Engineering Mechanics

Statics and Dynamics

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Published in India by
Oxford University Press
Ground Floor, 2/11, Ansari Road, Daryaganj, New Delhi 110002, India

First published in 2019

ISBN-13: 978-0-19-949461-3
ISBN-10: 0-19-949461-4

Typeset in TimesLTStd-Regular
by Archetype, New Delhi 110063
Printed in India by Magic International (P) Ltd., Greater Noida

Preface

The subject of mechanics became popular among practising engineers, scientists, and academicians after Newton (1642–1727) established his three fundamental laws. Almost all problems of mechanics (both statics and dynamics) generally fit into these three laws. Any physical phenomenon with reference to the balance of force, moment, energy, and momentum, etc., and satisfying the conservation principles can be analysed or modelled using the laws of mechanics.

Many problems in universe are of simple engineering common sense. Design of any component, structure, or system, which may be subjected to static and dynamic loads, requires a thorough knowledge of engineering mechanics and other subjects derived from it. One who understands this subject can become a successful practising mechanical, structural, or process engineer or even a successful software engineer, who would still need mathematical logic to comprehend problems.

ABOUT THE BOOK

Engineering Mechanics: Statics and Dynamics is specially designed for undergraduate engineering students pursuing a one-semester course on engineering mechanics offered by various technological universities across the country.

The book deals with both statics and dynamics, the essential parts of rigid body mechanics. The content of the book, which was first published in 2010, is now updated with different types of additional problems with relevant illustrations and each chapter is provided with multiple-choice questions with answers. It starts with the discussion of elementary topics, including units, dimensions, and fundamental laws of mechanics, which form the basis of subsequent concepts such as dynamics, friction, and rigid body mechanics covered in the later part of the book. Students often find it difficult to understand and assimilate the concepts of mechanics, which are applied to solve engineering problems. Hence, a strong command of the fundamentals becomes essential.

Comprehensive and student-friendly in its treatment of concepts, the book provides numerous solved problems, multiple-choice questions with answers, exercises with hints, objective-type questions with answers, and solved model question papers given as additional online resources. It is recommended that all students follow the simple algebraic (addition and summation with respect to sign convention) approach to solve the problems.

SALIENT FEATURES

- The language of the text is simple and lucid with self-explanatory line diagrams, terminologies, and nomenclatures throughout the book to enable the students to understand the concepts easily.
- The chapters have been reorganized with independent parts dedicated to Statics and Dynamics.

- Two exclusive chapters present on properties of surfaces and solids, which delve deep into centroid, centre of gravity, moment of inertia, and mass moment of inertia.
- The worked-out problems interspersed throughout the text illustrate a systematic solution, starting with free-body diagram, moving on to novel quadrant approach to resolve the forces on plane, followed by representation of equilibrium equations with sign conventions, etc. Some of the problems are preferred to be solved by both vector and simple algebraic approaches. All exercise problems are supported with hints.
- At the end of each chapter, important concepts are summarized in recapitulation.
- University question paper (May 2017) is provided with complete solution and relevant illustrations.

ONLINE RESOURCES

The following resources are available to support the faculty and students using this text:

For faculty	For students
• Chapter-wise lecture PPTs	• Objective-type questions with answers to tackle both university examinations and relevant competitive examinations • Model question paper with complete solution, which will be helpful to the students and will also serve as a reference for teachers • Additional chapter-wise solved problems

CONTENT AND STRUCTURE

Each chapter commences with a list of learning objectives, which inform the reader what he/she is going to learn in the chapter. All the chapters are organized in such a way that the reader gets a thorough knowledge of the fundamentals, terminologies, and the advanced concepts clearly. This book is divided into two parts—Statics and Dynamics, each comprising a set of chapters related to that concept.

Part I – Statics comprises the following seven chapters:

Chapter 1 introduces the subject and discusses conceptualization of a body as a particle, plane rigid body, and three-dimensional rigid body (the actual body), depending upon the types of forces applied. Further, it deals with important laws and principles of mechanics. It also touches upon the units for various quantities and dimensional analysis to check the balance of units on both sides of an equation. Vector mathematics is introduced to represent the force and moment. Different vector operations are dealt with for addition and subtraction of forces. Calculation of moment about a point due to the force acting at another point using the cross product of two vectors (position vector and force vector) is also discussed.

Chapter 2 covers coplanar, concurrent forces. For a system of such forces, rigid body can be treated as a particle. Different methods, both analytical and polygon laws of graphical approach, for resolving a force are introduced in this chapter. Novel quadrant technique for resolving an inclined force along *x*- and *y*-directions on a plane, which is a simple method, is explained elaborately. Inclined quadrant approach is further discussed to resolve the forces along the edges of an inclined quadrant, which can be applied to inclined plane problems. Conditions of equilibrium of a system of concurrent, coplanar forces acting on a particle with sign conventions are also dealt with.

Chapter 3 deals with forces in space, that is, concurrent, non-coplanar forces. Vector representations of a force in space, finding the equivalent system of concurrent, non-coplanar forces, and equilibrium conditions with sign convention are discussed.

Chapter 4 focuses on plane rigid body on which non-concurrent, coplanar forces are externally applied. The chapter starts with explaining the method of drawing a free-body diagram of a plane rigid body and then moves on to discuss different types of supports and their reactions as well as different types of loads acting on plane and 3-dimensional structures. After explaining the requirement for stable equilibrium, the chapter discusses moment of a force (about an axis and about a point) and its vector representation and introduces the couple that gives pure rotational effect. Equivalence of different types of forces into force and moment at any point is then explained. Further, the conditions of equilibrium equations with sign conventions and their applications to a system of non-concurrent, coplanar and non-concurrent, non-coplanar forces are dealt with.

Chapter 5 deals with centroids of surfaces/areas/laminas of regular and compound regular sections. This chapter further discusses determination of surface areas and volumes using the Pappus–Guldinus theorems and centre of gravity for simple and compound solids.

Chapter 6 deals with area moments. Area moments of inertia about centroidal axis of areas and about other axes parallel to it are discussed in detail in this chapter. Polar moment of inertia, radius of gyration, perpendicular axis theorem, and mass moment of inertia for standard solids are also explained.

Chapter 7 describes the frictional forces experienced by the surfaces of contact at just start of motion. Laws of Coulomb friction are dealt with in this chapter along with frictional forces between two bodies in contact, wedge friction, and belt friction with respect to concurrent, coplanar forces. The chapter also discusses ladder friction and rolling resistance, in which the nature of forces is non-concurrent, coplanar.

Part II – Dynamics comprises chapters 8–10. *Chapters 8* and *9* deal with kinematics and kinetics of a particle respectively.

In *Chapter 8*, the motion of a particle, that is, position, velocity, and acceleration, with respect to time is introduced. Further, motion curves, different types of motion, that is, rectilinear motion, curvilinear motion, and projectile motion are explained. The chapter also includes motion of several particles on a straight line.

Chapter 9 presents various problems of kinetics of a particle, which can be solved by three different methods, viz., force method, energy method, and impulse-momentum method. Newton's second law, law of conservation of momentum, Newton's law of gravitation, and work-energy principle are important topics covered in this chapter. All these topics are explained with equations and sign conventions, which can be easily understood. Impulse-momentum principles and different types of collision of elastic bodies are also further discussed.

Chapter 10 is on kinematics and kinetics of rigid bodies. Rigid body motion in translation and rotation about an axis and general plane motion are independently elaborated in this chapter. Absolute and relative velocity and acceleration of a body in plane motion and instantaneous centre of rotation of a rigid body are also touched upon. The chapter discusses in detail kinetics of rigid body for translatory motion, rotation about an axis, and general plane motion. The work-energy principles for these different cases are also explained.

The *Appendix* contains the University question paper (May 2017) with the complete solution for practice.

ACKNOWLEDGEMENTS

I express my sincere and heartfelt gratitude to Prof. M.K. Surappa, the respected Vice-Chancellor, Anna University, Chennai, who, with his great academic interest motivated me in bringing out this book.

I thank Prof. Dr J. Kumar, the Registrar in-charge, Anna University, Chennai, for his sustained interest in the publication of this book to benefit undergraduate students of Engineering programmes who are taking up this course.

I express my sincere thanks to Prof. T.V. Geetha, Dean, College of Engineering, Guindy campus and Prof. Dr A. Rajadurai, Dean, Madras Institute of Technology, Chrompet, Anna University for their constant encouragement.

I am indebted to all the past and present Directors of the Centre for Academic Courses, Centre for Faculty Development, Chairmen of Faculty of Mechanical Engineering, Anna University, and former Heads of the Department of Mechanical Engineering, Prof. Dr B. Mohan, Prof. Dr S. Sampath Kumar, and Prof. Dr G. Thanigaiyarasu, College of Engineering, Guindy, for providing continuous opportunities and support to deliver training lectures on Engineering Mechanics through Faculty Development Programmes to the faculty of Anna University colleges and also to disseminate this knowledge through various web and EDUSAT lectures of Anna University. I also thank Prof. Dr P. Narayanasamy, the Former Dean and the Former Director of Center for Faculty Development and currently working as Dean–Networking and Computing, PSG College of Technology, Coimbatore for the recommendation of the web content video lectures on Engineering Mechanics delivered by me to the Government of Tamil Nadu for installing them in the free laptops distributed to school and college students by the Government.

I sincerely thank Prof. Dr R. Raju, Director, Centre for Academic Courses – Anna University, Chennai for his consistent encouragement.

I thank Prof. L. Karunamoorthy, Chairman, Faculty of Mechanical Engineering, Anna University and Prof. Dr S. Rajendra Boopathy, Head of the Department of Mechanical Engineering and Dr K. Shanmuga Sundaram, Head and Professor In-charge, Engineering Design Division, Department of Mechanical Engineering, Guindy, for constantly instilling interest in me to develop this book.

I sincerely thank Prof. Dr A. Elayaperumal, (Former Director, Student Affairs, Anna University, Chennai) of the Department of Mechanical Engineering, for motivating me to write this book and inspiring me with interesting suggestions during the course of preparing content for this book.

I sincerely thank all our colleagues in the Departments of Mechanical and Civil Engineering and Alagappa College of Technology, Guindy Campus and Madras Institute of Technology, Chrompet, Anna University, Chennai who have shown their interest and support at every stage of the development of this book.

I am deeply grateful to former professors—Dr N.S. Parthasarathy, Dr G. Ramaiyan, Dr K. Chandrasekaran, Dr V. Sundareswaran, Dr K. Srinivasan, and Dr C. Jebaraj—for their continued interest in this book.

I wholeheartedly thank Prof. Dr E. Balaguruswamy, Former Vice-Chancellor, Anna University, Chennai who has a pioneering vision, for giving me an opportunity to serve Anna University, Chennai, and thus introducing me to the teacher and student community.

I sincerely thank Prof. Dr P. Mannar Jawahar, Former Vice-Chancellor, Anna University, Chennai who wishes to refer me with high academic standard on this subject.

My discussions with all the professors, especially Dr M. Sekar, Professor of Civil Engineering and Former Dean of College of Engineering, Guindy, Anna University, Chennai, Dr Latha Nagendran,

Director-AUFRG Institute for CAD/CAM, Anna University, Dr R. Dhanaraj, Professor of Eminence, Department of Aerospace Engineering, Madras Institute of Technology, Chromepet, Dr D. Samuel Raj, Assistant Professor of Mechanical Engineering, and Dr M. Vasumathi, Assistant Professor of Mechanical Engineering of College of Engineering, Guindy were useful while framing the concepts for solving some of the problems.

I wish to place on record my respect and gratitude to my PhD supervisor Dr Fook Siong Chau, and professors Dr Cheng Li, Dr T.E. Tay, Dr C.J. Tay, Dr Li Xiaoping, and Dr Rahman Mustafizur of Department of Mechanical Engineering, National University of Singapore, under whom I have studied various mechanics-based subjects while doing my PhD during 1998–2002.

My compliments would become incomplete if I forget to thank my undergraduate project guide, Prof. K. Rajgopal, JNTU College of Engineering, and Former Vice-Chancellor, JNTU, Hyderabad, and postgraduate project guides, Prof. A.C. Bhaskar Naidu, Prof. B.V. Somasekara, and Prof. Harinath, Former Principal of BMS College of Engineering, Bangalore, for their valuable guidance during my undergraduate and postgraduate courses of study respectively.

I also recall, with a note of thanks, all my students and colleagues who have reposed faith in me to bring out this book on the subject that I have been teaching for more than 18 years.

I wholeheartedly thank my students, Mr U. Elangovan and Mr Vivek Shankar, who have helped in the development of illustrations for this book.

I sincerely thank the editorial team of Oxford University Press, India for their patient cooperation in bringing out this book.

This book would not have come to life, but for the critical reviews, comments, and suggestions by the esteemed reviewers, whose role though invisible, was vital. I owe my sincere thanks to them.

I also thank my wife, Mrs Lakshmi, son, Tharun Kumar, and daughter, Tharanya, for their patience during the time I spent in writing this book.

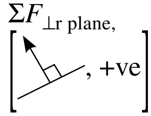
I request all the readers to provide their valuable suggestions.

