Preface

Economics and mathematics are highly interrelated subjects. Models of economic theories are developed using integration, differentiation, and probability. In the context of macroeconomics, it is often important to formulate strategies to combat phenomena like inflation or frame fiscal and monetary policies to stabilize the economy as well as boost economic growth. The use of mathematics is crucial to the development of these strategies and policies. The University of Calcutta has combined these two subjects into one paper that is offered in the third year of the B Com (Hons.) curriculum with the express intention of providing a comprehensive overview of these subjects, which will help the students grasp and apply the concepts learnt in their overall study of commerce.

About the Book

This book is designed to provide a comprehensive coverage with an easy-to-understand treatment of macroeconomic theory and advanced business mathematics, covering the syllabus for undergraduate honours students of commerce in the University of Calcutta.

Macroeconomics examines and analyses the performance of the economy as a whole. The major macroeconomic issues related to national income, inflation, employment, interest rate, and price level are dealt with in a very articulate manner in terms of verbal logic, graphical illustration, and mathematical proofs. While it contains the necessary topics completely covering the syllabus of the University of Calcutta, it also captures well the syllabi prescribed by other leading universities in India.

The second part of the book, Advanced Business Mathematics covers calculus, algebra, and probability theory. Keeping in mind that this book is for undergraduate commerce students, the theoretical portions have been discussed in a simple manner for better understanding.

Key Features

- The contents and sequence of topics in the book have been prepared by following the syllabus given by the University of Calcutta.
- The extent of each chapter has been decided keeping in mind the number of lectures assigned in the syllabus so that it would be easy for the teachers to frame their lesson plans.
- Practical examples are inserted to make the theoretical concepts easily understandable for the students.
- Numerical problems are provided for easy understanding of the different aspects of macroeconomic theory.
- In the advanced mathematics portion, a large number of problems are solved including those from different years' C.U. question papers.
- Solutions to the university questions as well as model questions are provided, which will help the students prepare for their examinations.

Organization of the Book

The first part containing macroeconomic theory is divided into six chapters by following the sequence of the syllabus prescribed by the University of Calcutta.

Chapter 1 provides a basic idea of the different aspects of macroeconomics. This chapter outlines some fundamental concepts and the scope of macroeconomics.

Chapter 2 begins with the national income accounts. The concept and meaning of national income, its measurement, components, and different methods of computing have been discussed in a very simple way. Examples with problems and solutions are given for conceptual clarity.

Chapter 3 deals with the determination of national income under the assumption of a fixed price of final goods and a constant rate of interest in the economy. This chapter presents the basic ideas developed in Keynes's theory to explain how effective demand determines national income.

Chapter 4 covers the Keynesian theory of aggregate demand when investment depends on the interest rate. The Keynesian theory of aggregate demand is constructed on the assumption that two facts are true simultaneously. First, the quantity of savings supplied equals the quantity of investment demanded; in other words, the product market is in equilibrium. Second, the quantity of money demanded is equal to the quantity of money supplied. In the Keynesian theory, we study the determination of equilibrium in these two markets simultaneously.

In *Chapter 5*, we discuss the concept and functions of money. We also talk about the demand for money, quantity theory of money, supply of money, credit creation process of commercial banks, and the different components of money supply. In this connection, we also discuss the concept of money multiplier and the process of controlling the supply of money by the central bank of a country.

Chapter 6 deals with inflation. As inflation affects the everyday lives of the citizens of a country and the overall situation of an economy, it is essential to discuss the concept, causes, and impacts of inflation. It is also necessary to differentiate between demand-pull and cost-push inflation. In this connection, it is essential to know the costs of inflation and the relation between inflation and unemployment. Finally, we should be conversant with the measures to control inflation.

The second part of this book deals with advanced business mathematics. This part is divided into nine chapters.

Chapter 1 introduces the basic concepts of number system, function and its classification, properties of functions, some special functions, and application of function in business and economics.

Chapter 2 explains the concept of limit of a variable and function, limit of some standard functions, and the concept of continuity and discontinuity with the graphical representation.

In *Chapter 3*, we discuss the differentiability of a function. There are three sections. In the first section, first order differentiability of a function is discussed. Here, differentiation of different types of functions—differentiation of composite functions, parametric functions, implicit function, logarithmic functions etc.—are discussed. In the second section, the concept of second order derivative of a function is introduced. The third section expounds on the concept of more than one variable, that is, function of several variables, partial derivative of a function, and Euler's theorem on homogeneous function and derivative of a function using total differential.

After introduction of elementary *differential calculus* in *Chapter 4*, the application of calculus is discussed. The significance of derivatives, their meaning in different aspects such as rate measures and economics is discussed. In addition, increasing and decreasing functions and maxima and minima of a function are elucidated here.

Integration is an important concept in calculus. In *Chapter 6*, a part of indefinite integration is introduced.

In *Chapters 6*, *7*, and *8*, important notions in algebra—matrix and determinant—are introduced. Some basic definitions related to matrix and determinant, their algebras and finally the system of linear equations solved using matrices and determinants are described.

Lastly, in *Chapter 9*, we introduce probability theory. Here, the classical definition of probability is introduced. Important theorems such as total probability theorem, joint probability theorem, and conditional probability theorem are stated.

Online Resources

The following resources are available to faculty and students using this text:

- Solutions to Chapter-end Exercises
- Solutions to Model Question Papers

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Readers are welcome to share their feedback and suggestions with us at p.das.wbes@gmail.com, asengupta15@yahoo.com, and samantapmju@gmail.com.

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Brief Contents

Detailed Contents

CHAPTER

4

Commodity and Money Market Equilibrium

This chapter is about the Keynesian theory of aggregate demand when investment depends on the interest rate. We will show that aggregate demand depends not only on the money supply, as in the classical theory, but also on fiscal policy and on the expectations of households and firms. The Keynesian theory of aggregate demand is constructed on the assumption that two facts are true simultaneously. First, the quantity of savings supplied equals the quantity of investment demanded; in other words, the product market is in equilibrium. Second, the quantity of money demanded is equal to the quantity of money supplied. In other words, the existing stock of money in circulation is willingly held. In the Keynesian theory, we have to study the determination of equilibrium in these two markets simultaneously.

In Chapter 3, we discussed national income determination and investment multiplier by assuming that private investment is autonomous. This is the subject matter of product market analysis, where equilibrium national income is determined by equating aggregate demand to aggregate supply of output. We also add the investment function, where private investment depends on both income and interest rates. In this chapter, we extend our analysis by taking the interest rate as an important determinant of investment; a reduction in the interest rate raises investment demand. Now, we have to find out what determines the interest rate. This search extends our analysis further to include the markets where interest rate is determined. This is the money market or market for financial assets where interest rate is determined. The interest rate, however, is not determined from the money market in an isolated manner. This forces us to study the interaction of the product and money markets. National income and interest rates are jointly determined by equilibrium in the product and money markets. The analysis in this chapter qualifies some conclusions we came to in Chapter 3.

4.1 Introduction

Keynes contended that monetary policy was powerless to boost the economy out of a depression because it depended on reducing interest rates, and in a depression, interest rates were already close to zero. Increased government spending, on the other hand, would not only boost demand directly, but also set off a chain reaction of increased demand from workers and suppliers whose incomes had been increased by the government's expenditure. Similarly, a tax cut would put more disposable income in the wallets of consumers, and that too would boost demand. The appropriate fiscal policy during periods of high unemployment was to run a budget deficit. These ideas flew in the face of the conventional wisdom that budget deficits were always bad. Historical research, however, pioneered by Milton Friedman has convinced many economists that the Depression was mainly the result of incompetent monetary policy in both Britain and the United States rather than the inability of monetary policy to influence the economy. Many economists had expected a resumption of the Great Depression when World War II ended, but instead the US economy experienced an era of spectacular growth. To the surprise of almost everyone, the most

aggravating problem of the post-war economy has been inflation, while recessions have been relatively brief and mild.

In Keynes's model, equilibrium national income is originated from the product market and interest rate is determined in the money market. Later on, John Hicks devised the IS-LM model (IS stands for investment-savings and LM for liquidity-money). He explained that income and interest rates are jointly determined by the interaction of the product market and money market. A part of the aggregate demand (consumption) in the product market depends on income, while another part (investment) depends on interest rate. In the money market also, a part of money demand depends on income and another part depends on interest rate. Thus, income and interest rate are determined jointly by the product market and money market equilibrium. This is the essence of Hicks's IS-LM model. The IS-LM model emphasizes the interaction between the product and money markets. In Chapter 3, we discussed national income determination by arguing that income affects spending, which in turn determines output and income. In this chapter, we consider the effects of interest rates on investment and aggregate spending, and ultimately on income. Thus, we need the dependence of the money market on income. Higher income raises money demand, and thus interest rates. Higher interest rates lower spending, and thus income. Spending, interest rates, and income are determined jointly by equilibrium in the product and money markets. Fiscal policy works by changing government expenditure or tax rates or both through the product market, but monetary policy works by changing money supply through the money market.

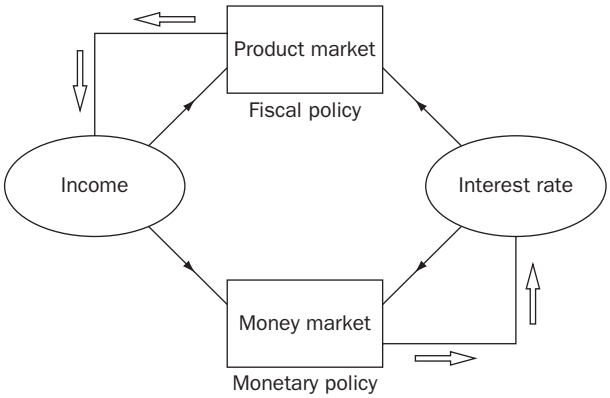

Fig. 4.1 Logical structure of the chapter

Figure 4.1 shows the logical structure of the chapter.

The chapter starts, after a few introductory remarks, with a discussion of the link between interest rates and aggregate demand by analysing the investment function once again in Section 4.2. We extend our analysis from Chapter 3 to include the interest rate as a determinant of aggregate demand and derive the key relationship between income and interest rates in the form of the IS curve, which explains equilibrium in the product market. In Section 4.3, we turn to the money market. We show that the demand for money depends on interest rates and income. There are combinations of interest rates and income levels for which the money market is in equilibrium in the form of the LM curve. In Section 4.4, we deal with the two-sector Keynesian model of national income determination by combining the IS and LM schedules to study the joint determination of interest rates and income. Section 4.5 lays out the multiplier process and the effectiveness of monetary and fiscal policies in this framework.

4.2 Product Market Equilibrium

This section explains how the interest rate is related to income in the Keynesian theory of the product market. In the classical theory, income is determined on the assumption that there is no unemployment. Given this assumption, the determination of the real interest rate is relatively straightforward since, once they had determined full employment output, the classical economists were able to represent savings by an upward sloping supply schedule. The equilibrium real interest rate could then be found as the rate at which the quantity of savings supplied was equal to the quantity of investment demanded.

