is a vector quantity and is defined as the force $\vec{F}$ exerted on a tiny positive test charge q_0 at that point divided by the magnitude of the test charge. The SI unit of electric field is Newton per coulomb (N/C).
- In a uniform electric field, the lines are straight, parallel, and uniformly spaced. In non-uniform fields, the field lines are curved and spaced non-uniformly.
- For a uniform field, the equipotential surfaces are a family of planes perpendicular to the electric field lines and thus to *E*, which is always tangent to these lines.

- The potential energy U of a test charge q at any point is equal to the negative of the work done $-w_{\infty f}$ on the test charge by the electric field. It is measured in joules.
- The potential energy per unit charge U/q is called *electric potential, which* is a scalar quantity.
- The component of E in any direction is expressed as the negative of the rate of change of electric potential with distance in that direction.
- Magnetic field is defined as the magnetic force exerted per unit charge per unit velocity and the SI unit of B is Tesla or wb/m^2.
- In longitudinal uniform electric field, the charged particle moves along straight line. The longitudinal electric field is used to impart kinetic energy to the charged particles.
- In transverse electric field, the charged particle moves along the parabola. The transverse electric field is used to deflect the charged particles horizontally or vertically.
- The deflection sensitivity, '*S*' is defined as the deflection produced 'D' per unit deflecting voltage 'V_d' applied to deflecting plates. It is inversely proportional to the accelerating potential V_A.
- The trajectory of an electron projected into a uniform electric field is a parabola. Various parameters of projected motion of a charged particle are similar to the projectile in a gravitational field.
- A static magnetic field does not change kinetic energy of the moving charged particles.
- A static magnetic field does not exert any force on a charged particle at rest or moving parallel or anti-parallel to the field.
- The trajectory of a charged particle in transverse uniform magnetic field is circle. The application of transverse magnetic field is to change the path of the charged particle along the curved path.
- A charged particle projected in a uniform magnetic field at an acute or obtuse angle will move along a helical path. The helical motion in non-uniform magnetic field is used for focusing the charged particle in an electron microscope.
- The crossed field configuration is used as a velocity filter or selector to find unknown velocity or to filter out the charged particles with the desired velocity.

Self-study Questions

1. Show that an electron moves along a parabolic path when it enters a uniform electric field applied perpendicular (transverse) to its motion.
2. What is the force experienced by a charged particle in (a) electric field and (b) magnetic field? Show that the kinetic energy of a charged particle remains constant when it moves in the magnetic field.
3. Describe the motion of an electron when projected at an acute angle with the direction of uniform electric field and determine:
   (a) Maximum distance reached by the electron in the direction of the field.
   (b) Time taken to reach maximum distance.
   (c) Range of charged particle
4. If a moving electron has no deflection in passing through certain region of space, can we be sure that there is no magnetic field in the region? If it is deflected sideways can we be sure that a magnetic field exists in that region?
5. State the shapes of trajectories of a charged particle of velocity $\vec{v}$ moving in an electric field $\vec{E}$ when
   (a) $\vec{v}$ is perpendicular to $\vec{E}$.
   (b) $\vec{v}$ makes acute angle with $\vec{E}$.
   What are these shapes if $\vec{E}$ is replaced by magnetic field $\vec{B}$?
6. State the conditions under which a charged particle moves in a straight line in
   (a) an electric field $\vec{E}$
   (b) a magnetic field $\vec{B}$
   (c) in a region having both $\vec{E}$ and $\vec{B}$
7. In a transverse uniform magnetic field, show that the radius of curvature of the path of a charged particle is proportional to its momentum.
8. How is it possible for a charged particle to pass through a combination of electric and magnetic fields without any deviation? What will be its

velocity? What will happen if the particles are moving slowly or faster than this velocity?

9. When and why does a charged particle entering magnetic field follow a helical path?

10. Magnetic field changes the velocity of a charged particle without changing its speed—explain.

Answers

Checkpoint 1
1(c), 2(a), 3(c), 4(a)

Checkpoint 2
1(a), 2(c), 3(a), 4(b), 5(c), 6(b), 7(b), 8(c), 9(c), 10(a)

Checkpoint 3
1(c), 2(b), 3(c), 4(d), 5(a), 6(a), 7(a), 8(c), 9(b), 10(c), 11(a), 12(a), 13(d)