Vela Murali

Brief Contents

Detailed Contents

List of Symbols

$\theta, \alpha, \beta, \gamma$	Used to represent angles
θ	Used to represent angular displacement
ω	Angular velocity
α	Angular acceleration
Σ	Symbol used for algebraic sum
$\boldsymbol{\lambda}$	Used to represent unit vector
μ	Coefficient of friction
μ_s	Coefficient of static friction
μ_k	Coefficient of kinetic friction
ρ	Density of material
$\rightarrow$, +ve	Force direction along the positive x-axis is taken as positive and opposite to it is taken as negative
$\uparrow$, +ve	Force direction along the positive y-axis is taken as positive and opposite to it is taken as negative
$\curvearrowleft$, +ve	Moment in anticlockwise direction is taken as positive and clockwise moment is taken as negative
a	Acceleration
a_O	Absolute acceleration of point O
$(a_{A/O})_t$	Relative acceleration of point A with respect to O in tangential direction
$(a_{A/O})_n/(a_{A/O})_r$	Relative acceleration of point A with respect to O in normal/radial direction
$a_{A/O}$	Total acceleration of point A with respect to O
a_t	Tangential component of acceleration
a_n/a_r	Normal/radial component of acceleration
c	Used to represent centroid
e	Coefficient of restitution
F	Force
F_{as}	Average spring force
F_f	Frictional force
F_{kf}	Kinetic frictional force
ΣF_x, ($\rightarrow$, +ve)	Algebraic sum of forces along the x-axis (force acting along the positive x-axis is taken as +ve and opposite to it is taken as –ve)
ΣF_y, ($\uparrow$, +ve)	Algebraic sum of forces along the y-axis (force acting along the positive y-axis is taken as +ve and opposite to it is taken as –ve)
ΣF_z, ($\swarrow$, +ve)	Algebraic sum of forces along the z-axis (force acting along the positive z-axis is taken as +ve and opposite to it is taken as –ve)
$\Sigma F_{\text{along plane}}$, ($\nearrow$, +ve)	Algebraic sum of forces along plane (force acting in the direction up the plane is taken as +ve and opposite to it is taken as –ve)
$\Sigma F_{\perp\text{r plane}}$, 	Algebraic sum of forces perpendicular to the plane (force acting in the direction outward normal to the plane is taken as +ve and opposite to it is taken as –ve)

$\Sigma F_{\text{along motion}}$	Algebraic sum of forces along motion (force acting in the direction of motion is taken as +ve and opposite to it is taken as –ve)
g	Acceleration due to gravity
G	Universal gravitational constant, centre of gravity, centroid
H_O	Angular momentum about point O
$\boldsymbol{i}, \boldsymbol{j}, \boldsymbol{k}$	Unit vectors along x-, y-, and z-axes
I	Used to represent area moment of inertia or mass moment of inertia, impulse / instantaneous centre
I_M	Mass moment of inertia
I_{xx}	Area moment of inertia about centroidal xx-axis
I_{yy}	Area moment of inertia about centroidal yy-axis
J	Polar moment of inertia
K_{xx}	Radius of gyration about xx-axis
K_{yy}	Radius of gyration about yy-axis
K	Spring constant
M	Moment
$\Sigma M_{x\text{-axis}}$, ($\curvearrowleft$, +ve)	Algebraic sum of moments about the x-axis (moment in anticlockwise direction is taken as positive and clockwise moment is taken negative)
$\Sigma M_{y\text{-axis}}$, ($\curvearrowleft$, +ve)	Algebraic sum of moments about the y-axis (moment in anticlockwise direction is taken as positive and clockwise moment is taken as negative)
$\Sigma M_{z\text{-axis}}$, ($\curvearrowleft$, +ve)	Algebraic sum of moments about the z-axis (moment in anticlockwise direction is taken as positive and clockwise moment is taken as negative)
$\Sigma M_{z\text{-axis taken at any point}}$ ($\curvearrowleft$, +ve)	Algebraic sum of moments about z-axis taken at any point on a plane (moment in anticlockwise direction is taken as positive and clockwise moment is taken as negative)
M_O	Moment about point O due to the force F acting at a point, say A, defined with the position vector $\boldsymbol{r}_{OA}$
$\Sigma M_{\text{along rotation}}$	Algebraic sum of moments along rotation (moment acting in the direction of rotation is taken as +ve and opposite to it is taken as –ve)
MM and NN	Used to represent principal coordinate axes
N	Used to represent normal reaction force
Ox', Oy'	Used to represent reference coordinate axes
P, Q, R, O, etc.	Points used to represent positions
$\int P dt$	Impulse during deformation
$\boldsymbol{r}$	Used to represent position vector
R	Used to represent normal reaction force
$\int R dt$	Impulse during reaction
s	Distance
SS, TT	Used to represent reference coordinate axes
t	Time
u	Initial velocity
V	Velocity
V_O	Absolute velocity of point O
$V_{A/O}$	Relative velocity of point A with respect to O
WD	Work done

Part I

Statics

1. Fundamentals of Mechanics
2. Statics of Particle: Coplanar, Concurrent Forces
3. Statics of Particle: Concurrent, Non-coplanar Forces (Forces in Space)
4. Statics of Rigid Body: Non-concurrent, Coplanar Forces and Non-concurrent, Non-coplanar Forces
5. Properties of Surfaces and Solids: Centroid and Centre of Gravity
6. Properties of Surfaces and Solids: Moment of Inertia and Mass Moment of Inertia
7. Friction

CHAPTER 1

Fundamentals of Mechanics

CHAPTER OBJECTIVES

After reading this chapter, the readers will be able to understand
- *the fundamentals of mechanics of rigid bodies*
- *different laws and principles to study the conditions of equilibrium, motion, and inertial effects on a body*
- *how vector mathematics can be used to solve the problems of mechanics*
- *how dimensional analysis can be useful in understanding equations of engineering problems*

1.1 INTRODUCTION

Mechanics is the branch of science that describes and predicts the conditions of inertia and motion of bodies due to the action of forces.

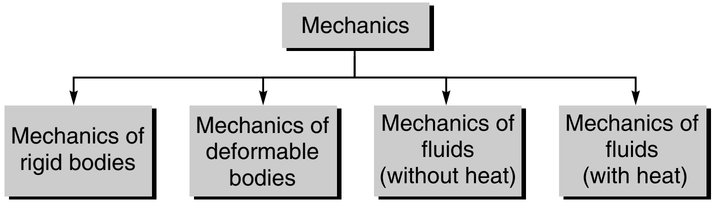
1.1.1 Mechanics and its Classification

Depending upon the nature of the body, the transmission of forces may cause the body to deform internally or may not produce any deformation but may cause the body to move. Accordingly, the field of mechanics can be broadly classified into

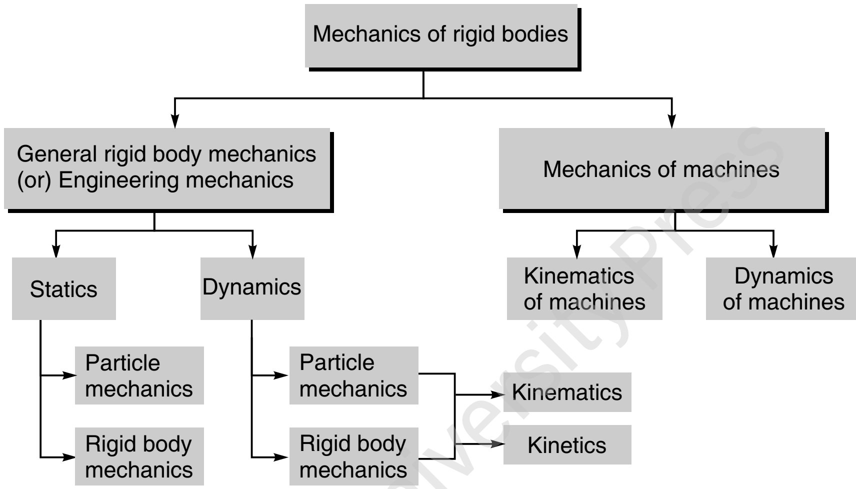
(a) Mechanics of rigid bodies
(b) Mechanics of deformable bodies
(c) Mechanics of fluids

The broad classification of mechanics is shown in Fig. 1.1(a). *Rigid body mechanics* is the field of mechanics in which the body is assumed to be perfectly rigid. In practice, structures and machines are never rigid and undergo small deformation under the action of external loads. Since the deformations are very small, they do not appreciably affect the condition of equilibrium, which may be under static equilibrium or under dynamic equilibrium due to external loads or forces applied over it including self-weight and support reaction forces.

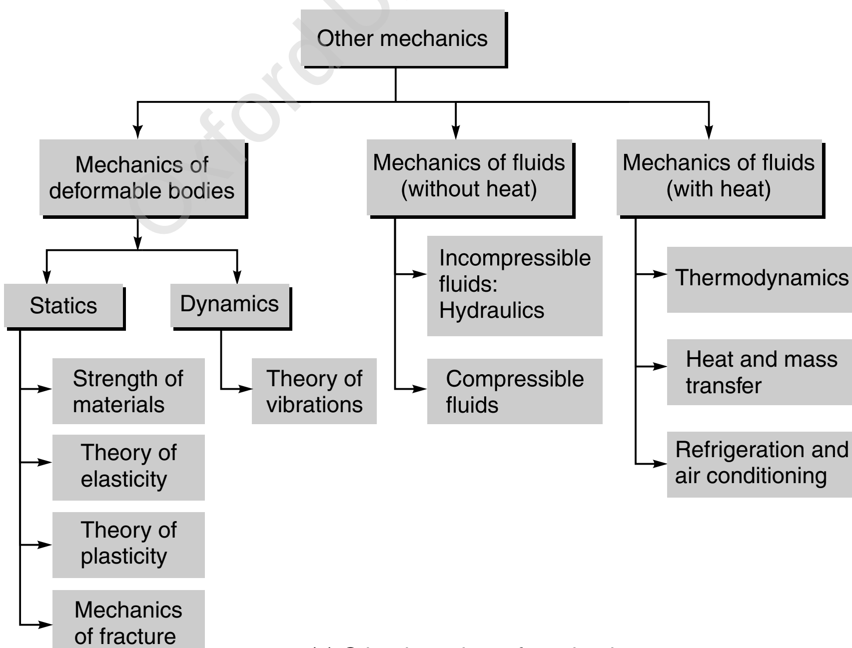
Figure 1.1(b) depicts the classification of mechanics of rigid bodies into general rigid body mechanics (also called as *engineering mechanics*) and *mechanics of machines*. In mechanics of machines, we study kinematics and dynamics, which deal with desired motion of rigid bodies by transmission of forces (e.g., single slider crank mechanism that converts reciprocatory motion of the piston into rotary motion of the crank shaft).



(a) Broad classification of mechanics



(b) Mechanics of rigid bodies



(c) Other branches of mechanics

Fig. 1.1 Mechanics and its classification

In general rigid body mechanics, bodies are considered as rigid and transfer the forces or moments due to forces to their supports, and body may be either in static condition or in dynamic condition. The study of rigid body mechanics is further divided into two groups. The first branch is *particle mechanics*, in which a system of concurrent forces is applied at a point in the body due to which the body experiences only force effect. To study the condition of the body, it is sufficient to consider the point of the body where all these forces act upon. Whereas in *rigid body mechanics*, a system of non-concurrent forces act on the rigid body, and the dimensions of the body are important to evaluate the moments due to forces and hence to evaluate the condition of the body.

In *statics* we deal with finding the resultant of a system of forces, equivalent of a given force into a force–moment system at any point, equivalent of a force–moment system into a single force at a point, conditions of equilibrium for the body under the action of system of forces and moments due to forces, or the reaction forces and reaction moments required to be developed by the supports to keep the body in equilibrium condition.

In *dynamics* we study how the body tends to move, i.e. either with constant velocity or constant acceleration/deceleration or variable acceleration/deceleration or projectile motion or motion of a body along a curve under the action of forces and moments due to forces. Dynamics is further divided into two groups, namely kinematics and kinetics. *Kinematics* deals with geometry of the motion, i.e. position, velocity, acceleration of bodies with respect to time, irrespective of the cause of the motion. Further, it deals with different types of motions namely rectilinear motion, projectile motion, and curvilinear motion. *Kinetics* deals with geometry of the motion, i.e. position, velocity, acceleration with respect to the cause of the motion and the relation between the acceleration and the cause of the motion which is force in case of translatory motion and moment in case of rotatory motion.

Figure 1.1(c) depicts mechanics of deformable bodies and mechanics of fluids.

As far as the resistance of a structure to failure is concerned, it is important to relate deformation to external loading and geometry of the structure and these are studied in a separate subject called mechanics of deformable bodies. Strength of materials and theory of elasticity deal with deformable bodies under static loading and recoverable shapes after unloading.

Theory of plasticity deals with deformable bodies under static loading and irrecoverable shapes after unloading. Mechanics of fracture deals with deformable physical bodies containing a definite size of crack and subjected to static or dynamic loading and the recoverable or irrecoverable shapes after unloading. Theory of vibrations deals with deformable bodies under dynamic loading and recoverable shapes after unloading.

Another branch of mechanics is mechanics of fluids, which deals with the case in which there is no heat transfer into or out of the system consisting of static fluid or fluid flow. Again, this particular subject can be subdivided into the study of incompressible fluids (hydraulics, which deals with problems involving liquids) and compressible fluids. Thermodynamics, heat and mass transfer, and refrigeration and air conditioning deal with fluids considering the effect of heat transfer into or out of the system.

1.1.2 Historical Development of Mechanics

Many researchers have contributed to the development of various concepts of rigid body mechanics to establish this particular course, which is studied as rigid body mechanics or engineering mechanics at the undergraduate level.

Among various researchers, Archimedes (287–212 BC) developed the concept of buoyancy forces. Kepler (1571–1630) established the fundamentals of astronomy for planetary motion and the principles were named after him as Kepler's laws.

Newton (1642–1727) was the first person in history to establish the laws of mechanics applicable to solids and his principles were named as Newton's three laws and the mechanics developed by him was called Newtonian mechanics.

Bernoulli (1667–1748) developed the principle of virtual work, which is applied in fluid mechanics as Bernoulli's equation for the total energy at any point in a fluid flow.

D'Alembert (1717–1783) established a very important principle for a dynamic system to be brought to equilibrium. His principle was named as D'Alembert's principle applied to a dynamic system.

1.1.3 Fundamental Concepts

In Newtonian mechanics, space, time, and mass are absolute quantities and independent of each other.

Space is associated with the conception of the position of a point, say P. Three coordinates, with reference to a particular point or the origin in three mutually perpendicular directions, may define the position of P and are called coordinates of P.

The **time** of an event in case of the dynamic condition of a point is to be defined.

Mass is used to quantify the amount of resistance that is exerted by a body while changing its state of rest or motion.

Force is defined as the ability to translate a body into action or as the action of one body on another (e.g. gravitational forces, magnetic forces, and so on).

Force can be characterized with magnitude, point of application, and direction. It is a vector quantity. A force in the xy-plane can be represented as

$$\boldsymbol{F} = F_x\,\boldsymbol{i} + F_y\,\boldsymbol{j} \tag{1.1}$$

The magnitude of the resultant force $\boldsymbol{F}$ is $\sqrt{F_x^2 + F_y^2}$ and its direction with respect to the x-axis is $\theta = \tan^{-1}(F_y/F_x)$ as shown in Fig. 1.2.