In the Keynesian model, it is no longer true that there is a unique level of income. The fact that income may fluctuate over the business cycle will affect the interest rate because households will be willing to save more if they are rich than if they are poor. If income is high and unemployment is low, there will be a relatively high supply of savings. Firms will not need to offer a high rate of interest to attract lenders and so a high value of income will be associated with a low equilibrium interest rate. If instead income is low and unemployment is high, investors will compete with each other to borrow a small pool of savings and they will bid up the rate of interest. A low value of income will be associated with a high equilibrium interest rate. In the Keynesian model, we summarize this idea by deriving a schedule, the IS curve, that plots the nominal interest rate on the vertical axis and the level of income on the horizontal axis. At every point on the IS curve, the product market is in equilibrium.

4.2.1 Derivation of IS Curve

The product market is in equilibrium when the aggregate demand for output equals aggregate supply of it. In a closed economy, aggregate demand has three components: consumption expenditure, investment expenditure, and government expenditure. Consumption expenditure depends on disposable income (national income less taxes), investment expenditure depends on the interest rate, and government expenditure is autonomous. Aggregate supply in the product market is gross national product (GNP) at factor cost or national income. Thus, the equilibrium condition in the product market is given by

$$Y = C\big(Y - t(Y)\big) + I\big(r\big) + G, \quad 0 < C' < 1, \quad 0 < t' < 1, \quad I' < 0 \tag{4.1}$$

Here, $C' = \dfrac{\mathrm{d}C}{\mathrm{d}Y} = \text{mpc}$. The tax function is $T = t(Y)$, and $t' = \dfrac{\mathrm{d}T}{\mathrm{d}Y}$ is the marginal tax rate.

$I' = \dfrac{\mathrm{d}I}{\mathrm{d}r}$ measures interest elasticity of investment. The alternative form of equilibrium is

$$I(r) + G = S\big(Y - t(Y)\big) + t(Y) \tag{4.2}$$

Here, $S' = \dfrac{\mathrm{d}S}{\mathrm{d}Y} = \text{mps}$

The alternative form of the equilibrium condition states that investment (I) equals savings (S) under the assumption that government budget is balanced. Thus, the product market equilibrium schedule is popularly known as the IS curve, the schedule along which investment equals savings. The product market equilibrium, either in equation 4.1 or 4.2, consists of several combinations of income (Y) and interest rate (r) that keep equality between aggregate demand and aggregate supply, or between savings and investment.

The geometric representation of product market equilibrium is the IS curve. The IS curve is a locus of several combinations of income and interest rate that maintain the product market in equilibrium in a sense that aggregate demand equals aggregate supply, or investment equals savings. The product market equilibrium schedule, the IS curve, is an extension of national income determination with a 45°-line diagram as shown in Chapter 3. Now investment is no longer fully exogenous but is also determined by the interest rate.

By following the trick we have applied in Chapter 3 the derivation of the IS curve is shown in Fig. 4.2. Panel B of Fig. 4.2 shows the investment function, which relates investment to the interest rate. Panel A of Figure 4.2 locates equilibrium points corresponding to different interest rates, and the IS curve is drawn in Panel C. The negatively sloped investment curve implies that investment demand rises when the interest rate falls and vice versa. In Chapter 3, we discussed why investment is inversely related to the interest rate. The level

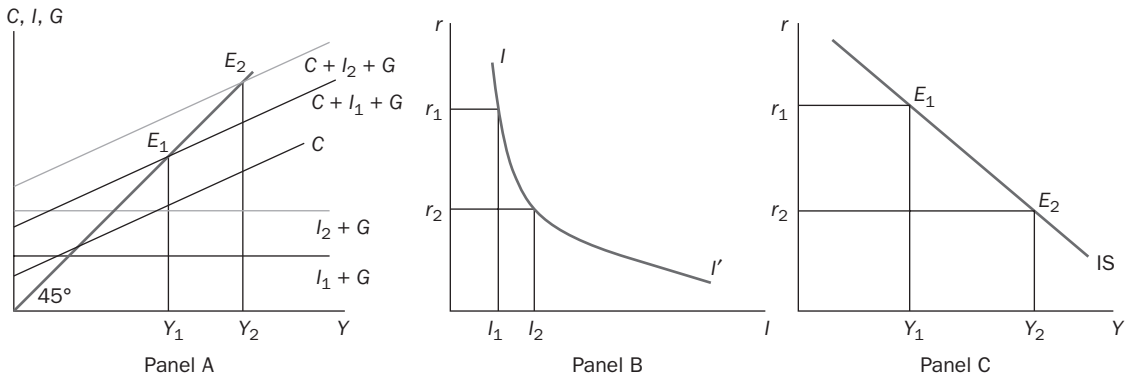

Fig. 4.2 Derivation of IS curve: aggregate demand approach

of investment is I_1 when the interest rate is r_1. This level of investment produces aggregate demand for output $C + I_1 + G$ shown in Panel A. Given this level of aggregate demand, the product market is in equilibrium at point E_1, where equilibrium national income is determined at Y_1 as shown in Panel A of Fig. 4.2. Thus, when the interest rate is at r_1, income will be at Y_1, satisfying equilibrium in the product market. Measuring income along the horizontal axis and the interest rate along the vertical axis, this (Y_1, r_1) combination which maintains equilibrium in the product market is plotted at point E_1 in Panel C of Fig. 4.2. When the interest rate falls to r_2, investment rises to I_2 and the equilibrium point shifts to E_2, where aggregate demand $(C + I_2 + G)$ equals income (Y_2). Therefore, income level Y_2 corresponds to the interest rate r_2, satisfying equilibrium in the product market. This (Y_2, r_2) combination point is placed at E_2 in Panel C. By joining all points such as E_1, and E_2, we can draw the IS curve.

A given reduction in the interest rate, from r_1 to r_2, raises the intercept of the aggregate demand curves by the same vertical distance, as shown in Panel A of Fig. 4.2. However, the implied change in income is larger than the change in investment demand because of the multiplier effect. The larger the mpc, the larger the change in income produced by a given change in the interest rate.

Let us assume a linear consumption function and a linear investment function as we used in Chapter 3:

$C(Y) = a + bY_d = a + b(Y - T) = a + b(1 - t)Y,\ T = tY$

$I(r) = a_1 - b_1 r$

The equation of the IS curve becomes

$$\begin{aligned} &Y = a + b(1-t)Y + a_1 - b_1 r + G \\ &\text{or,} \\ &\{1 - b(1-t)\}Y = a + a_1 - b_1 r + G \end{aligned} \tag{4.3}$$

From equation 4.3, it is clear that an increase in the interest rate reduces aggregate demand.

Alternatively, by using savings–investment equality, the equation of the IS curve is

$$\begin{aligned} &S + T = \bar{I} + G \\ &\text{or,} \\ &-a + \{1 - t - b(1-t)\}Y + tY = a_1 - b_1 r + G \\ &\text{or,} \\ &[1 - b(1-t)]Y = a + a_1 - b_1 r + G \end{aligned} \tag{4.4}$$

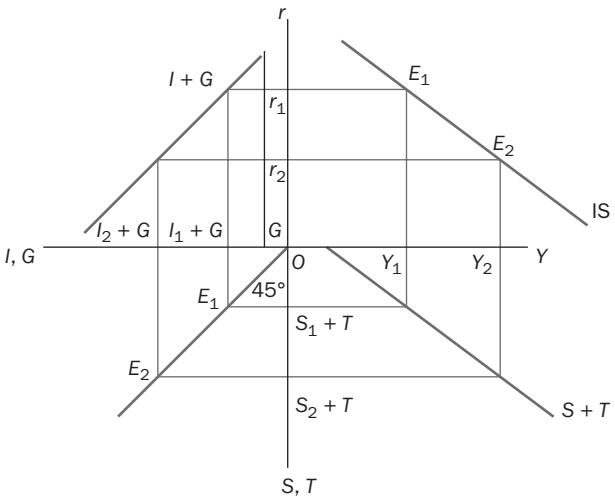

Fig. 4.3 Derivation of IS curve: investment–savings approach

The derivation of the IS curve for the linear investment schedule is illustrated in Fig. 4.3 in terms of a four-quadrant figure. In quadrant 2, we have shown the linear investment schedule and government expenditure by measuring them along the horizontal axis from right to left with reference to the origin O. The interest rate is measured along the vertical axis. The investment schedule is negatively sloped and government expenditure is not dependent on the interest rate, shown by a vertical line. The savings plus tax schedule is shown in quadrant 4 by an upside-down straight line. The linear savings schedule is obtained from the linear consumption schedule. In quadrant 3, a 45° diagonal line is used to find out equilibrium in the product market. The IS curve is derived in quadrant 1.

At an interest rate r_1, investment plus government expenditure is $I_1 + G$. Point E_1 in quadrant 3 determines equilibrium in the product market by equating $I_1 + G$ to $S_1 + T$ at income level Y_1. Therefore, the combination of Y_1 income and r_1 interest rate keeps the product market in equilibrium, which is shown by point E_1 in the first quadrant. Points E_1 shown in the first and third quadrants represent the same equilibrium. A fall in the interest rate to r_2 raises investment to I_2, increasing the level of spending at each income level. The new equilibrium point E_2 requires higher savings at S_2 corresponding to higher income level Y_2. In the first quadrant, point E_2 records the new equilibrium in the product market corresponding to (Y_2, r_2) combination of income and interest rate. We can apply the same procedure to all conceivable levels of the interest rate and thereby generate all the points that make up the IS curve. Thus, by joining all combination points such as E_1 and E_2, we can draw the IS curve.

4.2.2 Slope of IS Curve

The IS curve is negatively sloped reflecting the increase in aggregate demand associated with a reduction in the interest rate. Therefore, the lower the rate of interest, the higher the income by maintaining the product market in equilibrium. We can calculate the slope of the IS curve from either equation 4.3 or 4.4:

$$\frac{dr}{dY} = \frac{1-b(1-t)}{-b_1} \tag{4.5}$$

As $0 < b < 1$, $0 < t < 1$, and $b_1 > 0$, the slope of the IS curve, $\frac{dr}{dY} < 0$

The steepness of the curve depends on how sensitive investment spending is to changes in the interest rate. Suppose that investment spending is very sensitive to the interest rate, so that b_1 in equation 4.5 is large, that is, a given change in the interest rate produces a large change in investment demand and also aggregate demand that produces ultimately a large change in equilibrium income. If investment is very sensitive to the interest rate, the IS curve is very flat. Conversely, if b_1 is small and investment spending is not very sensitive

to the interest rate, the IS curve is relatively steep. If investment is perfectly interest elastic or the investment schedule is horizontal, the IS curve will be horizontal to the income axis. On the other hand, if investment is perfectly interest inelastic, that is, the investment schedule is vertical, the IS curve will be vertical.