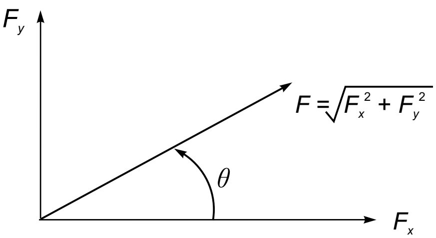


Fig. 1.2 Graphical representation of a force in the xy-plane

Similarly, a force in the space can be represented with three components in the x-, y-, and z-directions in vector form as

$$\boldsymbol{F} = F_x\,\boldsymbol{i} + F_y\,\boldsymbol{j} + F_z\,\boldsymbol{k} \tag{1.2}$$

The magnitude of the resultant force $\boldsymbol{F}$ is $\sqrt{F_x^2 + F_y^2 + F_z^2}$ and its directions with respect to the x-, y-, and z-axes respectively are $\theta_x = \cos^{-1}(F_x/F)$, $\theta_y = \cos^{-1}(F_y/F)$, and $\theta_z = \cos^{-1}(F_z/F)$, as shown in Fig. 1.3, where $\cos\theta_x$, $\cos\theta_y$, and $\cos\theta_z$ are called *directional cosines* and are also represented as l, m, and n respectively. Mathematically, it can be shown that

$$l^2 + m^2 + n^2 = 1 \tag{1.3}$$

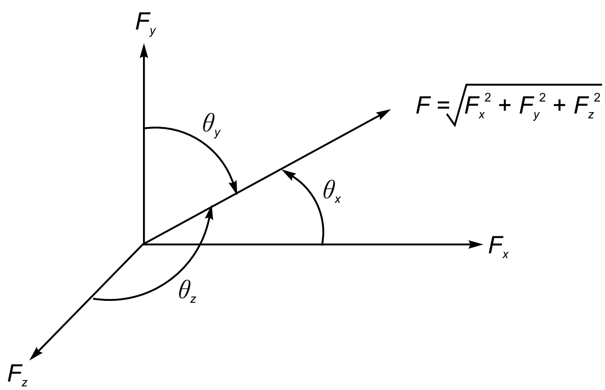


Fig. 1.3 Graphical representation of a force in space

Momentum of a body is the quantity of motion possessed by a moving body and is denoted by $\boldsymbol{M}$, which is directly proportional to velocity, and hence it is also called as linear momentum or simply momentum. Its unit is kg-m/s. It is equal to the product of mass and velocity, i.e.

$$\boldsymbol{M} = \text{mass} \times \text{velocity} = m\boldsymbol{V} \tag{1.4}$$

Conceptualization of rigid body mechanics Rigid bodies are made up of atoms and molecules and these can be physically defined by their shape. At micro-level, the behaviour of these atoms and molecules is too complex to study and hence mass can be assumed to be continuously distributed within the body. The body's behaviour can be measured with its dimension or position with respect to certain coordinate system and time. This method of description of a body at its macro-level is called *continuum*. It can be rigid or deformable, holding its shape or continuously deforming and changing its shape depending upon the matter under study or consideration.

Rigid body mechanics A body that does not undergo any deformation due to the action of the forces applied on it is considered to be rigid. Or, in other words, if the deformation of a body due to the external forces applied on it is negligible as compared with its dimension/shape, then the body can be considered as rigid. Hence, the system of external forces and moments due to the forces applied over the body and its support reaction keep the body in equilibrium under static condition.

A system of external forces and moments due to the forces applied over the body will be in equilibrium with its inertia forces and inertia moments respectively under dynamic condition of the body.

Generally, a system of non-concurrent forces, which may be coplanar or non-coplanar (discussed later in this chapter), is discussed under rigid body mechanics.

Particle mechanics A body is idealized as a particle when the whole mass of the body is concentrated at its centroid and lines of action of a system of forces, including the support reactions applied on the body, pass through the centroid. These forces have only the force effect on the body and no moment effect. Generally, a system of concurrent forces, which may be coplanar or non-coplanar, is discussed under particle mechanics.

1.2 LAWS OF MECHANICS

The following are the fundamental laws/principles of mechanics:

- Newton's laws: I, II, and III
- Lami's theorem
- Parallelogram law for addition of forces
- Triangular law of forces
- Polygon's law
- Principle of transmissibility: sliding vector
- Newton's law of gravitation

1.2.1 Newton's Laws

Newton's first, second, and third laws are discussed below.

Newton's first law of motion If a body is in a state of rest or uniform motion, then it will continue to be in the same state of condition until and unless an external force influences it.

Newton's second law of motion When a body is under acceleration or deceleration, then the rate of change of momentum of the body in the direction of the motion is equal to the algebraic sum of the forces acting along the same direction of the motion. For the case of a rigid body,

$$\sum F_{\text{along motion}} = \frac{d}{dt}(mV) = ma_{\text{along motion}} \tag{1.5}$$

Newton's third law of motion The action of force and the reaction developed have the same magnitude and are opposite to each other, and they lie along the same line of action. In simple terms, for every action, there is an equal and opposite reaction. For example, when a gun is fired, the spring force on the bullet that is in contact with the surface of the barrel develops the reaction force, which kicks the shoulder.

1.2.2 Lami's Theorem

Lami's theorem states that 'if three forces acting on a particle are in equilibrium, then each force is proportional to the sine of the angle between the other two forces'.

Explanation

Three concurrent, coplanar forces, which are acting at a point in the body with their directions represented away from the point, keep the body in static equilibrium. According to Lami's theorem, for the forces as shown in Fig. 1.4, each force is proportional to the sine of the angle included between the other two forces. This is basically the trigonometric sine rule applicable to a triangle.

Mathematically, Lami's theorem can be written as

$$\frac{F_1}{\sin\theta_1} = \frac{F_2}{\sin\theta_2} = \frac{F_3}{\sin\theta_3} \tag{1.6}$$

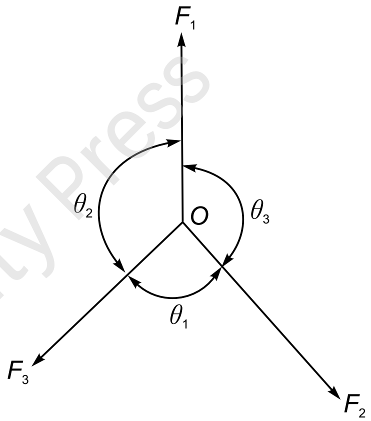


Fig. 1.4 Equilibrium of three concurrent, coplanar forces: Lami's theorem

1.2.3 Parallelogram Law for Addition of Forces

According to this law, if an equivalent single force, which is called resultant, can replace the two forces acting on a particle, then the resultant can be found by drawing the diagonal of the parallelogram, which has sides equal to the given forces. This is illustrated in Fig. 1.5.

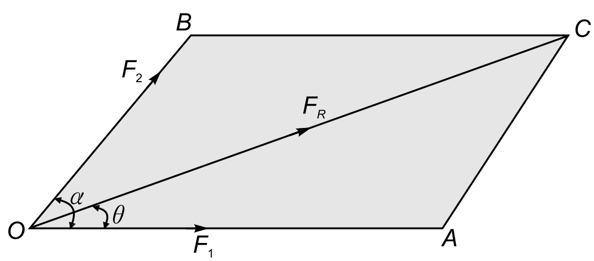


Fig. 1.5 Two forces and their resultant represented in a parallelogram

1.2.4 Triangular Law of Forces

This law states that if $\boldsymbol{F}_1$ and $\boldsymbol{F}_2$ are two forces acting on a particle which can be represented by the two sides of a triangle in the magnitude and direction taken one after the other, then the side that closes the triangle represents the resultant in opposite direction. This is illustrated in Fig. 1.6.

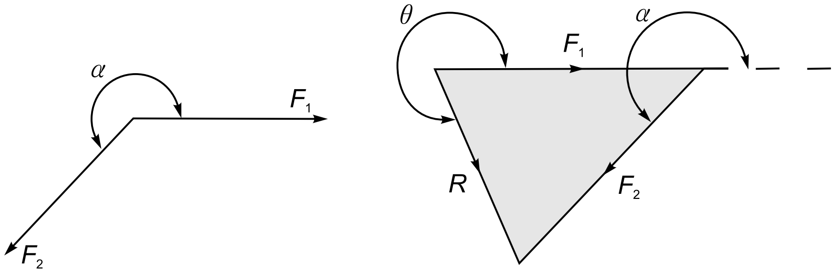


Fig. 1.6 Finding resultant of two forces by the triangular law of forces

1.2.5 Polygon Law of Forces

It states that if $\boldsymbol{F}_1, \boldsymbol{F}_2, \boldsymbol{F}_3 \ldots$ form a system of more than two concurrent, coplanar forces which are acting on a particle, the magnitude and direction of the resultant can be found by drawing a force polygon.

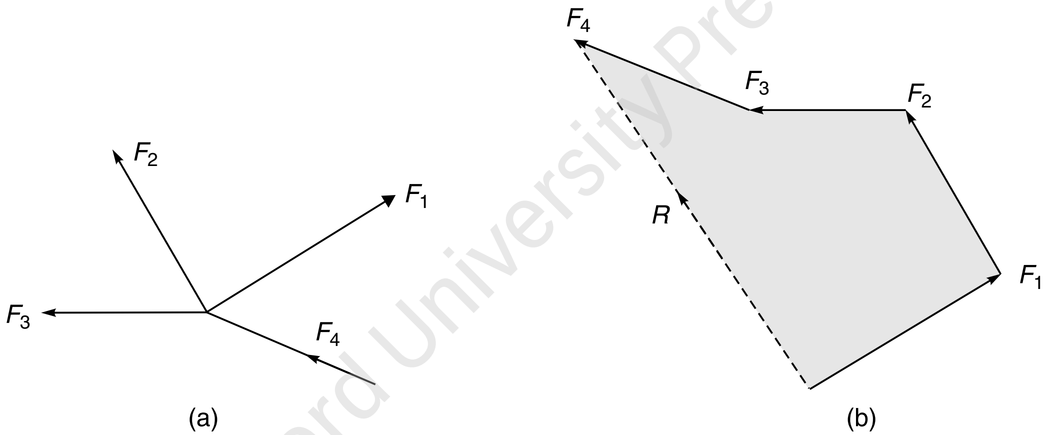


Fig. 1.7 (a) A system of concurrent, coplanar forces and (b) the force polygon for the forces in (a)

For a given system of forces shown in Fig. 1.7(a), the force polygon can be drawn by taking a suitable scale. The sides of the polygon are to be constructed with the magnitude and direction of various forces taking one after the other. After drawing all the sides, two ends of the force polygon or the closing side of the force polygon gives the magnitude and direction in opposite order as shown in Fig. 1.7(b).

1.2.6 Principle of Transmissibility

According to this principle, 'the condition of equilibrium or motion of a rigid body will remain unchanged if the force acting at a point on the rigid body is transmitted to another point on the same line of action in the same direction'. For example, pushing a vehicle from behind has the same effect as when it is pulled from front with the force of same magnitude and along the same line of action as shown in Fig. 1.8.

A sliding vector is a vector which acts along the same line in the space with same quantity. When we deal with the external action of a force on a rigid body, the force may be applied at any point along its line of action without changing its effect on the body as a whole. This force vector is called a sliding vector. In Fig. 1.8, force $\boldsymbol{F}$ is considered to be the sliding vector.

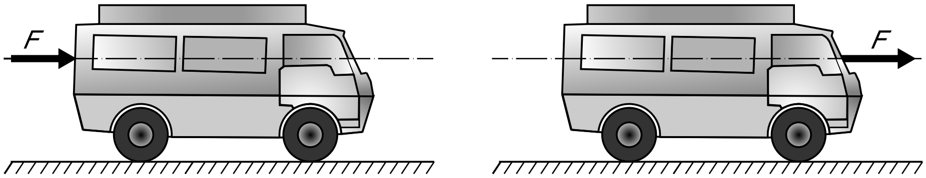


Fig. 1.8 Principle of transmissibility

1.2.7 Newton's Law of Gravitation

It states that the gravitational force of attraction between two bodies is proportional to the product of the masses of the two bodies and inversely proportional to the square of the distance between them. This law is illustrated in Fig. 1.9.

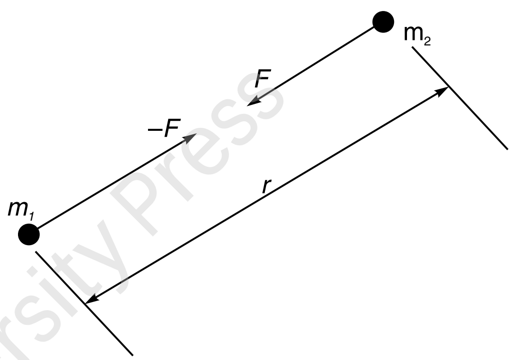


Fig. 1.9 Newton's law of gravitation

Mathematically, Newton's law of gravitation can be written as

$$F \propto \frac{m_1 m_2}{r^2} \Rightarrow F = G\frac{m_1 m_2}{r^2} \tag{1.7}$$

where G is the universal constant or constant of gravitation. Its value is equal to 66.73×10^{-12} m^3/kg-s^2.

If the particle lies on Earth, then Eq. (1.7) can be rewritten for the force of attraction by Earth on the particle as

$$F = G\frac{Mm}{R^2} \tag{1.8}$$

where M is the mass of Earth in kg, which is equal to 5.98×10^{24} kg, m is the mass of the particle, and R is the distance between the centre of Earth and the centre of the particle and is equal to the radius of Earth (6.378×10^6 m). Equation (1.8) can be further rewritten to calculate the weight of a body W of mass m, which is on Earth, or the force of attraction F of Earth on the particle as follows:

$$F = m\left(\frac{GM}{R^2}\right) \Rightarrow W = mg \tag{1.9}$$

where g is called the acceleration due to gravity and is equal to

$$\left(\frac{GM_{\text{Earth}}}{R^2_{\text{Earth}}}\right) = 9.81 \text{ m/s}^2 \tag{1.10}$$

Similarly, the mass of Moon is $M_{\text{Moon}} = 7.35 \times 10^{22}$ kg, about 1.2% of Earth's mass and its radius R_{Moon} is equal to 1,737 km or 1,737,000 m. Using Eq. (1.9), weight of a body W of mass m, which is on Moon, or the force of attraction F of Moon on the particle can be given as follows:

$$F = m\left(\frac{GM_{\text{Moon}}}{R^2_{\text{Moon}}}\right) = mg_{\text{Moon}} \tag{1.11}$$

where g_{Moon} is called the acceleration due to gravity on Moon and is equal to

$$g_{\text{Moon}} = \left(\frac{GM_{\text{Moon}}}{R^2_{\text{Moon}}}\right) = 1.622 \text{ m/s}^2 \tag{1.12}$$

Similarly, $g_{\text{Mars}} = 3.711\text{m/s}^2$, $g_{\text{Jupiter}} = 24.79 \text{ m/s}^2$, etc., can be calculated.

All the principles discussed in this section will be introduced in subsequent chapters as and when they are needed.

Example 1.1 Assume that a tunnel is dug along the diameter of Earth of mass M and radius R. What is the force on a particle of mass m placed in the tunnel at a distance r from the centre of Earth?

Solution From Fig. 1.10, and from Newton's law of gravitation, the force of attraction when the mass m is placed on Earth of mass M and radius R is

$F = m\left(\frac{GM_{\text{Earth}}}{R^2_{\text{Earth}}}\right)$, where G is universal constant

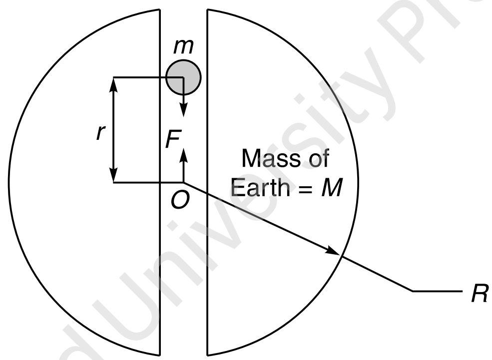


Fig. 1.10 A tunnel dug along a diameter of Earth

From Fig. 1.10

For the Earth radius R, the force on mass = $m\left(\frac{GM}{R^2}\right)$

For the radius r, the force = ?

$\therefore$ Force $F = \left(\frac{GMm}{R^3} r\right)$ (*Ans*)

1.3 SCALAR AND VECTOR QUANTITIES

Scalars are quantities which are described only by magnitude, independent of direction, i.e. they are one dimensional measurement of quantity. Temperature, mass, length, height, time, energy, volume, density, mass, area, pressure, work, power, etc., are all scalar quantities.

Vectors are quantities which are described by both magnitude and direction and have more than one value associated with them. For example, velocity has a magnitude, called as speed, as well as a direction like south or northwest or 15 degrees east of south, etc. Here there are two values associated with the vector: one is speed and another is direction. Vectors are expressed with reference to x-, y- and z-coordinate system when the quantities are referred in the space and they are expressed with reference to x- and y-coordinate system when they are referred in the plane.

For example x-component of velocity of the particle in a plane is given as 14.4 kmph (i.e. 4 m/s) and its y-component is 10.8 kmph (i.e. 3 m/s).

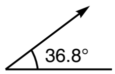
$$V_x = 14.4 \text{ kmph (i.e. 4 m/s)}; \; V_y = 10.8 \text{ kmph (i.e. 3 m/s)}$$

The magnitude of velocity is

$$V = \sqrt{V_x^2 + V_y^2} = 5 \text{ m/s} \tag{1.13}$$

The direction with respect to the x-axis would be given by

$$\theta = \tan^{-1}\left(\frac{V_y}{V_x}\right) = 36.8° \tag{1.14}$$

Now the velocity can be represented as $V = 5$ m/s, 

Different scalar and vector quantities are given in Table 1.1

Table 1.1 Scalar and vector quantities

Scalars		Vectors	
S. No.	**Quantity**	**S. No.**	**Quantity**
1.	Length	1	Displacement
2.	Area	2	Velocity
3.	Volume	3	Acceleration
4.	Speed	4	Momentum
5.	Density	5	Force: Lift force, Drag force, Thrust force, Weight
6.	Pressure		
7.	Temperature		
8.	Energy, Work		
9.	Power		

1.3.1 Free and Fixed (or Bound) Vectors

Free vector is a vector that can be moved anywhere in the space by maintaining the same magnitude and sense of movement. Alternatively, it can be defined as a vector which does not possess specific point of application and it is free to be moved to another point in the body to produce same effect.

Example: $\boldsymbol{F}_1$ and $\boldsymbol{F}_2$ are two forces, acting on a rigid body at points 1 and 2 respectively, translate the body along their line of action as shown in Fig. 1.11(a). These forces are free vectors which are shifted to another two points 3 and 4 respectively without changing their magnitude and the direction and produce the same translatory effect for the body as shown in Fig. 1.11(b).

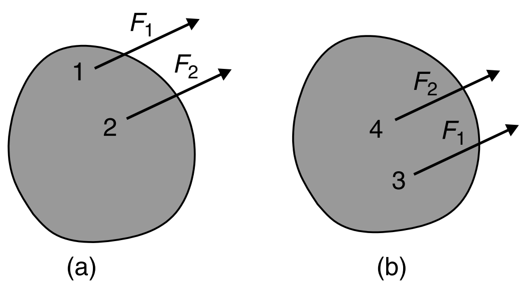


Fig. 1.11 Free vector

Another example is shown in Fig. 1.12(a), (b) and (c) in which forces $\boldsymbol{F}_1$ and $-\boldsymbol{F}_1$, $\boldsymbol{F}_2$, and $-\boldsymbol{F}_2$, and $\boldsymbol{F}_3$ and $-\boldsymbol{F}_3$ are pairs of parallel forces respectively placed at a distance of $\boldsymbol{S}_1$, $\boldsymbol{S}_2$, and $\boldsymbol{S}_3$, and produce same couple of $\boldsymbol{M} = \boldsymbol{F}_1 \times \boldsymbol{S}_1 = \boldsymbol{F}_2 \times \boldsymbol{S}_2 = \boldsymbol{F}_3 \times \boldsymbol{S}_3$ in anticlockwise direction. These forces are called as free vectors.

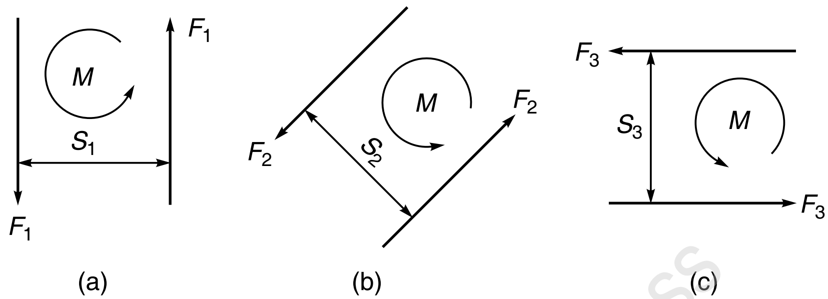


Fig 1.12 Free vectors producing same couple

Bound or fixed vector has a specific point of application with required magnitude and along a particular line of action. This vector is fixed or bound to a particular location and hence is called as bound or fixed vector.

Example: A deformable body at a point P is hammered using a sledge hammer with force $\boldsymbol{F}$ along the line of action as shown in Fig. 1.13 so that it undergoes change in shape at this particular point which is fixed or bound. Hence this force is called as bound or fixed vector.

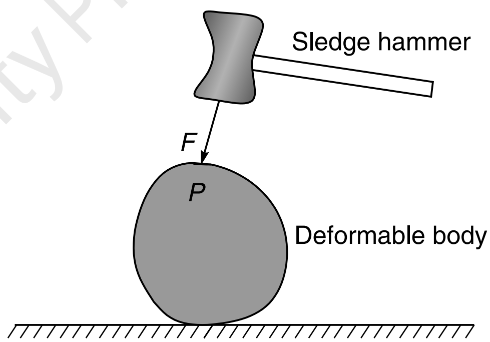


Fig. 1.13 Fixed or bound vector

1.4 VECTOR OPERATIONS

In this section, we will deal with vector operations and vector representation of quantities.

1.4.1 Addition and Subtraction

First we will discuss how vectors are added.

Addition

The parallelogram law or the traingular law can be used for vector addition. Two vectors $\boldsymbol{P}$ and $\boldsymbol{Q}$ as shown in Fig. 1.14(a) can be added using the parallelogram law. The vectors are joined at their tails and parallel lines are drawn (shown with dotted lines) from the arrowhead of each vector as shown in Fig. 1.14(b) to form a parallelogram. The diagonal of the parallelogram represents the resultant force vector $\boldsymbol{R}$ of the two vectors, which is given by

$$\boldsymbol{R} = \boldsymbol{P} + \boldsymbol{Q}$$

Using the triangular law, the head of vector $\boldsymbol{P}$ is connected to the tail of $\boldsymbol{Q}$ as shown in Fig. 1.14(c). The resultant $\boldsymbol{R}$ can also be obtained by connecting the head of vector $\boldsymbol{Q}$ to the tail of $\boldsymbol{P}$. In other way, $\boldsymbol{R}$ can be obtained from Fig. 1.14(d).

Hence, vector addition is commutative, i.e.,

$$\boldsymbol{R} = \boldsymbol{P} + \boldsymbol{Q} = \boldsymbol{Q} + \boldsymbol{P} \tag{1.15}$$

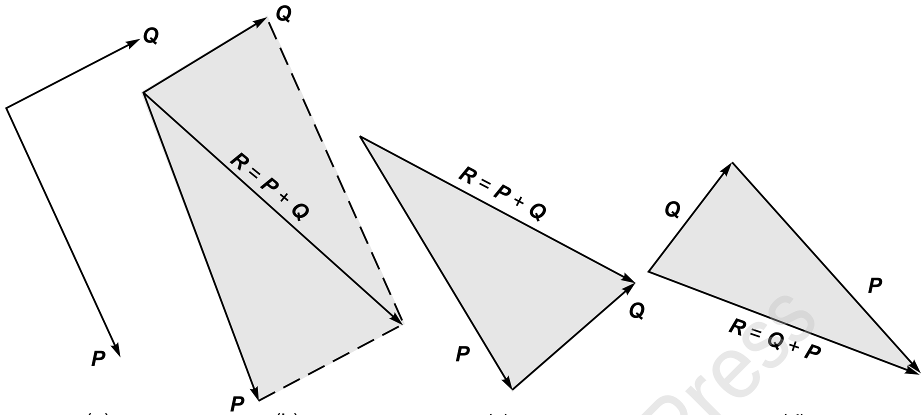


Fig. 1.14 Vector addition: Finding resultant using the parallelogram and triangular laws

Subtraction

The parallelogram law or the triangular law can be used for vector subtraction.

From Fig. 1.15(a), (b), and (c),

$$\begin{aligned}\boldsymbol{R} &= -\boldsymbol{P} + \boldsymbol{Q} = (-\boldsymbol{P}) + \boldsymbol{Q}\\ &= \boldsymbol{Q} + (-\boldsymbol{P})\end{aligned}$$

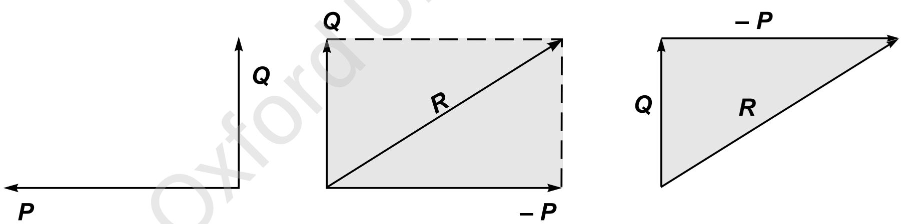


Fig. 1.15 Vector subtraction: Finding resultant using the parallelogram and triangular laws

Vector subtraction is also commutative, similar to vector addition.

Representation of a vector quantity (1D, 2D, or 3D) using unit vectors Unit vectors, denoted by $\boldsymbol{i}$, $\boldsymbol{j}$, and $\boldsymbol{k}$, are used to represent the direction along the x-, y-, and z-axes respectively and their magnitude is equal to 1. A one-dimensional vector is represented with a unit vector of $\boldsymbol{i}$. A two-dimensional vector is represented with unit vectors of $\boldsymbol{i}$ and $\boldsymbol{j}$. Similarly, a three-dimensional vector is represented with unit vectors of $\boldsymbol{i}$, $\boldsymbol{j}$, and $\boldsymbol{k}$. For example,

$$\text{Velocity, } \boldsymbol{V} = 5\boldsymbol{i} + 2\boldsymbol{j} \text{ m/s} \quad \text{(two-dimensional)}$$

$$\text{Force, } \boldsymbol{F} = 3\boldsymbol{i} + 4\boldsymbol{j} + 5\boldsymbol{k} \text{ N} \quad \text{(three-dimensional)}$$

Example 1.2 If $\boldsymbol{P} = 3\boldsymbol{i} + 2\boldsymbol{j} - 5\boldsymbol{k}$, $\boldsymbol{Q} = 5\boldsymbol{i} + 3\boldsymbol{j} - 3\boldsymbol{k}$, and $\boldsymbol{R} = 3\boldsymbol{i} - 5\boldsymbol{j} + 2\boldsymbol{k}$, find the vector $3\boldsymbol{P} + \boldsymbol{Q} - 3\boldsymbol{R}$ and its magnitude.