4.2.3 Shifts of the IS Curve

The IS curve will shift when the autonomous parts of aggregate demand or other parameters of the IS curve equation is changed. Let us see what happens if government expenditure goes up. Assume that government goods and services cannot be easily substituted for private consumption goods so that the private savings schedule is unaffected by an increase in government purchases. The effect of an increase in government purchases, when we make this assumption, is illustrated in Fig. 4.4. When government expenditure increases, the $I + G$ line shifts outwards to $I + G_1$. Given the interest rate at r_1, investment plus government expenditure is more. To keep the product market in equilibrium, savings plus tax will also be more, which corresponds to higher income Y_{11}. Therefore, the new combination of income and interest rate (Y_{11}, r_1) produces equilibrium in the product market at the increased level of government expenditure. An increase in aggregate demand due to higher government spending shifts the aggregate demand curve up, raising the equilibrium level of output at interest rate r_1. At each level of the interest rate, equilibrium income is now higher. Accordingly, the IS schedule in the first quadrant shifts rightwards from IS to IS_1. We have shown below that the horizontal shift of the IS schedule is equal to the multiplier times the increase in government spending. Thus, an increase in government purchases or transfer payments will shift the IS curve out to the right, with the extent of the shift depending on the size of the multiplier. Similarly, we can show that a reduction in transfer payments or in government purchases shifts the IS curve to the left.

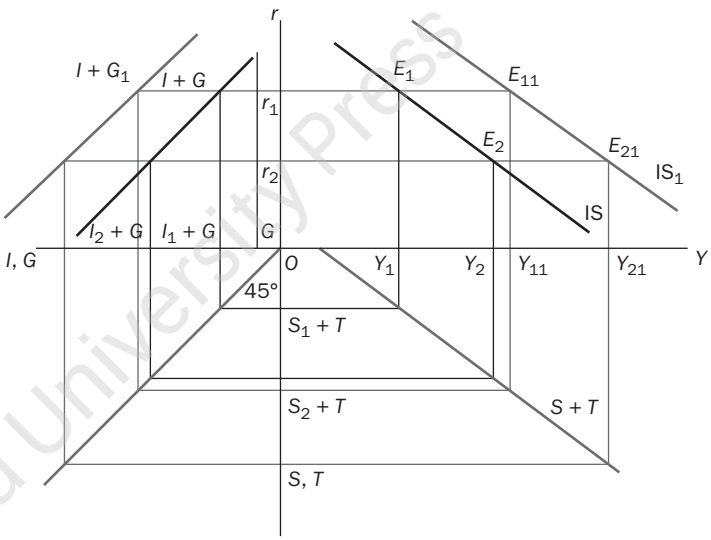

Fig. 4.4 Shifts of the IS curve due to an increase in government expenditure

The position of the IS curve changes with the change in tax rate. We can analyse the effect of a change in net taxes on the IS curve. We assume that taxes are levied as a lump sum on households and firms, and that net taxes affect savings only through their effect on disposable income. The IS curve shifts to the left when net taxes go up. For the case of a change in net taxes, we need to consider two effects of an increase in taxes on the product market. First, if net taxes increase, the $S + T$ schedule, shown in the southeast quadrant, will shift directly to the left. Second, there is an indirect effect that follows from the fact that if net taxes increase, households will have less disposable income and their supply of savings will fall. Since the direct effect is more than the indirect effect, the $S + T$ schedule will shift to the left, shifting the IS curve also to the left. The leftward shift of the IS curve when taxes increase is smaller than the rightward shift when government purchases increase because, in the case of taxes, the savings curve shifts to partially offset the change in the demand for funds by the government.

We have shown the shifts of the IS schedule owing to an increase in mpc, or a decrease in mps, in Fig. 4.5. A fall in mps shifts the $S + T$ schedule rightwards. This is because when mps declines, other things remain the same, people will save less at the given level of income. In other words, a fall in mps or a rise in mpc means a rise in aggregate demand through more consumption, which produces more income. Therefore, a given rate of interest corresponds to higher income to keep the product market in equilibrium. Thus, a fall in mps shifts the IS schedule rightwards. However, in this case, the slope of the IS schedule also changes, implying that the change in mps or mps does not produce a parallel shift.

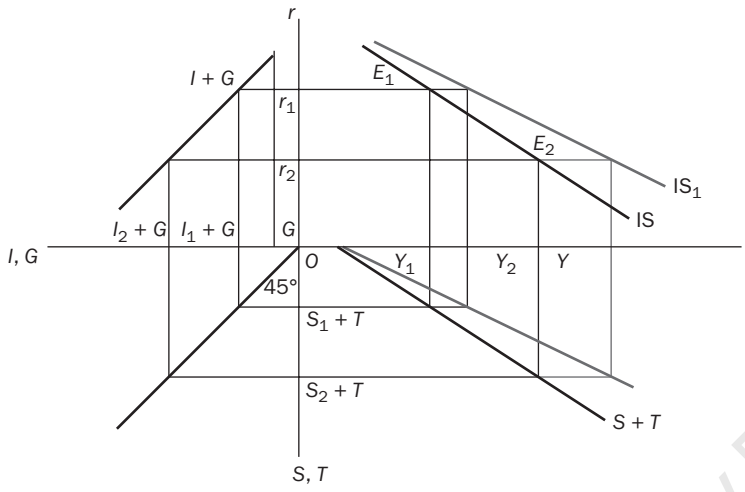

Fig. 4.5 Shifts of the IS curve due to an increase in mpc

4.2.4 Points off the IS Curve

It is clear that any combination point of income and interest rate on the IS curve satisfies the equilibrium condition in a sense that planned output is exactly equal to planned demand, and, hence, no unintended inventory changes appear on the curve. Now, let us understand the implication of a combination point off the IS curve. This is illustrated in Fig. 4.6. Positions off the IS curve imply disequilibrium in the product market. In Fig.4.6, points E_1 and E_2 are on the IS schedule, but point E_3 lies below the curve, while point E_4 is above the curve. At point E_3, we have the same level of income, Y_1 as at E_1, but the interest rate is lower. Therefore, the demand for investment is higher than at E_3, and the demand for goods is higher than at E_1. This means that aggregate demand exceeds the level of output, and so there is an excess demand for goods generating a fall in inventory. On the other hand, at point E_4, the interest rate is higher than at E_2. Thus, investment demand and aggregate demand are lower than at E_2. There is an excess supply of goods at point E_4, raising the inventory level. Therefore, points above the IS schedule correspond to an excess supply of goods, and points below relate to an excess demand for goods.

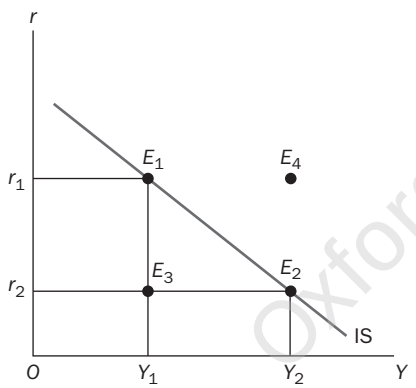

Fig. 4.6 Disequilibrium in the product market

Problem 4.1 Find an equation of the IS curve in a closed economy where the consumption, investment, and government expenditure are

$$C = 350 + 0.3Y; I = 120 - 40r; G = 120$$

What value of the real interest rate clears the goods market when $Y = 600$? Calculate the slope of the IS curve.

Solution The IS curve represents equilibrium in the product market. Therefore, the equation of the IS curve is

$$Y = C + I + G$$
$$\text{or, } Y = 350 + 0.3Y + 120 - 40r + 120$$
$$\text{or, } 0.7Y + 40r = 590$$
$$\text{When } Y = 600,\ 40r = 590 - 420 = 170 \text{ or, } r = 4.25$$

$$\text{Slope of the IS curve, } \frac{dr}{dY} = -0.0175$$

4.3 Money Market Equilibrium

The nominal money demand is the individual's demand for cash in hand. The real money demand, on the other hand, is the demand for money in terms of the number of units of goods that money will buy. The real money demand is equal to the nominal money demand divided by the price level. If the nominal money demand is ₹500 and the price level is ₹5, then the real money demand is 100. The real money demand is called the demand for real balances. People create demand for real balances because money has some important functions. We will discuss in detail different functions of money in Chapter 5. In the context of demand for money, we are now mentioning two important functions of money: money functions as a medium of exchange, and money functions as a store of value. Money is the means by which we purchase goods and services. Money facilitates the exchange, because everyone is willing to accept money as a medium of exchange for whatever it is that one might want to buy or sell. It is also very easily divisible to the scale of what is being exchanged. Money is a store of value because it is a liquid asset. The liquidity of an asset refers to how quickly the asset can be turned into cash, and since money is already cash, it is the most liquid asset possible. This is probably the reason that so many people hold onto cash as a store of value.

Given our explanations of the functions of money, it will not be surprising that there are three different types of demand for money. In Keynes's theory, money is demanded due to three main motives: transactions motive, precautionary motive, and speculative motive. The transactions motive gives rise to the transactions demand for money, which refers to the demand for cash by the people for making current transactions of all kinds. The precautionary motive induces the public to hold money to provide for contingencies requiring sudden expenditure and for unforeseen opportunities of advantageous purchases. This demand arises because of uncertainties. The speculative motive giving rise to the speculative demand for money is the most important contribution Keynes made to the theory of the demand for money. The transactions and precautionary motives are related to the medium of exchange function of money. The asset demand for money is connected with the speculative motive.

4.3.1 Transactions Demand for Money

The transactions demand for money is using money as a medium of exchange. People want to hold money for transactions purposes to bridge the gap between the earning point and the spending point. If there was a perfect match between the moments we receive money in transactions and the moments we use money, we would not need to hold any money at all for transactions. However, in the real world, there is not going to be an exact match between when we receive money and when we need to make payments. Suppose, you receive ₹50,000 every month as payment for your work. This payment comes once a month. However, you need to pay for several necessary items over the month. In the course of the month, you have to hold some money for spending on these items. How much money is needed for this purpose depends on the volume of transactions, which in turn depends on

income. Therefore, the transactions demand for money depends on income. If the volume of income and output produced in the goods markets increases, then clearly there will be a larger volume of transactions and exchanges taking place. People will need to hold a larger volume of money to meet all these transactions and make payments. The transactions demand for money is a demand for active balances because it is used as a means of payments in national income-generating transactions.

4.3.2 Precautionary Demand for Money

People often demand money as a precaution against an uncertain future. Unexpected expenses, such as medical or car repair bills, often require immediate payment. The need to have money available in such situations is referred to as the precautionary motive for demanding money. The precautionary demand for money arises because of uncertainties. According to Keynes, precautionary demands for money are those which are held *to provide for contingencies requiring sudden expenditure and for unforeseen opportunities of advantageous purchases, and also to hold an asset of which the value is fixed in terms of money to meet a subsequent liability in terms of money*. This demand for money depends on income (Y).