Solution We have

$$\boldsymbol{P} = 3\boldsymbol{i} + 2\boldsymbol{j} - 5\boldsymbol{k},\ \boldsymbol{Q} = 5\boldsymbol{i} + 3\boldsymbol{j} - 3\boldsymbol{k}, \text{ and } \boldsymbol{R} = 3\boldsymbol{i} - 5\boldsymbol{j} + 2\boldsymbol{k}$$

So,
$$3\boldsymbol{P} + \boldsymbol{Q} - 3\boldsymbol{R} = (9\boldsymbol{i} + 6\boldsymbol{j} - 15\boldsymbol{k}) + (5\boldsymbol{i} + 3\boldsymbol{j} - 3\boldsymbol{k}) - (9\boldsymbol{i} - 15\boldsymbol{j} + 6\boldsymbol{k})$$
$$= 5\boldsymbol{i} + 24\boldsymbol{j} - 24\boldsymbol{k} \quad (Ans)$$

Magnitude of $3\boldsymbol{P} + \boldsymbol{Q} - 3\boldsymbol{R}$ is $\sqrt{5^2 + (24)^2 + (-24)^2} = \mathbf{34.31}$ (Ans)

Example 1.3 Determine the unit vector parallel to the resultant of vectors $\boldsymbol{P} = 3\boldsymbol{i} + 6\boldsymbol{j} - 9\boldsymbol{k}$ and $\boldsymbol{Q} = 3\boldsymbol{i} + 3\boldsymbol{j} + 5\boldsymbol{k}$.

Solution Resultant vector $\boldsymbol{R} = \boldsymbol{P} + \boldsymbol{Q} = 6\boldsymbol{i} + 9\boldsymbol{j} - 4\boldsymbol{k}$

Magnitude of the resultant, $\boldsymbol{R} = \sqrt{6^2 + 9^2 + (-4)^2} = 11.53$

The unit vector parallel to $\boldsymbol{R}$ is $\dfrac{\boldsymbol{R}}{|\boldsymbol{R}|} = \dfrac{6\boldsymbol{i} + 9\boldsymbol{j} - 4\boldsymbol{k}}{11.53}$

$$= 0.52\boldsymbol{i} + 0.78\boldsymbol{j} - 0.34\boldsymbol{k} \quad (Ans)$$

To check the answer, the magnitude of the unit vector should be 1, i.e.,

$$\sqrt{(0.52)^2 + (0.78)^2 + (-0.34)^2} = 1$$

Example 1.4 Determine the unit vector parallel to the line that starts at point A whose position is $(x_A, y_A, z_A) = (3, 2, -3)$ and passes through point B whose position is $(x_B, y_B, z_B) = (2, 1, 6)$.

Solution The unit vector $\boldsymbol{\lambda}_{AB}$ parallel to line $\boldsymbol{AB}$ is equal to $\dfrac{\boldsymbol{AB}}{|\boldsymbol{AB}|}$.

$$\boldsymbol{AB} = (x_B - x_A)\boldsymbol{i} + (y_B - y_A)\boldsymbol{j} + (z_B - z_A)\boldsymbol{k}$$
$$= (2 - 3)\boldsymbol{i} + (1 - 2)\boldsymbol{j} + [6 - (-3)]\boldsymbol{k} = -\boldsymbol{i} - \boldsymbol{j} + 9\boldsymbol{k}$$

Magnitude of $\boldsymbol{AB} = |\boldsymbol{AB}| = \sqrt{(-1)^2 + (-1)^2 + 9^2} = 9.11$

So, the unit vector $\boldsymbol{\lambda}_{AB} = \dfrac{-\boldsymbol{i} - \boldsymbol{j} + 9\boldsymbol{k}}{9.11}$

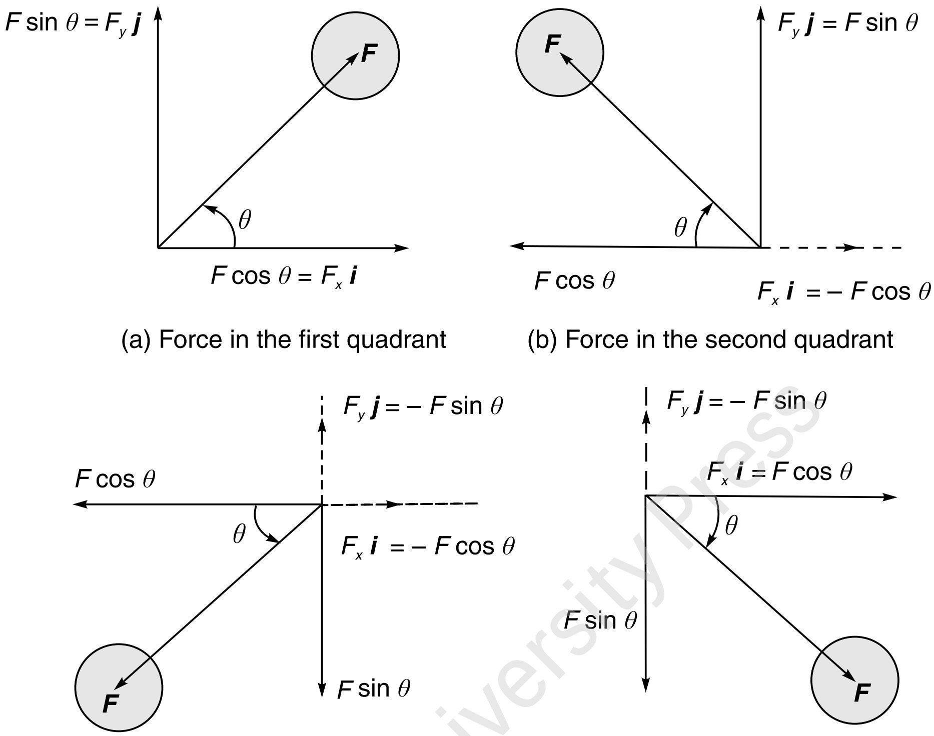
or $\quad \boldsymbol{\lambda}_{AB} = -0.1097\boldsymbol{i} - 0.1097\boldsymbol{j} + 0.988\boldsymbol{k} \quad (Ans)$

1.4.2 Vector Representation of a Force

First we will discuss vector representation of a force in a plane.

Force in a plane (two-dimensional force)

Figure 1.16 illustrates the two-dimensional vector representation of a force. In Fig. 1.16(a), the force vector $\boldsymbol{F}$, which is in the first quadrant, has two components along the x- and y-directions.



(a) Force in the first quadrant (b) Force in the second quadrant

(c) Force in the third quadrant (d) Force in the fourth quadrant

Fig. 1.16 Vector representation of a force in a plane (two-dimensional force)

This can be represented in the vector form as

$$\begin{aligned} \boldsymbol{F} &= F_x\boldsymbol{i} + F_y\boldsymbol{j} \\ &= (F\cos\theta)\boldsymbol{i} + (F\sin\theta)\boldsymbol{j} \end{aligned} \tag{1.16}$$

where $F_x = F\cos\theta$ and $F_y = F\sin\theta$.

Similarly, from Fig. 1.16(b),

$$\boldsymbol{F} = (-F\cos\theta)\boldsymbol{i} + (F\sin\theta)\boldsymbol{j} \tag{1.17}$$

where $F_x = -F\cos\theta$ and $F_y = F\sin\theta$.

From Fig. 1.16(c),

$$\boldsymbol{F} = (-F\cos\theta)\boldsymbol{i} + (-F\sin\theta)\boldsymbol{j} \tag{1.18}$$

where $F_x = -F\cos\theta$ and $F_y = -F\sin\theta$.

From Fig. 1.16(d),

$$\boldsymbol{F} = (F\cos\theta)\boldsymbol{i} + (-F\sin\theta)\boldsymbol{j} \tag{1.19}$$

where $F_x = F\cos\theta$ and $F_y = -F\sin\theta$.

Force in space (three-dimensional force)

Figure 1.17 illustrates the vector representation of a force in space.

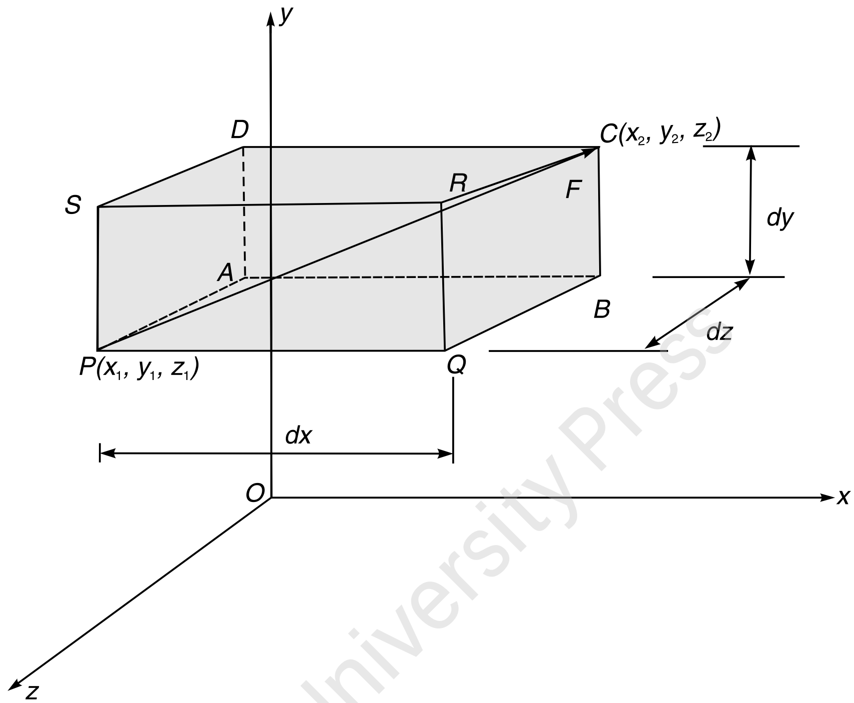


Fig. 1.17 Vector representation of a force in space (three-dimensional force)

In Fig. 1.17, the force vector $\boldsymbol{F}$ is acting from point P to point C. Here, $\boldsymbol{\lambda}_{PC}$ is a unit vector along the line PC.

Vector representation of the force $\boldsymbol{F}$ can be written as

$$\boldsymbol{F} = (\text{magnitude of } \boldsymbol{F})\, \boldsymbol{\lambda}_{PC} \tag{1.20}$$

where
$$\boldsymbol{\lambda}_{PC} = \frac{\text{Position vector of } \boldsymbol{PC}}{\text{Magnitude of } \boldsymbol{PC}} \tag{1.21}$$

$$\text{Position vector of } \boldsymbol{PC} = dx\, \boldsymbol{i} + dy\, \boldsymbol{j} + dz\, \boldsymbol{k} \tag{1.22}$$

$$\text{Magnitude of } \boldsymbol{PC} = \sqrt{(dx)^2 + (dy)^2 + (dz)^2} \tag{1.23}$$

Example 1.5 Express the force of 200 N acting from the point $P(x_1, y_1, z_1) = (0, 0, 0)$ to the point $C(x_2, y_2, z_2) = (2, 3, 4)$ cm in vector form.

Solution From Fig. 1.17 and Eq. (1.23),

$$\text{Magnitude of } \boldsymbol{PC} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

$$= \sqrt{2^2 + 3^2 + 4^2} = 5.38 \text{ cm}$$

$$\text{Unit vector } \boldsymbol{\lambda}_{PC} = \frac{dx\boldsymbol{i} + dy\boldsymbol{j} + dz\boldsymbol{k}}{\text{Magnitude of } \boldsymbol{PC}}$$

$$= \frac{2\boldsymbol{i} + 3\boldsymbol{j} + 4\boldsymbol{k}}{5.38}$$

or $\boldsymbol{\lambda}_{PC} = 0.37\boldsymbol{i} + 0.56\boldsymbol{j} + 0.74\boldsymbol{k}$

From Eq. (1.20),

$$\boldsymbol{F} = 200(0.37\boldsymbol{i} + 0.56\boldsymbol{j} + 0.74\boldsymbol{k})$$

or $\boldsymbol{F} = 74\boldsymbol{i} + 112\boldsymbol{j} + 148.5\boldsymbol{k}$ (*Ans*)

Example 1.6 Position vectors of points *A* and *B* shown in Fig. 1.18 are given by $\boldsymbol{r}_1 = 4\boldsymbol{i} + 3\boldsymbol{j} - 2\boldsymbol{k}$ and $\boldsymbol{r}_2 = 5\boldsymbol{i} + 4\boldsymbol{j} - 3.5\boldsymbol{k}$. Determine $\boldsymbol{AB}$ in terms of $\boldsymbol{i}, \boldsymbol{j}, \boldsymbol{k}$ and also find the distance between points *A* and *B*.

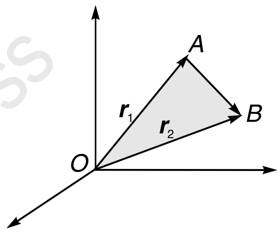


Fig. 1.18

Solution From closed polygon

$$\boldsymbol{r}_1 + \boldsymbol{AB} - \boldsymbol{r}_2 = 0$$

or $\boldsymbol{AB} = \boldsymbol{r}_2 - \boldsymbol{r}_1$

$$= (5\boldsymbol{i} + 4\boldsymbol{j} - 3.5\boldsymbol{k}) - (4\boldsymbol{i} + 3\boldsymbol{j} - 2\boldsymbol{k})$$

or $\boldsymbol{AB} = \boldsymbol{i} + \boldsymbol{j} - 1.5\boldsymbol{k}$ (*Ans*)

The distance $AB = \sqrt{1^2 + 1^2 + (-1.5)^2} = 2.06$ units (*Ans*)

Using vector mathematics, the problems in subsequent chapters are worked out.

1.4.3 Dot Product and Cross Product

We will first deal with dot product.