As both the transactions demand and precautionary demand for money depend on income, we can express these demands for money (L_1) together:

$$L_1 = L_1(Y), \quad L_1' = \frac{dL_1}{dY} > 0 \tag{4.6}$$

Keynes retained the influence of the Cambridge approach to the demand for money under which money demand is assumed to be a function of income (Y). The transactions demand for money in Keynes's version is similar to the Cambridge version of the classical money demand function. Thus, Keynes's transactions demand for money can be expressed in linear form as

$$L_1 = kPY \tag{4.7}$$

Here, k is constant and similar to income velocity of money, P is price level which is assumed to be constant in Keynes's framework. Therefore, the transactions and precautionary demand schedule will be an upward rising straight line passing through the origin as shown in Fig. 4.7.

4.3.3 Speculative Demand for Money

There is one other reason why people have a demand for holding money balances. This is called the speculative motive. People hold money as a financial asset just like stocks and bonds. Holding money as a liquid asset is using money as a store of value. Thus, the speculative demand for money is a demand for idle balances. An individual has to decide how to allocate the financial wealth between alternative types of assets. Decisions on the form in which to hold assets are portfolio decisions. For the sake of simplicity, Keynes assumed that perpetual bonds are the only non-money financial asset in the economy, which compete with money in the asset portfolio of the people. Bonds have returns in the form of interest rates, while money has no return. On the other hand, money has liquidity, but bonds have no liquidity. The more bonds held, the more interest received on total financial wealth. The more money held, the more likely the individual is to have liquidity. The portfolio decisions on how

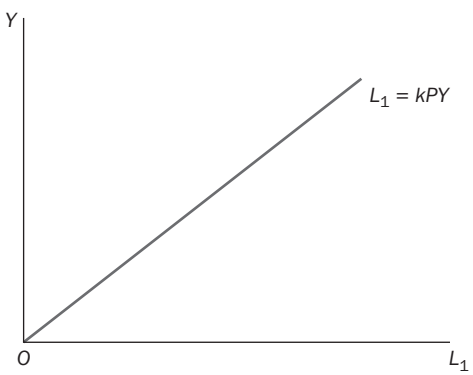

Fig. 4.7 Transactions and precautionary demand for money

much money to hold and on how many bonds to hold are really the same decision. This is because the sum of the individual's demand for money and demand for bonds has to add up to that person's total financial wealth. A decision to hold more money means a decision to hold less bonds. The wealth budget constraint implies that when the money market is in equilibrium, the bond market also is in equilibrium. This implication allows us to discuss assets markets entirely in terms of the money market.

Money does not earn any interest income. However, money is a certain asset in a sense that its capital value in terms of itself is always fixed. In other words, the nominal value of a ₹10 note is always ₹10. Money is a liquid asset. People create demand for money only because of its liquidity. Money is the most liquid asset and the cost of holding money as an asset is the foregone interest rate. Thus, there is an inverse relationship between the interest rate and the asset demand for money. If the interest rate goes up, the demand for liquidity goes down and vice versa.

Bonds, on the other hand, yield interest income to their holders adjusted by capital gain or capital loss. It can be shown algebraically that the price of a (perpetual) bond is given by the reciprocal of the market rate of interest times the coupon rate of interest. Suppose the coupon rate (i.e., interest payable on a bond) is ₹1 per year and the market rate of interest is 4% per year. Then, the market price of the bond will be ₹1/0.04 × 1 = ₹25. If the market rate of interest rises to 5% per year, the market price of the bond will fall to ₹1/0.05 × 1 = ₹20. Thus, bond price is seen as an inverse function of the interest rate. Bond prices keep on changing from time to time. Therefore, they are subject to capital gains or losses. Thus, to a bond-holder, the return from bond-holding per unit period (say a year) per ₹1 is the rate of interest ± capital gain or loss per year.

A bond is a promise by a borrower to pay the lender a certain amount (the principal) at a specified date (the maturity date of the bond) and to pay a given amount of interest per year in the meantime. A perpetuity is a bond which promises to pay interest forever, but not to repay the principal on the bond.

Equities or stocks are claims to a share of the profits of an enterprise. The shareholder, or stockholder, receives the return on equity in two forms: dividends and capital gain. The stockholders may receive a certain amount for each share they own in the form of dividends. Sometimes firms decide not to distribute profits to the stockholders but retain the profits and reinvest them by adding to the firms' stocks of machines and structures. In this case, the shares become more valuable, the price of the stock in the market will rise, and stockholders can make capital gains. A capital gain is an increase in the price of an asset per unit of time.

The speculators are of two kinds: *bulls* and *bears*. Bulls are those who expect the bond prices to rise in the future. Bears expect these prices to fall. In Keynes's model, these expectations are assumed to be held with certainty. Bulls, then, are assumed to invest all their idle cash into bonds and bears prefer more liquidity to bonds. To move to the aggregate speculative demand for money, Keynes assumed that different asset holders have different interest-rate expectations. Thus, at a very high rate of interest (and very low bond price), investors may behave like bulls. Then, the speculative demand for money will be equal to zero. However, at a very low rate of interest (very high bond price), investors will behave like bears and the demand for speculative balances will very high. Thus, Keynes's speculative demand for money is inversely related to the interest rate.

If we denote the speculative demand for money as L_2 and the interest rate as r, Keynes's speculative demand function can be written as

$$L_2 = L_2(r), \quad L_2' = \frac{dL_2}{dr} < 0 \tag{4.8}$$

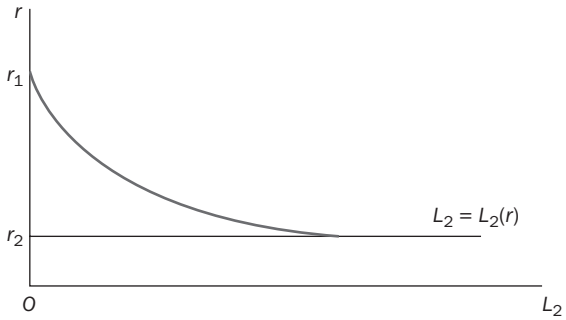

Fig. 4.8 Speculative demand for money

The geometric representation of Keynes's liquidity preference function or speculative demand for money function is shown in Fig. 4.8. Measuring L_2 along the horizontal axis and r along the vertical axis, the speculative demand curve is shown by a negatively sloped curve within a certain range of interest rates. It is vertical if the rate of interest is very high ($\geq r_1$), is negatively sloped if the rate of interest lies between r_1 and r_2, and becomes horizontal at the rate of interest r_2. As the rate of interest is very high at r_1, every investor behaves like bulls in the asset market and nobody prefers liquidity to bonds. Thus, at r_1, liquidity preference is zero and the speculative demand function is perfectly interest inelastic. The rate of interest is very low at r_2 and every investor in the asset market behaves like bears. As the interest rate is very low, everybody prefers liquidity and the liquidity preference function becomes perfectly interest elastic. This phase in the speculative demand function is known as *liquidity trap* in Keynes's theory and this trap appeared during the phase of deep depression.

The liquidity trap refers to a situation when at a certain rate of interest the speculative demand for money becomes perfectly elastic. This will come about when at that rate all the asset holders turn bears, so that none is willing to hold bonds and everyone wants to move into cash. At the rate of interest r_2, expansion of money supply cannot lower the rate of interest further. The people are willing to hold the entire extra amount of money in the form of cash at r_2. The extra liquidity created by the monetary authority gets trapped in the asset portfolios of the people without lowering r.

Keynes's theory of liquidity preference has been called into question by Tobin (1958).[1] In Keynes's theory, there is no uncertainty in interest-rate expectations. Unlike Keynes, Tobin assumes that an individual investor does hold his interest-rate expectations with uncertainty in the form of capital loss. Liquidity preference is analysed as behaviour towards risk under uncertainty. The degree of risk increases with the increase in the proportion of bonds in the asset portfolio. Asset holders need higher compensation in the form of rate of interest for undertaking higher risk. Thus, at higher interest rates, more bonds and less money will be held in the portfolio, and at lower interest rates, less bonds and more money will be preferred. The result is a diversified asset portfolio and a downward sloping asset demand curve for money with respect to the interest rate.

Keynes's theory of the speculative demand for money has also been criticized on the ground that bonds are taken as representative of all non-money financial assets. In reality, however, a large number of non-money financial assets are not similar to bonds. Examples of such assets include fixed deposits with commercial banks. Substitution between them and money does not entail Keynes's speculative motive, because they are not subject to variation in their nominal capital values. In their case, their rates of return influence as simple opportunity-cost variables without any element of speculation.

Also, as in Baumol–Tobin theory, the transactions demand for money also is interest elastic. The same can be argued for the precautionary demand for money too. The explanation of the speculative demand for money shows that this kind of demand will be an increasing function of total assets or wealth. If income is taken as a proxy for wealth, the speculative demand also becomes a function of both income and the rate of interest.

Keynes's money demand function can be specified as

$$L = L_1(Y) + L_2(r), \quad L_1' = \frac{dL_1}{dY} > 0, \quad L_2' = \frac{dL_2}{dr} < 0 \tag{4.9}$$

[1]Tobin, James (1958). Liquidity preference as behavior towards risk, *The Review of Economic Studies*, 25, 65–86.

Here, L is the total demand for money which is an additive demand function with two separate components.

The first part, $L_1(Y)$, represents the transactions and precautionary demand for money. Keynes made both an increasing function of the level of money income. In the Cambridge tradition, he tended to assume that $L_1(Y)$ had a proportional form of the kind represented in Fig. 4.7. The second component, $L_2(r)$, represents the speculative demand for money, which, as shown earlier, Keynes argued to be a declining function of interest rate. As shown in Fig. 4.8, this relation was not assumed to be linear.

Keynes's additive form of the demand function for money has been discarded by Keynesians and other economists. In addition, we can write the Keynesian money demand function as

$$L = L(Y, r), \quad \frac{\partial L}{\partial Y} > 0, \quad \frac{\partial L}{\partial r} < 0 \tag{4.10}$$

Here, it is hypothesized that L is an increasing function of Y and a decreasing function of r.

4.3.4 Supply of Money

In developing the LM curve, we assume that the entire money supply is an exogenous variable that is directly under the control of the central bank. The assumption that the money supply is exogenous is broadly correct, although it is oversimplified, since strictly speaking the central bank (e.g., Reserve Bank of India) can directly control only a small part of the stock of money. The theory of the demand for money is a theory about how the *real* value of money depends on income and the interest rate, but the theory of the money supply is a theory of how the *nominal* quantity of money is controlled by the central bank. The *real* supply of money depends not only on the behaviour of the central bank, but also on the price level. To complete our development of the LM curve, we will assume that the price level is exogenous.

4.3.5 Money Market Equilibrium: LM Curve

The demand for money is the purchasing power of money (real balances). The supply of money M is determined by the central bank, and its purchasing power is M/P, where P is the price level. It is assumed that the central bank has full control of the nominal money supply. The money market is in equilibrium when the demand for real balances equals the supply. We can write the equilibrium in the money market as

$$\frac{M}{P} = L(Y, r)$$

$$\text{or, } \frac{M}{P} = L_1(Y) + L_2(r) \tag{4.11}$$

Here, we have assumed the expected rate of inflation to be zero and therefore the nominal and real rate of interests are identical. Equation 4.11 shows several pairs of income (Y) and interest rate (r) that satisfy the equilibrium condition in the money market.

The LM curve represents the pairs of income and interest rate that will keep the money market in equilibrium with a given level of money supply and a given price level. The locus of all (y, r) combination points satisfying money market equilibrium as given in equation 4.11 is called the LM curve. In other words, the combinations of interest rates and income levels on the LM curve are such that the demand for real balances exactly matches the available supply. The LM schedule, or the money market equilibrium schedule, shows all combinations of interest rates and income levels such that the demand for real balances is equal to the supply. Along the LM schedule, the money market is in equilibrium.

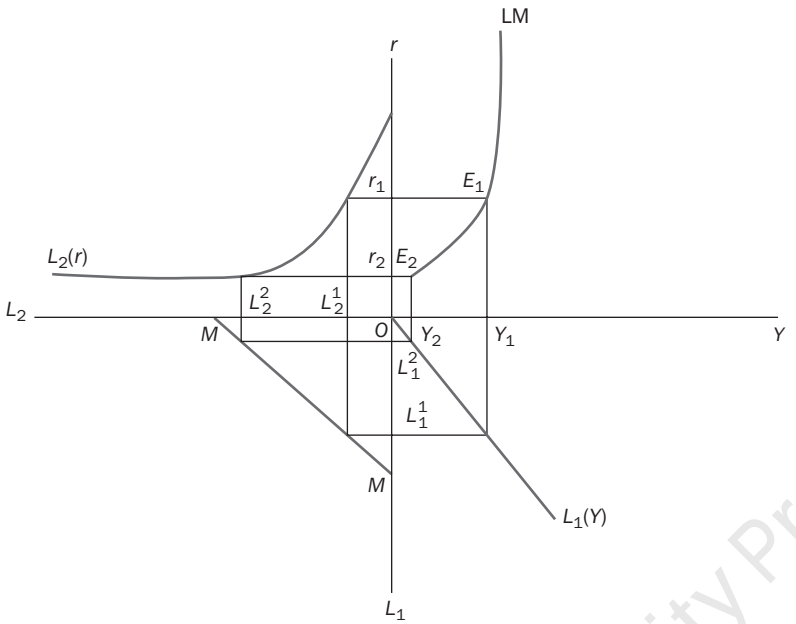

Fig. 4.9 Derivation of LM curve

The derivation of the LM curve in terms of a four-quadrant diagram is shown in Fig. 4.9. In the southeast quadrant of Fig. 4.9, the line $L_1(Y)$ gives the transactions and precautionary demand for money, which is an increasing function of income, measured downwards. In the northwest quadrant, $L_2(r)$ is the schedule representing the speculative demand curve showing the speculative demand for money is a decreasing function of interest rate. In the southwest quadrant, we find out equilibrium in the money market by equating the total demand for money to supply of it. A 45° line is drawn between the transactions demand axis and the speculative demand axis. The line is drawn at a distance from the origin on each axis equal to the total exogenously given real money supply, M/P. At any point on the 45° line, the transactions demand and the speculative demand add up to the total money supply on each axis. This 45° line directly represents the money market equilibrium condition.

We can now locate several pairs of (r, Y) that maintain the money market in equilibrium in the northeast quadrant. At a given level of income at Y_1, we can find L_1^1 transactions demand for money from the schedule $L_1(Y)$. We subtract this transactions demand from the money supply to find out the level of speculative demand at L_2^1 in maintaining money market equilibrium. This level of speculative demand shows the level of interest rate r_1 that will maintain money market equilibrium at income level Y_1. Thus, we have a pair (Y_1, r_1) at point E_1 in the northeast quadrant of Fig. 4.9. By repeating this process, we have another point E_2. By joining all such points, we can draw the LM schedule.

4.3.6 Slope of LM Curve

The LM curve is upward sloping because, given a certain amount of real balances, an increase in income requires an increase in the interest rate to keep the money market in equilibrium. For a particular LM curve, M and P do not change. Thus, by taking the total differential of equation 4.11, we have

$$0 = L_1' \mathrm{d}Y + L_2' \mathrm{d}r$$

$$\text{or, } \frac{\mathrm{d}r}{\mathrm{d}Y} = -\frac{L_1'}{L_2'} \tag{4.12}$$

Equation 4.12 provides the slope of the LM curve. Here, $L_1' = \partial L_1 / \partial Y$ is the slope of transactions demand for money and is positive; $L_2' = \partial L_2 / \partial r$ measures the slope of the speculative demand curve and is negative. Therefore, the slope of the LM curve, $\mathrm{d}r/\mathrm{d}Y > 0$, is positive. A rise in income increases the transactions demand

for money. The LM curve is positively sloped. An increase in the interest rate reduces the demand for real balances. To maintain the demand for real balances equal to the fixed supply, the level of income has to rise. Accordingly, money market equilibrium implies that an increase in the interest rate is accompanied by an increase in the level of income.

It is clear from equation 4.12 that the greater the responsiveness of the demand for money to income (L_1'), and the lower the responsiveness of the demand for money to the interest rate (L_2'), the steeper the LM curve. If the demand for money is relatively insensitive to the interest rate, so that L_2' is close to zero, the LM curve is nearly vertical. If the demand for money is very sensitive to the interest rate, so that L_2' is large, then the LM curve is close to horizontal. The horizontal part of the LM curve represents liquidity trap.

4.3.7 Points off LM Curve

We consider points off the LM schedule to characterize them as situations of excess demand or excess supply of money. Any point on the LM curve follows equilibrium in the money market in a sense that demand for real balances equals the supply. Points below and to the right correspond to an excess demand for real balances. Let us take a point E_1 on the LM curve where the combination of income and interest rate satisfies the money market equilibrium condition. Let income level be increased while the rate of interest remains the same as at point E_1. An increase in income moves the combination point from E_1 to A as shown in Fig. 4.10. As income is more at point A, demand for money is higher than its supply. Thus, at point A, there is an excess demand for money. By a similar argument, we can say that points above and to the left of the LM schedule correspond to an excess supply of real balances. Let us take another point E_2 on the LM curve where income and interest rate are determined by following money market equilibrium. Now, at point B, income is less while the interest rate is the same as at point E_2. Lower the income level, interest rate remaining the same, the lower will be the demand for money. Thus, at point B, there is an excess supply of money.

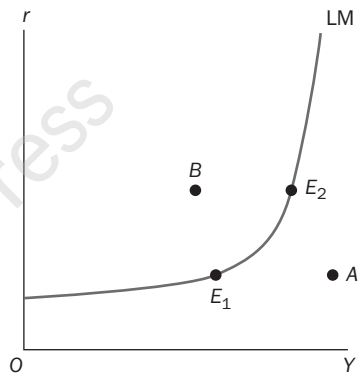

Fig. 4.10 Points off the LM curve

4.3.8 Shifts of LM Curve

The real money supply is held constant along the LM curve. It follows that a change in the real money supply will shift the LM curve. An increase in the money supply results in an excess of money at points on the initial LM curve and shifts the

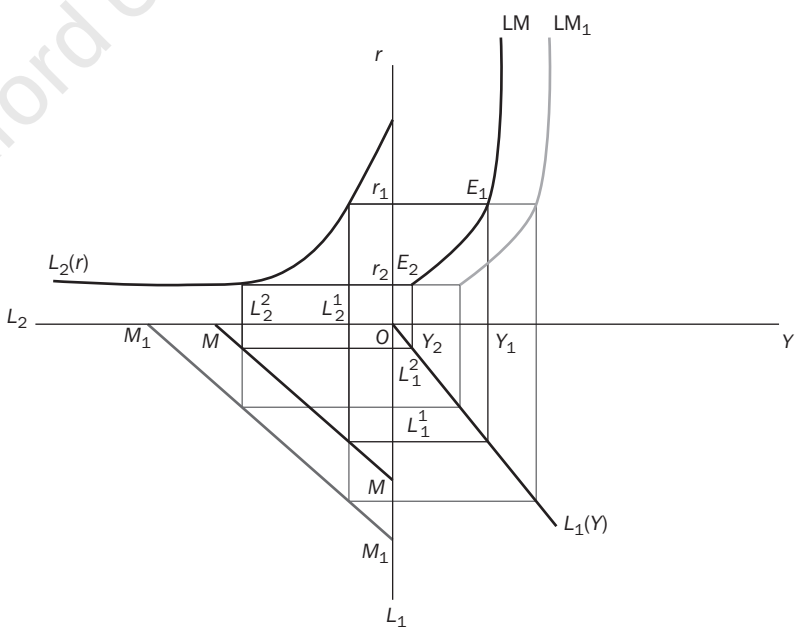

Fig. 4.11 Shift of the LM curve when money supply increases

LM curve to the right (Fig. 4.11). An increase in the stock of real balances shifts the 45° line outwards in the southwest quadrant from MM to M_1M_1. At the initial interest rate r_1, the equilibrium income will be higher to keep the money market in equilibrium. The equilibrium pair of income and interest rate shifts to the right and, as a result, the LM schedule will shift to the right in the northeast quadrant. At each level of the interest rate, the level of income has to be higher so as to raise the transactions demand for money and thereby absorb the higher real money supply.

Problem 4.2 Suppose that the real money demand function is

$L = 100 + 0.2Y - 200r$
Assume that $M = 300$, $P = 2$.
Find out the LM curve equation.
What is the value of the interest rate when $Y = 600$?

Solution The LM curve represents money market equilibrium. Therefore, the equation of the LM curve is

$$\frac{M}{P} = 100 + 0.2Y - 200r \text{ or, } 150 = 100 + 0.2Y - 200r$$

or, $0.2Y - 200r = 50$

When $Y = 600$, $120 - 200r = 50$ or, $r = 0.35$

The slope of the LM curve, $\frac{dr}{dY} = 0.001$

4.4 IS–LM Model

We are now ready to put together the two parts of the Keynesian theory of aggregate demand: the IS and LM curves. John Hicks, a Nobel laureate (1974), devised the IS-LM model based on Keynes's theory. IS stands for investment-savings and LM for liquidity-money. The IS-LM model describes the aggregate demand of the economy using the relationship between output and interest rates. Hicks allowed investment to be a function of interest rate. The IS equation shows an equilibrium relationship between the real income and the real rate of interest. The higher the rate of interest, the lower is investment, and consequently income. Hicks also considered the demand for and supply of liquidity or money, which depend on income and the nominal rate of interest. An increase in income results in a higher demand for money and an increase in the interest rate will decrease the demand. The reason is that the more income a nation has, the more transactions will be made; hence, the more money needed. However, the interest rate is the price of money, and an increase in the price will cut the demand.