Dot product (scalar)

Let us consider two vectors $\boldsymbol{P}$ and $\boldsymbol{Q}$ as shown in Fig. 1.19. The dot product of these vectors is given by

$$\boldsymbol{P} \cdot \boldsymbol{Q} = PQ \cos \theta \tag{1.24}$$

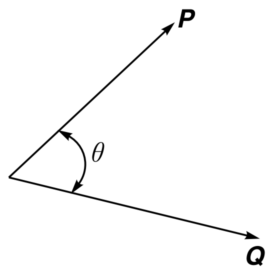


Fig. 1.19

Thus, the dot product of $\boldsymbol{P}$ and $\boldsymbol{Q}$ is equal to the magnitude of $\boldsymbol{P}$ multiplied by the component $\boldsymbol{Q} \cos \theta$ of vector $\boldsymbol{Q}$ in the direction of $\boldsymbol{P}$ or the magnitude of $\boldsymbol{Q}$ multiplied by the component $\boldsymbol{P} \cos \theta$ of $\boldsymbol{P}$ in the direction of $\boldsymbol{Q}$. It satisfies the commutative law, i.e.,

$$\boldsymbol{P} \cdot \boldsymbol{Q} = \boldsymbol{Q} \cdot \boldsymbol{P} \tag{1.25}$$

For unit vectors,

$$\boldsymbol{i} \cdot \boldsymbol{i} = \boldsymbol{j} \cdot \boldsymbol{j} = \boldsymbol{k} \cdot \boldsymbol{k} = 1 \text{ (since } \cos 0° = 1) \tag{1.26}$$

$$\boldsymbol{i} \cdot \boldsymbol{j} = \boldsymbol{j} \cdot \boldsymbol{i} = \boldsymbol{j} \cdot \boldsymbol{k} = \boldsymbol{k} \cdot \boldsymbol{j} = \boldsymbol{k} \cdot \boldsymbol{i} = \boldsymbol{i} \cdot \boldsymbol{k} = 0 \text{ (since } \cos 90° = 0) \tag{1.27}$$

The vectors $\boldsymbol{P}$ and $\boldsymbol{Q}$ can be expressed as

$$\boldsymbol{P} = P_x\boldsymbol{i} + P_y\boldsymbol{j} + P_z\boldsymbol{k}$$

and $\boldsymbol{Q} = Q_x\boldsymbol{i} + Q_y\boldsymbol{j} + Q_z\boldsymbol{k}$

So, $$\boldsymbol{P} \cdot \boldsymbol{Q} = P_x Q_x + P_y Q_y + P_z Q_z \tag{1.28}$$

Also $$\boldsymbol{P} \cdot \boldsymbol{P} = P_x^2 + P_y^2 + P_z^2 \tag{1.29}$$

Vectors also satisfy the distributive law, i.e.,

$$\boldsymbol{P} \cdot (\boldsymbol{Q} + \boldsymbol{R}) = \boldsymbol{P} \cdot \boldsymbol{Q} + \boldsymbol{P} \cdot \boldsymbol{R} \tag{1.30}$$

Cross product (vector)

The cross product $\boldsymbol{P} \times \boldsymbol{Q}$ of two vectors $\boldsymbol{P}$ and $\boldsymbol{Q}$ is another vector and its magnitude is given as

$$|\boldsymbol{P} \times \boldsymbol{Q}| = PQ \sin \theta \tag{1.31}$$

$$\boldsymbol{P} \times \boldsymbol{Q} = -\boldsymbol{Q} \times \boldsymbol{P} \tag{1.32}$$

The cross product satisfies the distributive law, i.e.,

$$\boldsymbol{P} \times (\boldsymbol{Q} + \boldsymbol{R}) = \boldsymbol{P} \times \boldsymbol{Q} + \boldsymbol{P} \times \boldsymbol{R} \tag{1.33}$$

The vectors $\boldsymbol{P}$ and $\boldsymbol{Q}$ can be expressed as

$$\boldsymbol{P} = P_x\boldsymbol{i} + P_y\boldsymbol{j} + P_z\boldsymbol{k}$$

and

$$\boldsymbol{Q} = Q_x\boldsymbol{i} + Q_y\boldsymbol{j} + Q_z\boldsymbol{k}$$

Applying Eq. (1.31) for unit vectors,

$$\boldsymbol{i} \times \boldsymbol{j} = \boldsymbol{k}, \quad \boldsymbol{j} \times \boldsymbol{i} = -\boldsymbol{k} \tag{1.34}$$

$$\boldsymbol{j} \times \boldsymbol{k} = \boldsymbol{i}, \quad \boldsymbol{k} \times \boldsymbol{j} = -\boldsymbol{i} \tag{1.35}$$

$$\boldsymbol{k} \times \boldsymbol{i} = \boldsymbol{j}, \quad \boldsymbol{i} \times \boldsymbol{k} = -\boldsymbol{j} \tag{1.36}$$

So, $\boldsymbol{P} \times \boldsymbol{Q} = (P_y Q_z - P_z Q_y)\,\boldsymbol{i} + (P_z Q_x - P_x Q_z)\,\boldsymbol{j} + (P_x Q_y - P_y Q_x)\boldsymbol{k}$ (1.37)

The cross product $\boldsymbol{P} \times \boldsymbol{Q}$ can be expressed as the determinant of matrix:

$$\boldsymbol{P} \times \boldsymbol{Q} = \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ P_x & P_y & P_z \\ Q_x & Q_y & Q_z \end{vmatrix} \tag{1.38}$$

1.5 CLASSIFICATION OF SYSTEM OF FORCES

A system of forces is classified according to its point of application as shown in Fig. 1.20. Further, different types of systems of forces are shown in Fig. 1.21.

Coplanar forces A set of various external forces that act in the same plane.

Non-coplanar forces A set of various external forces that act in different planes, i.e. they do not lie in the same plane.

Concurrent forces A set of various external forces whose lines of action act on or pass through the same point.

Collinear forces A set of various external forces whose lines of action act along the same line.

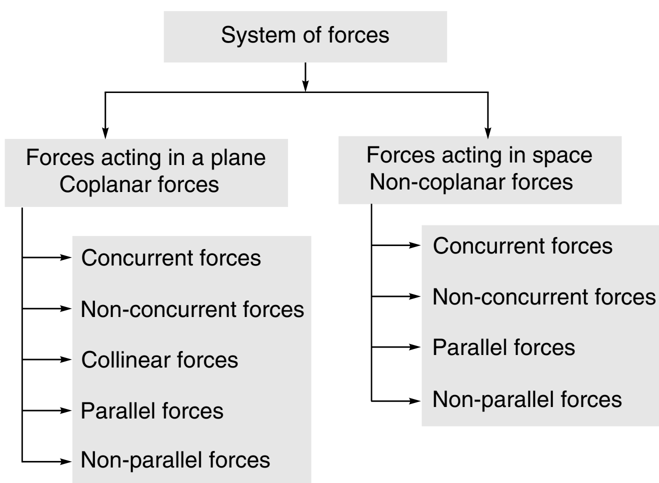
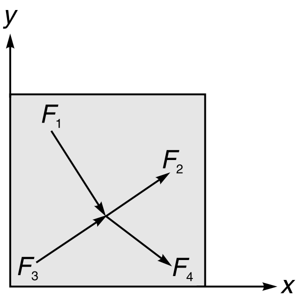
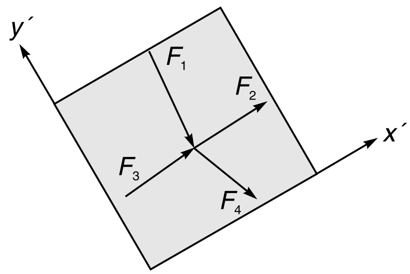


Fig. 1.20 System of forces

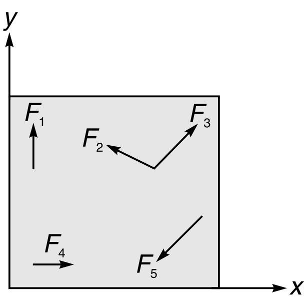
Parallel forces A set of various external forces whose lines of action are parallel to each other.
Non-parallel forces A set of various external forces whose lines of action are not parallel to each other.



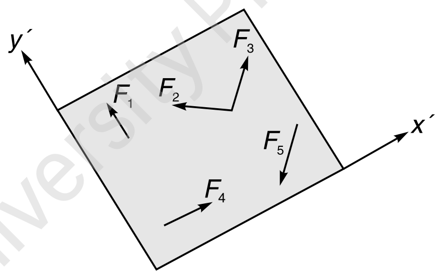
(a) Coplanar, concurrent forces in the *xy*-plane



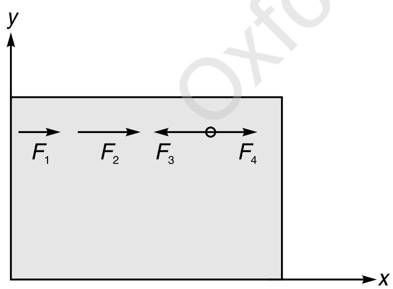
(b) Coplanar, concurrent forces in an inclined plane



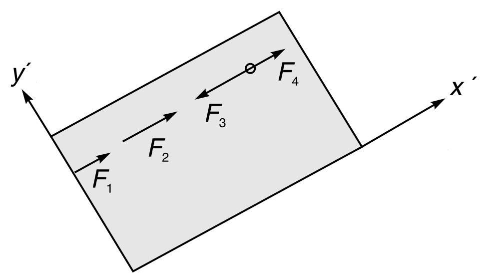
(c) Coplanar, non-concurrent forces in the *xy*-plane



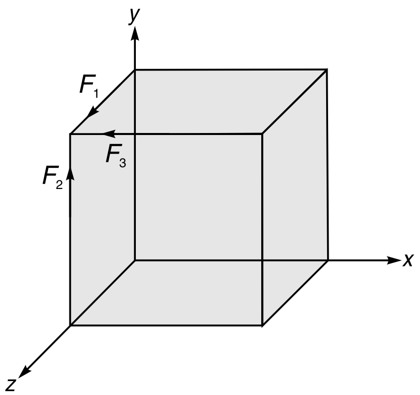
(d) Coplanar, non-concurrent forces in an inclined plane



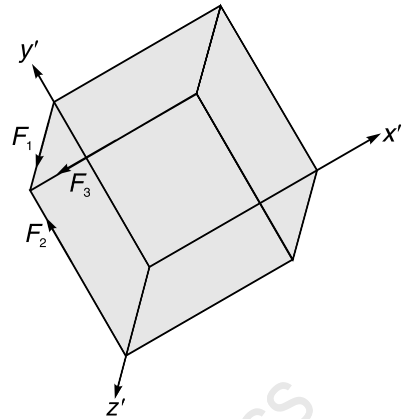
(e) Coplanar, collinear forces in the *xy*-plane



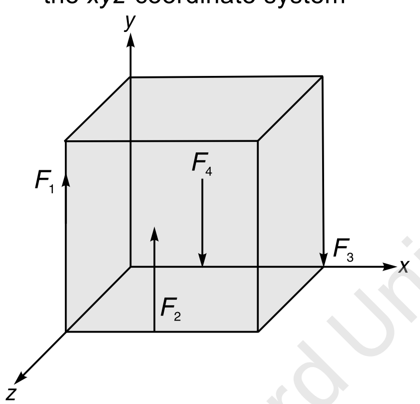
(f) Coplanar, collinear forces in an inclined plane



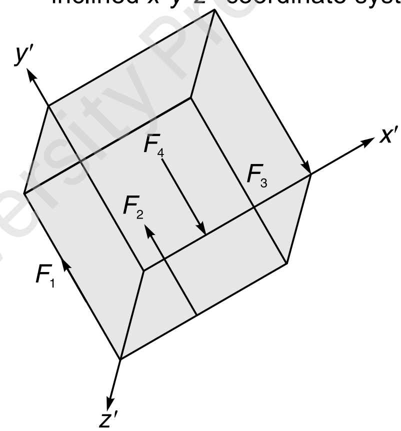
(g) Non-coplanar, concurrent forces in the *xyz*-coordinate system



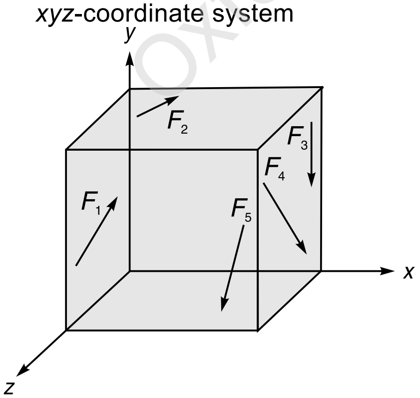
(h) Non-coplanar, concurrent forces in an inclined *x′y′z′*-coordinate system



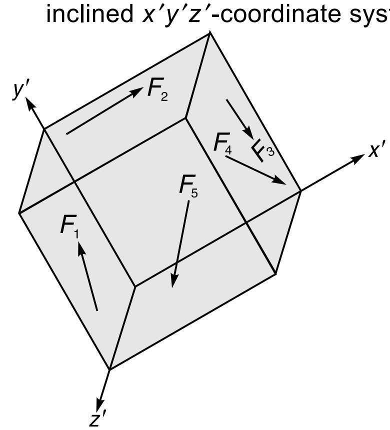
(i) Non-coplanar, parallel forces in the *xyz*-coordinate system



(j) Non-coplanar, parallel forces in an inclined *x′y′z′*-coordinate system



(k) Non-coplanar, non-concurrent, non-parallel forces in the *xyz*-coordinate system



(l) Non-coplanar, non-concurrent, non-parallel forces in an inclined *x′y′z′*-coordinate system

Fig. 1.21 Types of systems of forces

1.6 STATIC AND DYNAMIC EQUILIBRIUM CONDITIONS

1.6.1 Mechanics of Statics: Static Equilibrium Conditions

If in a body, the external forces, support reactions, moments due to forces, and reaction moments at supports, which are acting on the body, keep the body in equilibrium, then the body is said to be in static condition. A body moving with constant velocity is considered to be in static equilibrium. The equilibrium conditions for various cases are discussed below.