The task of the IS–LM model is to determine income and interest rate simultaneously by the interaction of the product and money markets. When we put together the IS and LM curves, we will be able to describe the simultaneous determination of the nominal interest rate and income in an IS–LM equilibrium. When we build the IS–LM model, we will take the price level as fixed. Thus, we may get different equilibrium points, the points of intersection between the IS and LM curves, corresponding to different price levels. The relationship between the price level and the equilibrium value of income in the IS–LM model is called the Keynesian aggregate demand curve.

Figure 4.12 shows that the interest rate and the level of output are determined by the interaction of the money (LM) and product (IS) markets. In Fig. 4.12, we have combined the LM curve with the IS curve. The IS curve, given in equation 4.1, represents values of the interest rate and income for which the product market is in equilibrium. Any point on the IS curve satisfies the product market equilibrium condition. The LM curve, presented in equation 4.11, denotes values of the interest rate and income for which the quantity of money demanded is equal to the quantity supplied. At any point on the LM curve, the money market is in equilibrium. The equilibrium of the IS–LM model occurs at point *E*, where the IS and LM curves intersect. This is the only point at which both the product market and the money market are in equilibrium simultaneously.

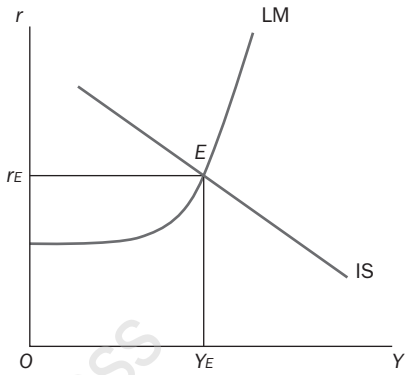

Fig. 4.12 Equilibrium in the IS–LM model

At point *E* in Fig. 4.12, the economy is in equilibrium, given the price level, because both the goods and money markets are in equilibrium. The demand for goods is equal to the level of output on the IS curve. In addition, on the LM curve, the demand for money is equal to the supply of money. Accordingly, at point *E*, firms produce their planned amount of output (there is no unintended inventory accumulation or rundown), and individuals have the portfolio compositions they desire.

What would happen if the economy were at a point other than point *E*? The answer is that there are two forces pulling the economy back to the point of intersection of the IS and LM curves. First, suppose that the economy is at a point that is below the IS curve. Points below the IS curve are points for which investment exceeds savings. Investors will bid up the interest rate in an attempt to secure funds and the interest rate will rise. A similar argument establishes that points above the IS curve are points for which savings exceed investment. Investors will be able to offer lower interest rates since there is an excess of savers in the market and the interest rate will fall.

At any point to the left of the LM curve, income is lower than a point that is on the LM curve but with the same interest rate. Since income is lower, the quantity of money demanded will also be lower. It follows that points to the left of the LM curve are points for which there is an excess supply of money. Households are holding more money than they need to finance their daily transactions and they will try to spend this money by demanding more commodities; the aggregate demand for goods and services will increase. As demand increases, firms will hire more workers and employment and income will rise until the economy is back on the LM curve. A similar argument establishes that if the economy is to the right of the LM curve, there is an excess demand for money. Households will buy fewer commodities and aggregate demand will fall. As demand falls, firms will lay off workers and employment and income will fall until the economy is back on the LM curve.

Income and interest rates adjust to the disequilibrium in goods markets and assets markets. Specifically, interest rates fall when there is an excess supply of money and rise when there is an excess demand. Income rises when aggregate demand for goods exceeds output and falls when aggregate demand is less than output. The system converges over time to the equilibrium at *E*. Thus, there are forces that move the economy towards point *E*, the IS–LM equilibrium point.

4.4.1 Derivation of Aggregate Demand Curve

The major assumption in IS–LM equilibrium is that the price level is constant and that firms are willing to supply whatever amount of output is demanded at that price level. Let us look at how equilibrium income in the IS–LM model is different for different values of the price level. The aggregate demand curve links the price

level to the aggregate quantity of commodities demanded. In the IS–LM model, the equilibrium output and the interest rate are determined under the assumption that the price level remains the same. When the price level changes and other factors remain the same, the position of the LM curve changes, and accordingly the equilibrium point will change. Figure 4.13 derives the aggregate demand curve. In Panel A, we plot the interest rate against income. We will use Panel A to show that the IS–LM equilibrium is different for different values of the price level. We must know the price level to construct this diagram since the real value of the supply of money depends on it. Given the price level P_1, the LM curve is LM_1. The equilibrium point is determined at E_1 where output and interest rate are determined at Y_1 and r_1, respectively. Tracing equilibrium income down to Panel B gives a point, E_1, corresponding to the equilibrium point, on the Keynesian aggregate demand curve.

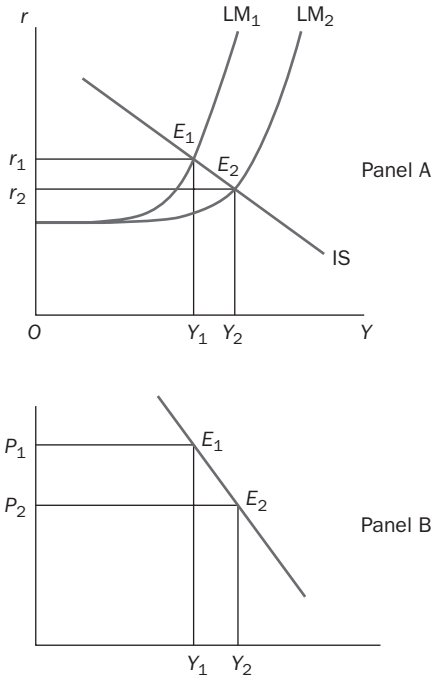

Fig. 4.13 Derivation of aggregate demand curve

Let the price level drops from P_1 to P_2. If price level falls to P_2, the supply of real balances will increase and the LM curve shifts in the rightward direction to LM_2 as shown in Panel A. Given the IS curve, the equilibrium point shifts to E_2, where output is determined at Y_2. The price output combination point is plotted at E_2 in Panel B. By joining all such points in Panel B, we have drawn a downward sloping curve. This curve resembles the aggregate demand curve. At every point on the aggregate demand curve, both the product market and the money market are in equilibrium.

4.4.2 Mathematical Formulation of IS–LM Model

The intersection of the IS and LM schedules in the diagrams corresponds to a situation in which both the IS and LM equations hold. The same interest rate and income levels ensure equilibrium in both the goods and the money market. In terms of the equations, this means that we have to solve the IS curve equation (4.1) and LM curve equation (4.11) simultaneously to get equilibrium values of Y and r. For simultaneous equilibrium, interest rates and income levels have to be such that both the goods market and the money market are in equilibrium. We reproduce here the IS curve equation and the LM curve equation as shown in equations 4.1 and 4.11, respectively, as

$$Y = C\left(Y - t(Y)\right) + I(r) + G\,;\ \frac{M}{P} = L_1(Y) + L_2(r)$$

By solving these equations, we will get equilibrium income and interest rate in the IS–LM model. To understand how to calculate equilibrium income and interest rate in the IS–LM framework, let us consider the following IS and LM curve equations:

$$Y = b(1-t)Y - b_1 r + G \tag{4.13}$$

$$m = \frac{M}{P} = l_1 Y - l_2 r \tag{4.14}$$

Here, m denotes real balances, l_1 is income sensitivity of money demand, and l_2 is interest sensitivity of money demand.

In matrix form, equations 4.13 and 4.14 can be expressed as

$$\begin{bmatrix} 1-b(1-t) & -b_1 \\ l_1 & -l_2 \end{bmatrix} \begin{bmatrix} Y \\ r \end{bmatrix} = \begin{bmatrix} G \\ m \end{bmatrix} \tag{4.15}$$

By using Cramer's rule, we can solve for equilibrium values of income and interest rate as

$$Y = \frac{-l_2 G + b_1 m}{-l_2\{1-b(1-t)\} + l_1 b_1} = \frac{G - \dfrac{b_1 m}{l_2}}{1-b(1-t) - \dfrac{l_1}{l_2} b_1} \tag{4.16}$$

Equation 4.16 shows that the equilibrium level of income depends on two exogenous variables: government spending (G) and the real money stock ($m = M/P$). Equilibrium income is higher the higher the level of government spending and the higher is the stock of real balances.

$$r = \frac{m\{1-b(1-t)\} - l_1 G}{-l_2\{1-b(1-t)\} + l_1 b_1} = \frac{-\dfrac{m}{l_2}\{1-b(1-t)\} + \dfrac{l_1}{l_2} G}{1-b(1-t) - \dfrac{l_1}{l_2} b_1} \tag{4.17}$$

Equation 4.17 shows that the equilibrium interest rate depends on the parameters of fiscal policy captured in the multiplier and the term G and on the real money stock, m. A higher real money stock implies a lower equilibrium interest rate.

4.5 Fiscal and Monetary Policies

Equilibrium in both the goods and money markets is simultaneously attained where the IS and LM curves intersect, that is, at point E in Fig. 4.12. Two points are worth emphasizing. First, the intersection of the two curves in Fig. 4.12 determines the values of the rate of interest and income which are consistent with equilibrium in both markets. Second, if the level of income is below that of full employment, then both fiscal and monetary policies have a potentially important role to play in stabilizing the economy. We now briefly review what determines the relative effectiveness of fiscal and monetary policies in influencing aggregate demand and therefore the level of output and employment.

4.5.1 Fiscal Policy Multiplier

The equilibrium levels of income and the interest rate change when either the IS or the LM curve shifts. The fiscal policy multiplier shows how much an increase in government spending, or a cut in tax rate, changes the equilibrium level of income, holding the real money supply constant. Figure 4.14 shows the effects of an increase in government expenditure on equilibrium income and interest rate. An increase in government spending shifts the IS schedule out and to the right from IS_1 to IS_2. The shift of the IS curve, given the LM curve at its initial position, results in a rise in the level of income from Y_1 to Y_2 and an increase in the interest rate from r_1 to r_2 by the movement of equilibrium from

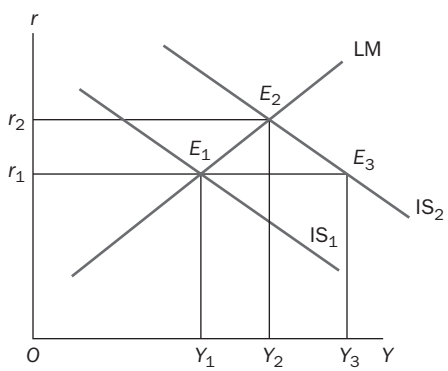

Fig. 4.14 Effects of an increase in government expenditure

E_1 to E_2. Here, the change in income is Y_1Y_2, which is clearly less than the horizontal shift in the IS curve. If, somehow the interest rate remains at r_1, the increase in income (Y_1Y_3) equals the length of the horizontal shift of the IS curve.

The increase in government spending does tend to increase the level of income. However, an increase in income increases the demand for money. With the supply of money fixed, the interest rate has to rise to ensure that the demand for money stays equal to the fixed supply. When the interest rate rises, investment spending is reduced because investment is negatively related to the interest rate. Accordingly, the change in equilibrium income is less than the change in the simple Keynesian model.

The government expenditure multiplier in the IS–LM framework can be obtained directly from the expression of equilibrium income shown in equation 4.16. We allow Y to change when G changes with fixed money supply and other parameters. Thus, by taking the total differential of equation 4.16 and keeping $dm = 0$, we have

$$dY = \frac{-l_2 dG}{-l_2\{1-b(1-t)\}+l_1 b_1} = \frac{dG}{1-b(1-t)-\frac{l_1}{l_2}b_1}$$

$$\text{or, } \frac{dY}{dG} = \frac{1}{1-b(1-t)-\frac{l_1}{l_2}b_1} \tag{4.18}$$

Similarly, the change in interest rate due to change in government spending is obtained from equation 4.17:

$$dr = \frac{-l_1 dG}{-l_2\{1-b(1-t)\}+l_1 b_1} = \frac{\frac{l_1}{l_2}dG}{1-b(1-t)-\frac{l_1}{l_2}b_1}$$

$$\text{or, } \frac{dr}{dG} = \frac{\frac{l_1}{l_2}}{1-b(1-t)-\frac{l_1}{l_2}b_1} \tag{4.19}$$

Equations 4.18 and 4.19 provide the rate of change of income and interest rate due to the change in government expenditure. The expression given in 4.18 is known as the government expenditure multiplier in the IS–LM model. If we compare the IS–LM multiplier with the simple Keynesian multiplier, it would be clear that the value of the multiplier in the IS–LM model is less than the value of multiplier in the simple Keynesian model. This is because of the additional term in the denominator, $-(l_1/l_2)\,b_1$, in equation 4.18. Here, l_1/l_2 is the slope of the LM curve given in equation 4.14. It gives the increase in r that is needed for money market equilibrium with the increase in Y. Since b_1, the slope of the linear investment function, gives the change in investment that comes from change in r, the expression $-l_1/l_2\,b_1$ then gives the decrease in investment that comes from the interest rate increase as Y and r rise along the LM curve.

The tax rate is another important fiscal instrument by which the government can control aggregate demand. To find out the tax cut multiplier, let us take the equilibrium condition in the product market as shown in

the IS curve equation (4.13) and the money market equilibrium condition is given in the LM curve equation shown in (4.14):

$$Y = b(1-t)Y - b_1 r + G$$

$$m = \frac{M}{P} = l_1 Y - l_2 r$$

To calculate the tax cut multiplier, we have to keep government expenditure and real money supply constant, and we can allow the tax rate (t) to change. Taking the total differential of equations 4.13 and 4.14, we have the following expressions:

$$\mathrm{d}Y = b(\mathrm{d}Y - t\mathrm{d}Y - Y\mathrm{d}t) + b_1 \mathrm{d}r$$

or,

$$\{1-b(1-t)\}\mathrm{d}y - b_1 \mathrm{d}r = -bY\mathrm{d}t \tag{4.20}$$

$$l_1 \mathrm{d}y - l_2 \mathrm{d}r = 0 \text{ or, } \mathrm{d}r = \frac{l_1}{l_2}\mathrm{d}y \tag{4.21}$$

Substituting the value of dr from (4.21) into (4.20) gives us

$$\{1-b(1-t)\}\mathrm{d}Y - b_1 \frac{l_1}{l_2}\mathrm{d}Y = -bY\mathrm{d}t \tag{4.22}$$

$$\text{or, } \mathrm{d}Y = \frac{-bY\mathrm{d}t}{1-b(1-t)-\frac{l_1}{l_2}b_1}$$

The numerator of equation 4.22 simply converts the tax change into the policy-induced change in consumer expenditure. The term $Y\mathrm{d}t$ is the change in disposable income that comes directly from the change in tax rate. Thus, the term $bY\mathrm{d}t$ is the change in consumption spending that comes from the change in disposable income. The negative sign implies that when tax rates go down, the policy-induced consumption expenditure goes up. By comparing equations 4.18 and 4.22, we can say that the tax cut multiplier is similar to the government expenditure multiplier.

4.5.2 Effectiveness of Fiscal Policy

An increase in government spending shifts the IS curve to the right, resulting in higher income and interest rate (shown in Fig. 4.14). The economic explanation is that the additional expenditure creates income for those who supply goods and services to the government. Again, the multiplier effect works to increase the aggregate income beyond the initial increase in government expenditure. However, there is a difference here. The increase in income increases the demand for money, which, given the supply of money, would increase the interest rate. Thus, there will be drop in investment, which to some extent will negate the initial effect of the expansionary fiscal policy. Nevertheless, because investment is not too sensitive to interest rate, this secondary effect would be small. A tax cut will also shift the IS curve to the right and will have an expansionary effect.

One can verify, from the expression for the fiscal policy multiplier as shown in equation 4.18, that fiscal policy will be more effective in influencing aggregate demand and therefore the level of output and employment

when the more interest-elastic is the demand for money (the flatter is the LM curve), and the less interest-elastic is investment (the steeper is the IS curve). In the limiting cases of a vertical LM curve (classical range), fiscal expansion will have no effect on income. In this phase, the rise in the rate of interest will reduce private investment by an amount identical to the increase in government expenditure producing full crowding out. At the other extreme, a horizontal LM curve (liquidity trap) provides the full multiplier effect of the simple Keynesian model.

How effective the fiscal policy is depends on the rate of fall of private investment owing to the rise in interest rate induced by the fall in speculative demand for money. The fall in investment due to the rise in interest rate along the LM curve induced by the increase in autonomous spending is known as the *crowding out effect*. The government expenditure multiplier is reduced by this crowding out effect. The effect depends on the slope of the LM curve. Given the slope of the IS curve, the crowding out is higher the more inelastic the LM curve. Given the slope of the LM curve, the crowding out is higher the more elastic the IS curve. An expansionary fiscal policy (increasing in G or decreasing in T) is more effective in affecting the level of real income, and the lower is the crowding out effect.

If the LM curve is horizontal, there will be no crowding out effect and full expansionary effect will be obtained (Fig. 4.15). The increase in government expenditure causes the level of income to rise by Y_1Y_2. This amounts to the full multiplier times the increase in G. If the increase in G is not tax financed and the money supply is fixed, the funds are obtained by borrowing from the non-bank private sector of the economy. This type of fiscal policy is a pure fiscal policy. As the interest elasticity of the demand for money is infinite, a very small increase in interest rate resulting from deficit financing will decrease the liquidity preference. Thus, a larger level of transactions has to be financed by a fixed money supply. In the new equilibrium, the rate of interest will be the same as before. As a result, private investment will be the same as before. Therefore, the increase in income will equal the full multiplier times the increase in government purchases. Thus, if the economy passes through deep depression characterized by the liquidity trap in the money market, the multiplier effect will be full. If the LM curve is horizontal, then the interest rate will not change when the IS curve shifts. This is the pure Keynesian situation where the fiscal policy will be fully effective.

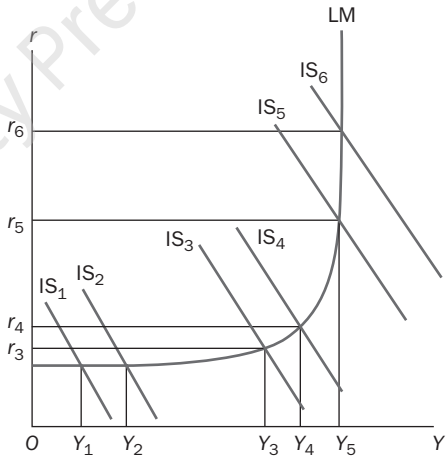

Fig. 4.15 Expansionary effects at different cyclical positions

If the LM curve is upward rising as in the case of a general Keynesian model, the crowding out effect is partial. In this case, the increase in G succeeds in increasing the equilibrium income from Y_3 to Y_4. However, in this intermediate range, the need to finance the increased volume of transactions forces the rate of interest up from r_3 to r_4. The level of private investment will fall because of this increase in interest rate. The expansionary effect of the fiscal policy is therefore partially negated.

If the LM schedule is vertical, the expansionary effect will be fully crowded out and the value of the fiscal policy multiplier is zero. The LM curve is vertical in a situation of full employment representing the classical case. Thus, in the classical range, the increase in G has no effect on the level of income. As there are no idle money balances available in the private sector of the economy, the government can borrow funds from the private sector only at the cost of the proportional reduction in private investment. Interest rates must rise enough to make the return on government bonds greater than the prospective yield on private investment. Any increase in G will be matched by an equal reduction in private investment. Thus, the crowding effect is full and fiscal policy has no effect on the level of income. Allocation of resources will change from the private sector to the public sector without any change in the level of income.

The orthodox Keynesian faith in the effectiveness of fiscal policy has been challenged by, among others, monetarists who typically argue that in the long run 'pure' fiscal expansion (i.e., expansion without any accommodating changes in the money supply) will result in the crowding out or replacement of components of private expenditure with relatively minor effects on aggregate demand, the level of income, and employment. A number of reasons as to why crowding out can occur in the IS–LM framework have been put forward in the literature, which do not rely on the demand for money being perfectly interest inelastic (a vertically sloped LM curve), including expectations and wealth effects (Carlson and Spencer).[2]

4.5.3 Monetary Policy Multiplier

The monetary policy multiplier shows how much an increase in the real money supply increases the equilibrium level of income, keeping fiscal policy unchanged.

$$\begin{aligned} &dY = b(dY - t\,dY) + b_1 dr \\ &\text{or,} \\ &\{1 - b(1-t)\}dY - b_1 dr = 0 \end{aligned} \tag{4.23}$$

$$l_1 dy - l_2 dr = dm \text{ or, } dr = \frac{l_1}{l_2} dy - \frac{1}{l_2} dm \tag{4.24}$$

By substituting dr from (4.24) into (4.23), we have

$$\{1 - b(1-t)\}dY - b_1\left(\frac{l_1}{l_2} dY - \frac{1}{l_2} dm\right) = 0$$

$$\text{or, } dY = \frac{\frac{b_1}{l_2} dm}{1 - b(1-t) - \frac{l_1}{l_2} b_1} \tag{4.25}$$

Equation 4.25 provides the monetary policy multiplier. The smaller l_2 and l_1 and the larger b_1, the more expansionary the effect of an increase in real balances on the equilibrium level of income. Large b_1 corresponds to a very flat IS schedule. The monetary policy multiplier is illustrated in Fig. 4.16. An increase in M will shift the LM curve to the right, by an amount given by the change in M. Let the initial LM curve be LM_1. After the increase in M, the new LM curve is LM_2. As we can see the effect is an increase in income from Y_1 to Y_2 and a reduction in interest rate from r_1 to r_2. Thus, an increase in money supply (everything else constant) will increase the equilibrium level of income and decrease the equilibrium interest rate. The idea is that an increase in the money supply, given the money demand, will create an excess of money in the money market. To eliminate this excess of money, the theory of liquidity preference says that the interest rate must decrease. However, if the interest rate decreases, investment will increase and therefore income will increase. As income increases, money demand increases, helping to restore the equilibrium in the money market. The process ends at the new equilibrium E_2 determining new income and interest rate at Y_2 and r_2, respectively. It should be evident that a

[2]Keith M. Carlson and Roger W. Spencer, "Crowding Out and Its Critics," Federal Reserve Bank of St. Louis *Review*, December 1975, pp. 2–17.