(i) A plane body idealized as a particle and applied with a system of concurrent, coplanar forces, as shown in Fig. 1.22, should satisfy the following equilibrium conditions:

$$\sum F_x = 0 \text{ and } \sum F_y = 0 \tag{1.39}$$

(ii) A plane body idealized as a particle and applied with a system of concurrent, non-coplanar forces, as shown in Fig. 1.23, should satisfy the following equilibrium conditions:

$$\sum F_x = 0, \sum F_y = 0, \text{ and } \sum F_z = 0 \tag{1.40}$$

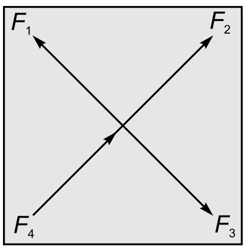


Fig. 1.22 A system of concurrent, coplanar forces

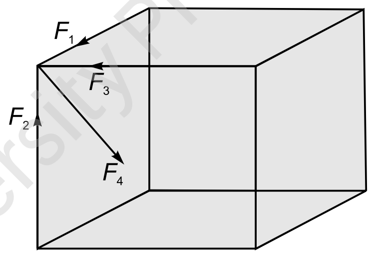


Fig. 1.23 A system of concurrent, non-coplanar forces

(iii) A rigid body idealized as a plane rigid body and applied with a system of non-concurrent, coplanar forces, as shown in Fig. 1.24, should satisfy the following equilibrium conditions:

$$\sum F_x = 0, \sum F_y = 0, \text{ and } \sum M_{\text{about } z\text{-axis taken at any point}} = 0 \tag{1.41}$$

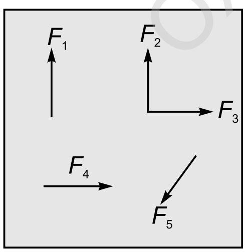


Fig. 1.24 A system of non-concurrent, coplanar forces

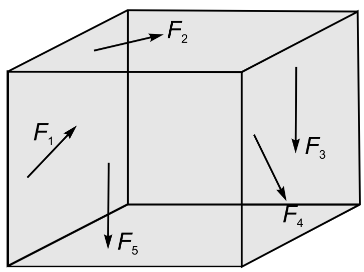


Fig. 1.25 A system of non-concurrent, non-coplanar forces

(iv) A rigid body applied with a system of non-concurrent, non-coplanar forces, as shown in Fig. 1.25, should satisfy the following equilibrium conditions:

$$\sum F_x = 0, \sum F_y = 0, \sum F_z = 0, \sum M_x = 0, \sum M_y, \text{ and } \sum M_z = 0 \tag{1.42}$$

1.6.2 Mechanics of Dynamics: Dynamic Equilibrium Conditions

In a body applied with external forces and support reactions, the moments due to forces and reaction moments at supports will cause the body to accelerate or decelerate in the direction along which it can move. However, a body getting translated or rotated or getting both translated and rotated (general plane motion) will be under equilibrium with its inertia forces and inertia moments. Dynamics is divided into two groups—kinematics and kinetics.

Under *kinematics*, only the geometry of the motion relating to the various motion parameters, such as position, velocity, and acceleration of the body, with respect to time is studied irrespective of the cause of the motion (which is either forces or the moments due to forces).

Under *kinetics*, the geometry of the motion such as acceleration relating to the cause of the motion, which is either force or moment due to force, is studied.

A body under constant acceleration with constraints has the following cases, which can be discussed for dynamic equilibrium over the period of time.

(a) A plane body idealized as a particle under the action of concurrent, coplanar forces should satisfy the following dynamic and static equilibrium conditions for different cases.

(i) For the case when the body is free to move in the x-direction and constrained to move in the y-direction as shown in Fig. 1.26, the dynamic and static equilibrium conditions can be written as

$$\sum F_x = ma_x \text{ and } \sum F_y = 0 \tag{1.43}$$

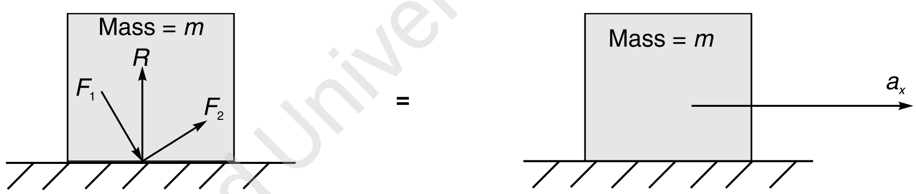


Fig. 1.26 A system of concurrent, coplanar forces causing the body to accelerate in the x-direction

(ii) For the case when the body is free to move in the y-direction and constrained to move in the x-direction as shown in Fig. 1.27, the dynamic and static equilibrium conditions can be written as

$$\sum F_x = 0 \text{ and } \sum F_y = ma_y \tag{1.44}$$

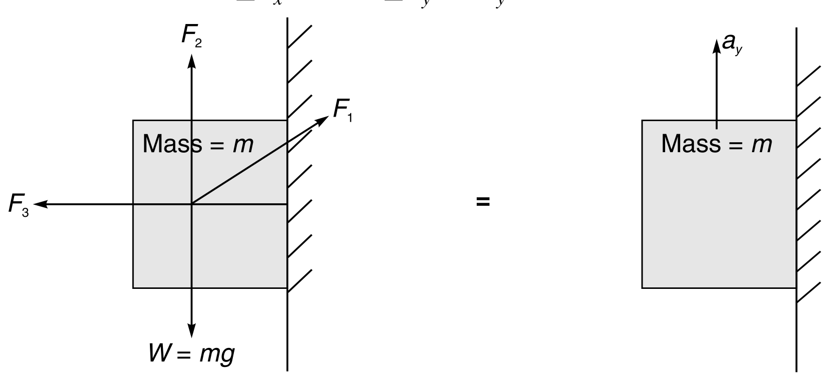


Fig. 1.27 A system of concurrent, coplanar forces causing the body to accelerate in the y-direction

(iii) If the body is free to move in an inclined direction and constrained to move in the direction perpendicular to the inclined plane as shown in Fig. 1.28, the equilibrium conditions can be written as

$$\Sigma F_{\text{along the motion}} = ma_{\text{along the motion}} \text{ and } \Sigma F_{\perp \text{ to the motion}} = 0 \quad (1.45)$$

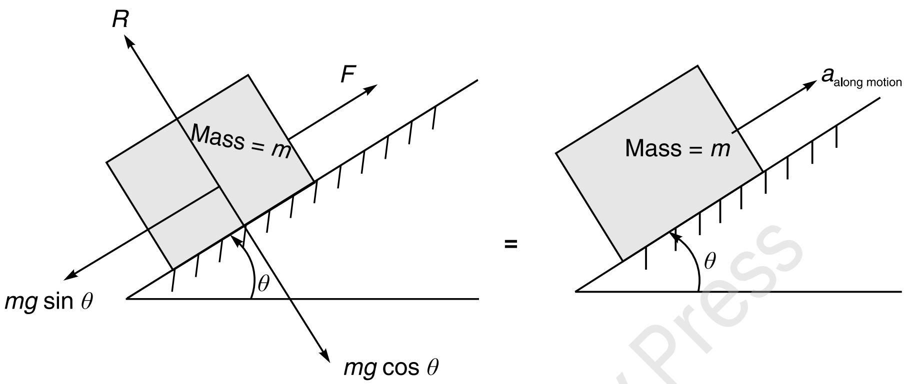


Fig. 1.28 A system of concurrent, coplanar forces causing the body to accelerate along the inclined plane

(b) A body idealized as a particle under the action of concurrent, non-coplanar system of forces should satisfy the following dynamic and static equilibrium conditions for the different cases.

(i) For the case when the body is free to move in the x-direction and constrained to move in the y- and z-directions, the dynamic and static equilibrium conditions can be written as

$$\Sigma F_x = ma_x,\ \Sigma F_y = 0,\ \text{and } \Sigma F_z = 0 \quad (1.46)$$

(ii) For the case when the body is free to move in the y-direction and constrained to move in the x- and z-directions, the dynamic and static equilibrium conditions can be written as

$$\Sigma F_x = 0,\ \Sigma F_y = ma_y,\ \text{and } \Sigma F_z = 0 \quad (1.47)$$

(iii) For the case when the body is free to move in the z-direction and constrained to move in other two perpendicular directions, the dynamic and static equilibrium conditions can be written as

$$\Sigma F_x = 0,\ \Sigma F_y = 0,\ \text{and } \Sigma F_z = ma_z \quad (1.48)$$

(iv) For the case when the body is free to move in an inclined direction and constrained to move in other two perpendicular directions to the inclined plane, the dynamic and static equilibrium conditions can be written as

$$\Sigma F_{\text{along motion}} = ma_{\text{along motion}},\ \Sigma F_{\perp\text{-I to the motion}} = 0,\ \text{and } \Sigma F_{\perp\text{-II to the motion}} = 0 \quad (1.49)$$

(c) A plane body under the action of non-concurrent, coplanar system of forces should satisfy the following dynamic and static equilibrium conditions for different cases.

(i) For the case when the body is free to translate in the x-direction and constrained to move in the y-direction and constrained to rotate about the z-axis, the dynamic and static equilibrium conditions can be written as

$$\Sigma F_x = ma_x,\ \Sigma F_y = 0,\ \text{and } \Sigma M_{\text{about } z\text{-axis taken at any point}} = 0 \quad (1.50)$$

(ii) For the case when the body is free to translate in the x-direction and constrained to move in the y-direction and free to rotate about the z-axis, the dynamic and static equilibrium conditions can be written as

$$\sum F_x = ma_x,\ \sum F_y = 0, \text{ and } \sum M_{\text{about } z\text{-axis and along rotation}} = I\alpha_{\text{along motion}} \tag{1.51}$$

where I is the mass moment of inertia of an axis about which the body is rotating.

Similarly, the equilibrium equations can be written for the case when the body is free to translate in the y-direction and constrained to move in the x-direction and either free to rotate or constrained to rotate about the z-axis.

(d) A solid body under the action of non-concurrent coplanar or non-coplanar system of forces should satisfy the equilibrium conditions according to the constraints. The static and dynamic equilibrium equations for the same can be written accordingly as discussed above.

1.7 UNITS AND DIMENSIONS

1.7.1 System of Units (SI Units)

In Section 1.1.3, the important fundamental concepts, namely space, time, mass, and force, were introduced. The space (which is referred in terms of length), time, and mass are called base units, whereas the unit of force is a derived one. Force is equal to the multiplication of mass and acceleration. A force of 1 N gives an acceleration of 1 m/s^2 for a body of mass of 1 kg (Fig. 1.29). Similarly, when a body of mass 1 kg falls freely, it accelerates with 9.81 m/s^2 and exerts the force of 9.81 N, which is called the weight of the body, as shown in Fig. 1.30.

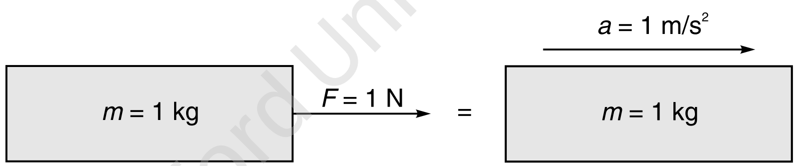


Fig. 1.29 One newton force accelerates the mass of 1 kg with 1 m/s^2

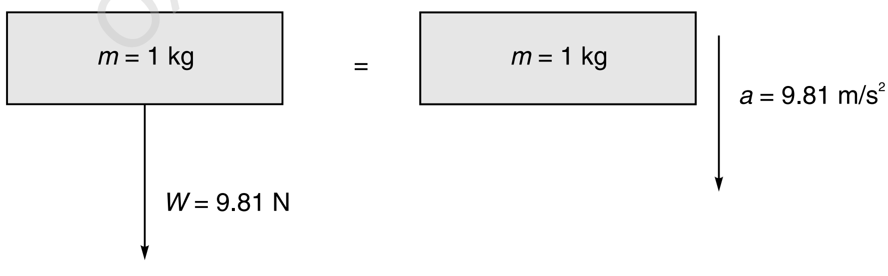


Fig. 1.30 A body of 1 kg mass accelerates with 9.81 m/s^2 and exerts a force of 9.81 N

The following four systems of units are generally followed:

(a) SI units: International system of units

(b) MKS units: Metre, kilogram, second system

(c) CGS units: Centimetre, gram, second system

(d) FPS units: Foot, pound, second system

Presently, the SI system of units is followed in most countries and hence in this textbook the same SI units are used. The important SI units used for various quantities that are used in mechanics are given in Table 1.2. In addition to the SI units for various quantities, some prefixes, which are given in Table 1.3, are also used under the SI system of units to represent very large or very small numbers. For example, Young's modulus of steel may be given as 200 GPa which is equal to 200×10^9 N/m^2. Yield strength of steel may be given as 250 MPa, which is equal to 250×10^6 N/m^2. Speed may be given, for example, as 100 kmph, which is equal to $100 \times (1000/3600)$ m/s.

Table 1.2 Various quantities used in mechanics and their SI units

S. No.	Quantity	Dimensions in terms of MLT	Unit	Representation of the unit
	Base units			
1.	Length	L	Metre	m
2.	Mass	M	Kilogram	kg
3.	Time	T	Second	s
	Derived units			
4.	Area	L^2	Square metre	m^2
5.	Acceleration	LT^{-2}	Metre per square second	m/s^2
6.	Angle	–	Radian	rad
7.	Angular velocity	T^{-1}	Radian per second	rad/s
8.	Angular acceleration	T^{-2}	Radian per square second	rad/s^2
9.	Area moment of inertia	L^4	(metre)^4	m^4
10.	Absolute viscosity	$ML^{-1}T^{-1}$	Newton-second per square metre or Pascal-second	N-s/m^2 or Pa-s
11.	Density	ML^{-3}	Kilogram per cublic metre	kg/m^3
12.	Energy/work	ML^2T^{-2}	Joule	J
13.	Flow rate or discharge	L^3T^{-1}	Cubic metre per second	m^3/s
14.	Frequency	T^{-1}	Hertz (cycles per second)	1/s
15.	Force/weight	MLT^{-2}	Newton	N
16.	Impulse/momentum	MLT^{-1}	Newton-second	N-s or kg-m/s
17.	Kinematic viscosity	L^2T^{-1}	Square metre per second	m^2/s
18.	Mass moment of inertia	ML^2	Kilogram-square metre	kg-m^2
19.	Modulus of elasticity/ modulus of rigidity	$ML^{-1}T^{-2}$	Pascal	N/m^2 or Pa
20.	Moment of force	ML^2T^{-2}	Newton-metre	N-m
21.	Pressure/stress	$ML^{-1}T^{-2}$	Pascal	N/m^2 or Pa
22.	Power	ML^2T^{-3}	Watt	W or J/s
23.	Specific weight	$ML^{-2}T^{-2}$	Newton per cubic metre	N/m^3
24.	Surface tension	MT^{-2}	Newton per metre	N/m
25.	Volume of a solid	L^3	Cubic metre	m^3
26.	Volume of a liquid	L^3	Litre	10^{-3} m^3 (1000 cubic cm)
27.	Velocity	LT^{-1}	Metre per second	m/s

Table 1.3 Various prefixes used in the SI system

S. No.	Prefix	Value	Symbol used
1.	Nano	10^{-9}	n
2.	Micro	10^{-6}	μ
3.	Milli	10^{-3}	m
4.	Kilo	10^{3}	k
5.	Mega	10^{6}	M
6.	Giga	10^{9}	G

1.7.2 Conversion of One System of Units to Another

Let us take an example. The SI unit of pressure is Pa or N/m^2. This unit can be converted into a different unit in terms of N/mm^2 as follows:

$$1 \text{ N/m}^2 = 1 \text{ N/(1000 mm)}^2 = 1 \times 10^{-6} \text{ N/mm}^2$$

or $$1 \text{ N/mm}^2 = 1\times10^{6} \text{ N/m}^2 = 1 \text{ MPa}$$

Thus, a unit in one system can be converted into its equivalent in other system. The US customary units and their SI equivalent are given in Table 1.4.