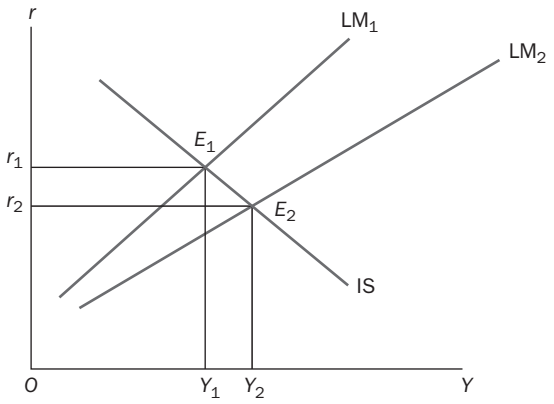

Fig. 4.16 Monetary policy multiplier

decrease in money supply will have the reverse effects. The interest rate will increase causing investment and income to decrease. Again, the reduction in income, to some extent, will modify the effects of the drop in money supply. The reason is that the lower income will reduce the demand for money.

4.5.4 Effectiveness of Monetary Policy

The effectiveness of monetary policy depends on the cyclical position of an economy. Within the orthodox Keynesian transmission mechanism, the effectiveness of monetary policy depends on the degree to which the rate of interest falls following an increase in the money supply, and the degree to which investment responds to a fall in the rate of interest. We can verify that monetary policy will be more effective in influencing aggregate demand and therefore the level of output and employment when the demand for money is more interest inelastic (the steeper is the LM curve), and the investment is more interest elastic (the flatter is the IS curve). In the extreme Keynesian cases of either a horizontal LM curve (liquidity trap) or a vertical IS curve (investment is completely interest inelastic), the transmission mechanism breaks down and monetary policy will have no effect on the level of income. In the case of the liquidity trap, the interest rate is at the lowest possible level and the additional liquidity would not have any effect on the interest rate and on investment.

Figure 4.17 shows how the monetary policy affects the level of income and the interest rate. If the IS schedule (IS_1) cuts the LM curve in the Keynesian liquidity trap region, the increase in the money supply will not affect the level of income and the interest rate at all. A very little fall in the interest rate produces an unlimited preference for liquidity so that all the added money balances move into idle holdings and none move into added transaction balances. In equilibrium, therefore, the rate of interest remains the same. Investment is not stimulated and the level of income remains unchanged. Thus, if liquidity trap prevails as in deep depression, monetary policy is totally ineffective in expanding the economy, or in controlling inflation.

If the IS curve (IS_2) cuts the LM curve in the upward rising part, the increase in the money supply succeeds in increasing the level of income along with the rise in interest rate. Thus, there will be crowding out effect due to the fall in investment because of the rise in interest rate induced by the increasing money supply. In this case, part of the increase in money supply is absorbed into active balance in the form of transactions demand for money and the rest into liquidity preference. The interest rate declines and investment will increase, but not to the full extent. The level of income will rise only partially as shown in equation 4.25. Therefore, if the LM curve is upward rising, known as the general Keynesian range, the monetary policy is partially effective.

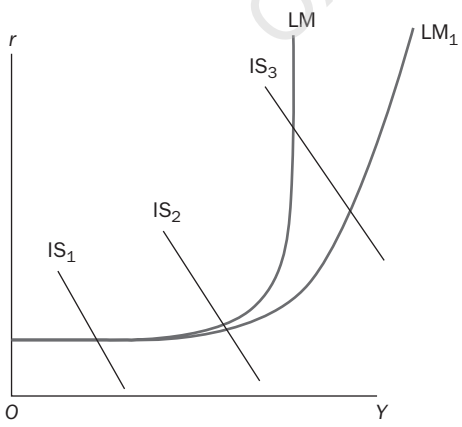

Fig. 4.17 Expansionary effects of monetary policy

If the IS curve (IS_3) cuts the LM curve in the vertical part of the LM curve representing the cyclical phase as described by classical economists, the monetary policy will be fully effective. In this phase, the prices of government bonds would be bid up enough that other assets are relatively more attractive than

government bonds. There will be no liquidity preference because of high interest rates and wealth holders will use added money balances to purchase other earning assets. In other words, the added money supply will move into investment in other earning assets such as new securities or new investment in physical capital in the real sector. New capital investment will raise the level of income, raising the transactions demand for money. Wealth holders will continue to compete for earning assets, bond prices continue to rise, and interest rates continue to fall until the point is reached where new investment raises the level of income exactly enough to absorb added money balances into transactions.

From the earlier discussion, it should be evident that, while both fiscal and monetary policies can, in normal circumstances, be used to influence the level of output and employment, the relative effectiveness of these two policy instruments depends on the structural parameters of the model, that is, the relative slopes of the IS and LM curves. Within the orthodox Keynesian approach, the demand for money has traditionally been viewed as being highly responsive to changes in the rate of interest (generating a relatively flat LM curve), while investment has been taken as being fairly unresponsive to changes in the rate of interest (generating a relatively steep IS curve).

SUMMARY

- The IS curve is the locus of combinations of the interest rate and level of income such that the product market is in equilibrium.
- The IS curve is negatively sloped, implying that an increase in the interest rate reduces planned investment spending and therefore reduces aggregate demand, reducing the equilibrium level of income.
- The less sensitive investment spending is to changes in the interest rate, the steeper the IS curve.
- The IS curve is shifted by changes in autonomous spending. An increase in autonomous spending, including an increase in government purchases, shifts the IS curve out to the right.
- At points to the right of the curve, there is excess supply in the goods market; at points to the left of the curve, there is excess demand for goods.
- In Keynes's theory, money is demanded due to three main motives: transactions motive, precautionary motive, and speculative motive.
- The transactions demand for money is using money as a medium of exchange. People want to hold money for transactions purposes to bridge the gap between the earning point and the spending point. The transactions demand for money is a demand for active balance because it is used as a means of payments in national income-generating transactions.
- Precautionary demands for money are those that are held to provide for contingencies requiring sudden expenditure and for unforeseen opportunities of advantageous purchases, and also to hold an asset of which the value is fixed in terms of money to meet a subsequent liability in terms of money.
- People hold money as a financial asset just like stocks and bonds. Holding money as a liquid asset is using money as a store of value. This is called the speculative motive. The speculative demand for money is a demand for idle balances. Keynes's speculative demand for money is inversely related to the interest rate.
- The liquidity trap refers to a situation when at a certain rate of interest the speculative demand for money becomes perfectly elastic.
- Money market equilibrium implies that an increase in the interest rate is accompanied by an increase in the level of income.
- The LM curve is the schedule of combinations of income and interest rates such that the money market is in equilibrium.
- The LM curve is positively sloped. Given the fixed money supply, an increase in the level of income, which increases the quantity of money demanded, has to be accompanied by an increase in the interest rate. This reduces the quantity of money demanded and thereby maintains money market equilibrium.
- The LM curve is shifted by changes in the money supply. An increase in the money supply shifts the LM curve to the right.
- At points to the right of the LM curve, there is an excess demand for money, and at points to its left, there is an excess supply of money.

- The interest rate and level of output are jointly determined by the simultaneous equilibrium of the goods and money markets. This occurs at the point of intersection of the IS and LM curves.
- The fiscal policy multiplier shows how much an increase in government spending, or a cut in tax rate, changes the equilibrium level of income, holding the real money supply constant.
- The fall in investment due to the rise in interest rate along the LM curve induced by the increase in autonomous spending is known as the crowding out effect. The effect depends on the slope of the LM curve.
- The monetary policy multiplier shows how much an increase in the real money supply increases the equilibrium level of income, keeping fiscal policy unchanged.
- If the LM curve is horizontal, there will be no crowding out effect and fiscal policy will be fully effective. If the LM curve is upward rising as in the case of a general Keynesian model, the fiscal policy effect is partial. If the LM schedule is vertical, the expansionary effect will be fully crowded out and the value of the fiscal policy multiplier is zero.
- The effectiveness of monetary policy depends on the degree to which the rate of interest falls following an increase in the money supply, and the degree to which investment responds to a fall in the rate of interest. If liquidity trap prevails as in deep depression, monetary policy is totally ineffective in expanding the economy, or in controlling inflation. If the LM curve is upward rising, known as the general Keynesian range, the monetary policy is partially effective. If the IS curve cuts the LM curve in the vertical part of the LM curve representing the cyclical phase as described by classical economists, the monetary policy will be fully effective.

EXERCISES

4.1 What is an IS curve?
4.2 Why is the IS curve negatively sloped?
4.3 What is an LM curve?
4.4 Why is the LM curve positively sloped?
4.5 Explain the concept of liquidity trap.
4.6 Explain how the IS curve is derived.
4.7 Explain how the LM curve is derived.
4.8 Show how equilibrium level of income and interest rate are simultaneously determined in the IS–LM model.
4.9 The relative effectiveness of the fiscal and monetary policies depends on the slopes of IS and LM curves. Explain.
4.10 Show with the help of a diagram that the extent of change in national income arising from an increase in government expenditure depends on the elasticity of demand for money. [C.U., 2012]
4.11 How are interest rate and national income affected by a rise in the supply of money? [C.U., 2012]
4.12 Indicate briefly the aspects of effectiveness and limitations of fiscal policy.
4.13 Discuss with the help of the IS–LM model the effectiveness of fiscal policy in increasing national income. [C.U., 2013]
4.14 Derive graphically the IS curve. [C.U., 2013]
4.15 Explain graphically how the LM curve shifts with a change in money supply. [C.U., 2013]
4.16 Discuss Keynes's theory of demand for money. [C.U., 2013]
4.17 Define LM curve. Derive and explain the different slopes of the LM curve in its different segments. [C.U., 2015]
4.18 Suppose that the government adopts an expansionary fiscal policy by way of increasing autonomous government expenditure. Show that the resulting expansionary impact on the equilibrium income will be less in the IS–LM model than that in the simple Keynesian model. [C.U., 2015]
4.19 Consider the following economy:

$C = 200 + 0.5Y$
$I = 200 - 500r$
$G = 150$
$L = 0.5Y - 250r$
$M = 4900$
$P = 10$

(a) Find out the equation of the IS curve.
(b) Find out the equation of the LM curve.
(c) Find out the equilibrium values of Y and r.