Table 1.4 US customary FPS units and their SI equivalent

S. No.	Quantity	US customary unit	SI equivalent
1.	Acceleration	ft/s^2	0.3048 m/s^2
		in/s^2	0.0254 m/s^2
2.	Area	ft^2	0.0929 m^2
		in^2	645.2 mm^2
3.	Energy	lb-ft	1.356 J
4.	Force	kip	4.448 kN
		lb	4.448 N
		oz	0.2780 N
5.	Impulse	lb-s	4.448 N-s
6.	Length	ft	0.3048 m

1.7.3 Dimensional Analysis

Dimensions of various quantities are given in Table 1.2 in terms of absolute MLT (mass, length, and time) system. There is another system called gravitational FLT (force, length, and time) system to represent various quantities. The absolute MLT system is generally followed. The variables governing the phenomena expressed in a mathematical equation give the relationship between the variables. The variables of the equation may be dimensional or non-dimensional. The qualitative description of a variable/quantity is known as *dimension* and the quantitative description is called *unit*. For example, the weight of a body, which is equal to the product of its mass and the acceleration due to gravity, has the dimension MLT^{-2}. If the mass of the body is 1 kg, then its weight equals 1 kg × 9.81 m/s^2 = 9.81 N, which is the quantitative value in the SI system of unit.

To be correct, the governing equation should satisfy the dimensions on both sides of the equation. This condition is called *dimensional homogeneity*.

Using dimensional analysis, the dimensions of unknown variables can be determined for physical phenomenon that is expressed in mathematical equation. The homogeneous equation of a physical phenomenon can be converted into a non-dimensional form. For example, the deflection of a helical spring is expressed as

$$\delta = \frac{8FD^3n}{Gd^4} \tag{1.52}$$

where δ is the deflection, F is the force, D is the mean diameter of the coil, n is the total number of turns of the coil, d is the diameter of the wire of the helical spring, and G is the shear modulus of the spring material.

Substituting the dimensions in Eq. (1.52),

$$\mathrm{L} = \frac{(\mathrm{MLT^{-2}})\,\mathrm{L^3}}{\dfrac{\mathrm{MLT^{-2}}}{\mathrm{L^2}}\mathrm{L^4}}$$

or

$$\mathrm{L} = \mathrm{L} \tag{1.53}$$

Example 1.7 Check the dimensional homogeneity for the equation $v^2 - u^2 = 2aS$, where S is the displacement travelled by the particle when the velocity of the particle changes from u to v with constant acceleration of a.

Solution Substituting the dimensions in the given equation,

$$(\mathrm{LT^{-1}})^2 - (\mathrm{LT^{-1}})^2 = (\mathrm{LT^{-2}})\mathrm{L}$$

or

$$\mathrm{L^2T^{-2}} - \mathrm{L^2T^{-2}} = \mathrm{L^2T^{-2}}$$

or

$$\mathrm{L^2T^{-2}} = \mathrm{L^2T^{-2}} \tag{1.54}$$

1.8 HOW TO SOLVE AN ENGINEERING MECHANICS PROBLEM

Problems in engineering mechanics are to be approached as an actual engineering situation, and with individual experience and common sense, it is easy to understand and formulate the problem. The first step involved in formulating the problem is stating the problem, which contains the given data and the details regarding what is to be determined.

The physical quantities are to be represented in a neat line diagram. The independent line diagrams for all the bodies representing the magnitudes and the directions of the forces acting on the body (which are known as free-body diagrams) are to be drawn.

The next step is the solution part, which is based on the fundamental principles/laws of mechanics stated in Sections 1.1.3 and 1.2. They are used further in later chapters to write a set of equations for a given numerical data of the problem. Thus, by solving the equations, the required unknown values could be found out. Use of suitable principles and correct computations in practical engineering problems are highly important since they influence the design safety of the structure and its behaviour as well as the manufacturing cost of the entire structure. Most static problems of rigid body mechanics are related to the mass, geometry, and type of the constraints/supports of the body. For these problems, the reaction forces and reaction moments that are developed can be determined by using the static equilibrium equations. After finding the support reactions, supports can be designed for the structure.

The dynamic problems of rigid body mechanics can be solved by two methods: force (or moment due to force) method and energy method.

The solution to the dynamic problems is also related to the mass and geometry of the body, geometry of the motion (rectilinear/curvilinear/projectile), type of the motion (uniform motion/uniformly accelerated motion/uniformly decelerated motion/combination of acceleration and deceleration, uniform motion, etc.), external forces, and reactions, and moments due to the forces.

Kinematics of a body can be expressed in terms of mathematical equations relating to its position, distance travelled, velocity, and acceleration with respect to time. Kinetics of a body can be expressed in terms of mathematical equations relating to the geometry of the motion (acceleration/deceleration) and mass of the body, to determine the inertia forces/inertia moments of the body under motion satisfying the condition of the dynamic equilibrium. Further, the body may be in static equilibrium in the directions perpendicular to the motion. From all these relations, a set of equations is formulated, which will be solved to determine the unknown quantities in a specific problem.

In a similar way, an alternate energy method can be utilized to formulate a set of equations to a specific problem from which the unknown quantities can be found.

Recapitulation

- Mechanics of bodies is classified as mechanics of rigid bodies, mechanics of deformable bodies, and mechanics of fluids.
- In engineering mechanics, depending on the nature of the external forces applied on the body, it can be idealized as particle and rigid body. Hence, this subject is studied under two groups—particle mechanics and rigid body mechanics.
- The fundamental laws/principles of mechanics are
  - (i) Newton's three laws of motion
  - (ii) Lami's theorem
  - (iii) Parallelogram law for addition of forces
  - (iv) Triangular law of forces
  - (v) Polygon's laws
  - (vi) Principle of transmissibility (sliding vector)
  - (vii) Newton's law of gravitation
- A body is idealized as a 'particle' in statics when its shape and size does not affect the solution to the given problem and the mass of the body is assumed to be concentrated to a specific point. When a system of concurrent forces is applied on a body, then the body can be idealized as a particle.
- An ideal situation for a rigid body is that when the shape and size of the body does not change at any condition of loading. A rigid body undergoes very small deformation and changes its shape under a system of external forces acting on it. The deformation and the change in shape are small and have negligible effect to develop the reactions required to maintain the equilibrium conditions of the body. And hence in a structure of engineering applications, the members of the structure are assumed to be rigid and the reactions at contact points are determined. In rigid body mechanics, the effects of a system of non-concurrent, coplanar or non-concurrent, non-coplanar forces acting on a body are studied.
- Statics of rigid bodies deals with a system of external forces applied on a body in equilibrium with support reactions along the constrained direction. Further, considering the unconstrained directions to be the directions perpendicular to the constrained directions along which the condition of equilibrium is to be justified.

- Dynamics of rigid bodies is studied under two groups, namely kinematics and kinetics. Kinematics deals with the geometry of the motion such as position/distance travelled, velocity, and acceleration of the body with respect to time irrespective of the cause of the motion (force or moment due to the force).
  In kinetics, the geometry of the motion, such as acceleration/deceleration, is related with the cause of the motion (force or moment due to the force) by which it satisfies the condition of the dynamic equilibrium.
- Any physical quantity is either a scalar or a vector. Scalar quantities have only magnitude, whereas vector quantities have both magnitude and direction. Force, momentum, and velocity are examples for vectors, whereas temperature, time, energy are examples of scalar quantities.
- Free vector is a vector which can be applied freely anywhere on the body to give same effect on the body. For example when a force applied in a certain direction with a certain magnitude on a point of body is moved to some other point, then it gives the same translatory effect. These forces are considered to be free vectors. Similarly, equal and opposite forces applied freely in the space on the body can produce same magnitude of couple and these forces are considered to be free vectors.
- Fixed or bound vector is restricted to be applied to a particular location in the body with the required magnitude and along particular line of action.
- Vector addition and subtraction can be expressed as $\boldsymbol{R} = \boldsymbol{P} + \boldsymbol{Q}$ and $\boldsymbol{R} = \boldsymbol{P} - \boldsymbol{Q}$.
- Vector representation of a force in one dimension, two dimensions (in a plane), and three dimensions (in space) can be expressed respectively as $\boldsymbol{F} = F_x\boldsymbol{i}$, $\boldsymbol{F} = F_x\boldsymbol{i} + F_y\boldsymbol{j}$, and $\boldsymbol{F} = F_x\boldsymbol{i} + F_y\boldsymbol{j} + F_z\boldsymbol{k}$, where F_x, F_y, and F_z are the forces along the x-, y-, and z-directions, respectively.
- The unit vector $\boldsymbol{\lambda}_{AB}$ of line AB whose coordinates are (x_A, y_A, z_A) and (x_B, y_B, z_B) is equal to $\boldsymbol{AB}/|\boldsymbol{AB}|$, where
  $\boldsymbol{AB} = (x_B - x_A)\,\boldsymbol{i} + (y_B - y_A)\,\boldsymbol{j} + (z_B - z_A)\,\boldsymbol{k}$
  and $|\boldsymbol{AB}| = \sqrt{(x_B - x_A)^2 + (y_B - y_A)^2 + (z_B - z_A)^2}$
  A force of magnitude F acting along line AB can be expressed in vector form as
  $$\boldsymbol{F} = F(\boldsymbol{\lambda}_{AB}) = \frac{\boldsymbol{F}}{|\boldsymbol{AB}|}\,[(x_B - x_A)\,\boldsymbol{i} + (y_B - y_A)\,\boldsymbol{j} + (z_B - z_A)\boldsymbol{k}].$$
- A system of forces according to point of application can be classified as coplanar (acting in the same plane) or non-coplanar (acting in different planes). Further they are classified as concurrent forces (acting at a particular point in the body) or non-concurrent forces (acting at different points in the body). Other system of forces are parallel forces (which are parallel to each other), non-parallel forces (which are not parallel to each other), and collinear forces (which are along the same line of action).
- Various systems of units are SI, MKS, CGS, and FPS.
- The problem-solving technique in engineering mechanics involves first drawing free-body diagram(s) showing the forces and the moments due to the forces with their directions. In case of static problems, a set of equations can be formulated by satisfying the equilibrium conditions and hence the unknown quantities of the problem can be found. In case of dynamic problems, a set of equations can be formulated in terms of kinematics and kinetics of the body satisfying the condition of the dynamic equilibrium along the direction of the motion and of the static equilibrium in the direction perpendicular to the motion. From these equations, the unknown quantities of the problem can be found.

EXERCISES

Review Questions

1. Differentiate between rigid body, deformable body, and fluid.
2. Differentiate between statics and dynamics of rigid body.
3. Define force and its units.
4. How is a force represented in vector form?
5. Differentiate between particle and rigid body.
6. Write the dimensions in MLT system for the following quantities:
   (i) Acceleration (ii) Force (iii) Moment
7. What is Lami's theorem?
8. Define unit vector.
9. How can the force shown in Fig. 1.31 be represented in vector form?

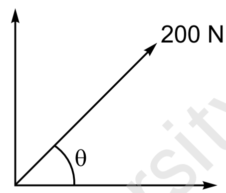


Fig. 1.31

Ans: (200 cos θ)$\boldsymbol{i}$ + (200 sin θ)$\boldsymbol{j}$

10. 'Most of the mechanics problems are governed by equilibrium conditions'. Do you agree? Justify your answer.

Multiple-choice Questions

1. A rigid body is treated as a particle when it is applied with
   (a) Concurrent forces (b) Non-concurrent forces
   (c) Coplanar forces (d) Non-coplanar forces
2. In rigid body mechanics, external forces applied on the body make the body
   (a) To undergo deformation (b) Not to undergo deformation
   (c) To deform continuously (d) To vibrate
3. Equilibrium of a rigid body in statics refers to
   (a) Balance of forces in static condition
   (b) Balance of both forces and moments in static condition
   (c) Balance of energy of the body
   (d) Balance of inertia force and inertia moment
4. A deformable body under its strength limit undergoes deformation till
   (a) It breaks
   (b) It elongates plastically
   (c) It buckles
   (d) It attains the balance of applied forces and moments with inertia forces and inertia moments
5. Kinematics of the rigid body is
   (a) Study of geometry of motion considering the cause of motion
   (b) Study of external forces acting on it without considering the geometry of motion

(c) Study of geometry of motion without considering the cause of motion
(d) Finding the reaction forces and moments at the supports

6. Weight of the body of mass 10 kg on Earth and Moon respectively are
(a) 16.22 N, 98.1 N (b) 162.2 N, 981 N
(c) 49.05 N, 8.11 N (d) 98.1 N, 16.22 N

7. Lami's theorem is applicable for
(a) Number of concurrent forces acting on the body
(b) Two equal and opposite forces acting on the body
(c) Three concurrent forces acting on the body
(d) Three non-concurrent forces acting on the body

8. Sliding vector
(a) Produces same effect on the body when it is moved to different locations along the same line of action
(b) Makes the body to slide
(c) Stops sliding the body
(d) Increases the sliding velocity of the body

9. Force in MLT system is represented by
(a) MLT^{-1} (b) LT^{-2} (c) MLT^{-2} (d) M

10. 10×10^6 N/m^2 is equal to
(a) 10 MPa (b) 10 N/mm^2 (c) 10×10^6 Pa (d) All of the above

Answers to Multiple-choice Questions

1. (a) **2.** (b) **3.** (b) **4.** (d) **5.** (c) **6.** (d)
7. (c) **8.** (a) **9.** (c) **10.** (d)